NONLINEAR MODEL PREDICTIVE CONTROL

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Contents

Summary

While linear model predictive control is popular since the 1970s, the 1990s have witnessed a steadily increasing attention from control theoreticians as well as control practitioners in the area of nonlinear model predictive control (NMPC). The practical interest is mainly driven by the fact that today's processes need to be operated under tighter performance specifications. At the same time more and more constraints, stemming for example from environmental and safety considerations, need to be satisfied. Often, these demands can only be met when process nonlinearities and constraints are explicitly taken into account in the controller. Nonlinear predictive control, the extension of the well established linear predictive control to the nonlinear world, is one possible candidate to meet these demands. This chapter reviews the basic

principle of NMPC, and outlines some of the theoretical, computational, and implementation aspects of this control strategy.

1. Introduction

Model predictive control (MPC), also referred to as moving horizon control or receding horizon control, is a control strategy in which the applied input is determined on-line at the recalculation instant by solving an open-loop optimal control problem over a fixed prediction horizon into the future. The first part of the obtained open-loop input signal is implemented until new measurements become available. Based on the new information the open-loop optimal control problem is solved again and the whole procedure is repeated. This recurrent (on-line) solution of the open-loop optimal control problem over a moving prediction window makes the key difference to other control methods. Obtaining the implemented input by a recurrent solution of an optimal control problem leads to a series of questions and problems like stability of the closed-loop and the efficient numerical solution of the optimal control problem. On the other side, since the applied input is based on an optimal control problem, it is possible to take specifications into account which are otherwise difficult to satisfy. For example, input and state constraints can be directly considered, the systematic handling of multivariable control problems is possible, and desired performance specifications can be optimized.

Basically, *linear* MPC and *nonlinear* MPC (NMPC) are distinguished (see also *Model Based Predictive Control for Linear Systems* and *Model Based Predictive Control*). Linear MPC refers to a family of MPC schemes in which linear models are used to predict the system dynamics, even though the dynamics of the closed-loop system might be nonlinear due to the presence of input and state constraints. The models used are often input-output models obtained through *Identification for control*. Linear MPC is by now a well established control strategy and is widely employed, especially in the process industry. Several thousand applications spanning from chemical to aerospace industry are reported. Many implementational and theoretical aspects of linear MPC are well understood. Important issues such as the efficient solution of the occurring quadratic program, the interplay between modeling, identification and control, as well as issues like stability are well addressed (see *Model Based Predictive Control for Linear Systems*).

Many systems are, however, inherently nonlinear. The inherent nonlinearity, together with higher product quality specifications and increasing productivity demands, tighter environmental regulations and demanding economical considerations require to operate systems over a wide range of operating conditions and often near the boundary of the admissible region. Under these conditions linear models are often not sufficient to describe the process dynamics adequately and nonlinear models must be used. This inadequacy of linear models is one of the motivations for the increasing interest in nonlinear model predictive control.

This chapter reviews the main principles underlying NMPC and outlines some of the theoretical, computational, and implementation aspects. Sections 1.1 and 1.2 introduce the basic principle of NMPC. In Section 2 theoretical aspects of NMPC like stability, robustness, and the output-feedback problem are reviewed. Solution methods for the open-loop optimal control problem that must be solved repeatedly in NMPC are presented Section 3.

The main focus of the chapter is on the direct use of nonlinear system models for prediction and optimization. The use of linear predictive control methods for nonlinear systems, for example based on piecewise linear approximations, is not considered here. Some remarks on this issue can be found in *Model Based Predictive Control* and in some of the references given at the end of the chapter. Note that in the text no direct references are given to make the chapter as self contained as possible. A bibliography is provided at the end of the chapter.

1.1. The Basic Principle of Model Predictive Control

Model predictive control is formulated as a repeated solution of a (finite) horizon open-loop optimal control problem subject to system dynamics and input and state constraints. Figure 1 depicts the basic principle of model predictive control.

Based on measurements obtained at time t, the controller predicts the dynamic behavior of the system over a prediction horizon T_p in the future and determines (over a control horizon $T_c \leq T_p$) the input such that a predetermined open-loop performance objective is minimized. If there were no disturbances and no model-plant mismatch, and if the optimization problem could be solved over an infinite horizon, then the input signal found at $t = 0$ could be open-loop applied to the system for all $t \geq 0$. However, due to disturbances and model-plant mismatch the actual system behavior is different from the predicted one. To incorporate feedback, the optimal open-loop input is implemented only until the next recalculation instant. The recalculation time between the new optimization can vary. Typically, it is however fixed, i.e the optimal control problem is reevaluated after the fixed, "recalculation time" δ. Using the new system state at time $t+\delta$, the whole procedure – prediction and optimization – is repeated, moving the control and prediction horizon forward.

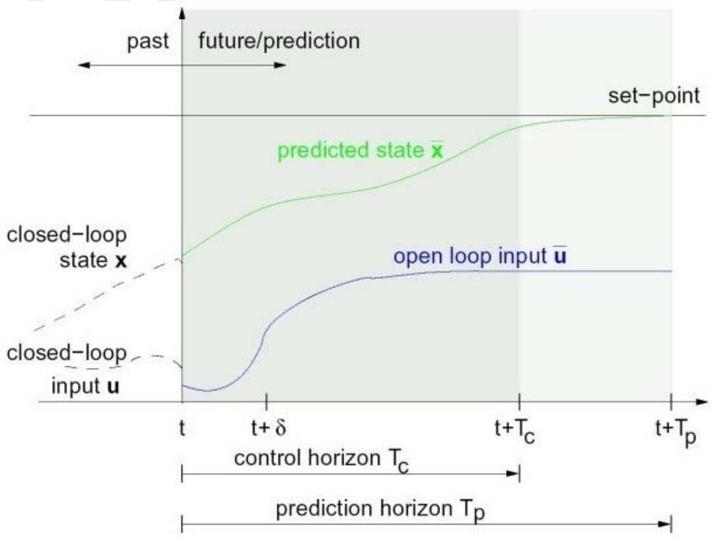

Figure 1: Principle of model predictive control.

In Fig. 1 the open-loop optimal input is depicted as arbitrary function of time. To allow a numerical solution of the open-loop optimal control problem the input is often parameterized by a finite number of “basis” functions, leading to a finite dimensional optimization problem. In practice often a piecewise constant input is used, leading to T_c/δ decision variables for the input over the control horizon.

The determination of the applied input based on the predicted system behavior allows the direct inclusion of constraints on states and inputs as well as the minimization of a desired cost function. However, since often a finite prediction horizon is chosen and thus the predicted system behavior will in general differ from the closed-loop one, precaution must be taken to achieve closed-loop stability and reasonable closed-loop performance. This issue is addressed in Section 2.

1.2. Mathematical Formulation of NMPC

Consider the class of continuous time systems described by the following nonlinear differential equation (Only the continuous time formulation of NMPC is presented. However, most of the approaches outlined have dual discrete time counterparts.)

$$\dot{\mathbf{x}}(t) = \mathbf{f}\left(\mathbf{x}(t), \mathbf{u}(t)\right), \quad \mathbf{x}(0) = \mathbf{x}_0 \tag{1}$$

subject to input and state constraints of the form:

$$\mathbf{u}(t) \in \mathcal{U}, \forall t \geq 0 \tag{2}$$

$$\mathbf{x}(t) \in \mathcal{X}, \forall t \geq 0. \tag{3}$$

Here $\mathbf{x}(t) \in \mathbb{R}^n$ and $\mathbf{u}(t) \in \mathbb{R}^m$ denote the vector of state and inputs, respectively. Furthermore, the input constraint set $\mathcal{U}$ is compact and $\mathcal{X}$ is connected. For example $\mathcal{U}$ and $\mathcal{X}$ are often given by box constraints of the form:

$$\mathcal{U} := \left\{\mathbf{u} \in \mathbb{R}^m \mid \mathbf{u}_{\min} \leq \mathbf{u} \leq \mathbf{u}_{\max}\right\} \tag{4}$$

$$\mathcal{X} := \left\{\mathbf{x} \in \mathbb{R}^n \mid \mathbf{x}_{\min} \leq \mathbf{x} \leq \mathbf{x}_{\max}\right\}, \tag{5}$$

with the constant vectors $\mathbf{u}_{\min}$, $\mathbf{u}_{\max}$ and $\mathbf{x}_{\min}$, $\mathbf{x}_{\max}$.

In NMPC the input applied to the system is usually based on the following finite horizon open-loop optimal control problem, which is solved at every recalculation instant:

Problem 1 *Find*

$$\min_{\bar{\mathbf{u}}(\cdot)} J(\mathbf{x}(t), \bar{\mathbf{u}}(\cdot))$$

With the cost functional $J(\mathbf{x}(t),\bar{\mathbf{u}}(\cdot)) := \int_t^{t+T_p} F(\bar{\mathbf{x}}(\tau),\bar{\mathbf{u}}(\tau))d\tau$ (6)

subject to:

$$\dot{\bar{\mathbf{x}}}(\tau) = \mathbf{f}\left(\bar{\mathbf{x}}(\tau),\bar{\mathbf{u}}(\tau)\right), \quad \bar{\mathbf{x}}(t) = \mathbf{x}(t) \tag{7}$$

$$\bar{\mathbf{u}}(\tau) \in \mathcal{U}, \quad \forall \tau \in [t, t+T_c] \tag{8}$$

$$\bar{\mathbf{u}}(\tau) = \bar{\mathbf{u}}(t+T_c), \ \forall \tau \in [t+T_c, t+T_p] \tag{9}$$

$$\bar{\mathbf{x}}(\tau) \in \mathcal{X}, \quad \forall \tau \in [t, t+T_p]. \tag{10}$$

Here T_p and T_c are the prediction and the control horizon with $T_c \le T_p$. The bar denotes internal controller variables and $\bar{\mathbf{x}}(\cdot)$ is the solution of (7) driven by the input signal $\bar{\mathbf{u}}(\cdot):[t,\, t+T_p] \to \mathcal{U}$ under the initial condition $\mathbf{x}(t)$. The distinction between the real system variables and the variables in the controller is necessary, since even in the nominal case the predicted values will not be the same as the actual closed-loop values. The difference in the predicted and the real values is due to determination of the applied input via a re-optimization (over a moving finite horizon T_c) at every recalculation instant.

The cost functional J is defined in terms of the stage cost F, which specifies the performance. The stage cost can for example arise from economical and ecological considerations. Often, a quadratic form for F is used:

$$F(\mathbf{x},\mathbf{u}) = (\mathbf{x}-\mathbf{x}_s)^T Q(\mathbf{x}-\mathbf{x}_s) + (\mathbf{u}-\mathbf{u}_s)^T R(\mathbf{u}-\mathbf{u}_s). \tag{11}$$

Here $\mathbf{x}_s$ and $\mathbf{u}_s$ denote a desired reference trajectory, that can be constant or time-varying. The deviation form the desired values is weighted by the positive definite matrices Q and R. In the case of a stabilization problem (no tracking), i.e. $\mathbf{x}_s = \mathbf{u}_s = const$, one can assume, without loss of generality, that $(\mathbf{x}_s,\mathbf{u}_s) = (0,0)$ is the steady state to stabilize.

The state measurement enters the system via the initial condition in (7) at the recalculation instant, i.e. the system model used to predict the future system behavior is initialized by the actual system state. Since all state information is necessary for the prediction, the full state must be either measured or estimated. Equation (9) fixes the input beyond the control horizon to $\bar{\mathbf{u}}(t+T_c)$.

In the following, optimal solutions of optimization Problem 1 are denoted by $\bar{\mathbf{u}}^*(\cdot;\mathbf{x}(t)):[t,t+T_p] \to \mathcal{U}$. The open-loop optimal control problem is solved repeatedly at the recalculation instants $t_j = j\delta,\, j = 0,1,\cdots$, and the input applied to the system is given by the sequence of optimal solutions of Problem 1:

$$\mathbf{u}(t) := \bar{\mathbf{u}}^*(t;\mathbf{x}(t_j)), \tag{12}$$

where t_j is the closest recalculation instant to t with $t_j \leq t$. Thus, the *nominal closed-loop system* is given by:

$$\dot{\mathbf{x}}(t) = \mathbf{f}\left(\mathbf{x}(t), \bar{\mathbf{u}}^*(t;x(t_j))\right), \tag{13}$$

The optimal cost of Problem 1 as a function of the state is referred to as *value function* V and is given by:

$$V(\mathbf{x}) = J(\mathbf{x}, \bar{\mathbf{u}}^\star(\cdot;\mathbf{x})). \tag{14}$$

The value function plays a central role in the stability analysis of NMPC, since it often serves as a Lyapunov function candidate.

1.3. Properties, Advantages, and Drawbacks of NMPC

From a theoretical and practical point of view, one would like to use an infinite prediction and control horizon, i.e. T_p and T_c in Problem 1 are set to ∞. This would lead to a minimization of the total occurring cost up to infinity.

However, often the solution of a nonlinear infinite horizon optimal control problem cannot be calculated (sufficiently fast). For this reason finite prediction and control horizons are considered. In this case the actual closed-loop input and states will differ from the predicted open-loop ones, even if no model plant mismatch and no disturbances are present (compare *Model Based Predictive Control*).

An analogy to this problem is somebody hiking in the mountains without a map. The goal of the hiker is to take the shortest route to his destination. Since he is often not able to see fare enough, the only thing he can do is to plan a certain route based on the current information (skyline/horizon) and then follow this route. After some time he will reevaluate his route based on the fact that he can see further.

Due to previously "invisible" obstacles the new route obtained might differ significantly from the previous one. Finite horizon NMPC shows many similarities to this analogy. At the recalculation instants the future is only predicted over the prediction horizon.

At the next recalculation instant the prediction horizon moves forward, allowing us to obtain more information. This is depicted in Figure 2, where the system can only move inside the shaded area as state constraints of the form $\mathbf{x}(\tau) \in \mathcal{X}$ are assumed to be present.

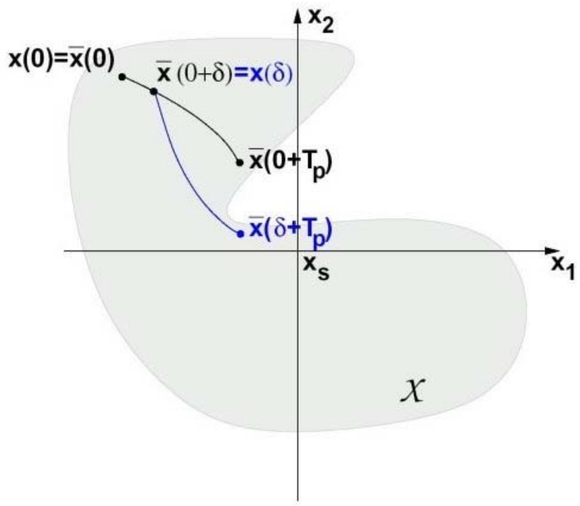

Figure 2: The difference between open-loop prediction and closed-loop behavior.

The difference of the predicted values and the closed-loop values has two immediate consequences. Firstly, the actual goal to compute a feedback such that the performance objective over the *infinite horizon* of the closed-loop is minimized is not achieved. In general it is by no means true the repeated minimization over a moving *finite horizon objective* leads to an optimal solution for the infinite horizon problem. The solutions will often differ significantly if a short finite horizon is chosen. Secondly, if the predicted and the actual trajectory differ, there is no guarantee that the closed-loop system will be stable. It is indeed easy to construct examples for which the closed-loop becomes unstable if a short finite horizon is chosen. Hence, when using finite prediction horizons the problem must be modified to guarantee stability, as outlined in Section 2.1.

The basic overall structure of a NMPC control loop is shown in Figure 3. Based on the applied input and the measured outputs a state estimate is obtained. This estimate is fed into the NMPC controller which computes a new input applied to the system. Often an additional reference/set-point or target calculation is added to the overall loop. However, the latter will not be covered in this note.

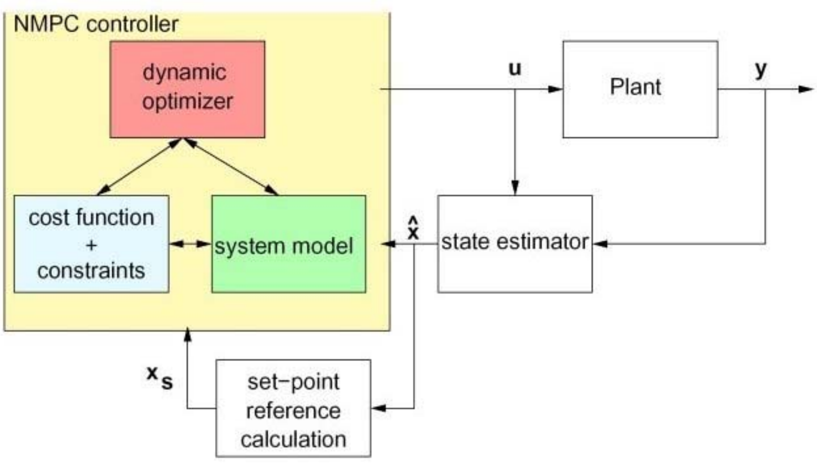

Figure 3: Basic NMPC control loop.

Summarizing, a standard NMPC scheme works as follows:

1. obtain measurements/estimates of the states of the system
2. calculate an optimal input **minimizing** the desired **cost function** over the **prediction horizon** using the **system model** for prediction
3. **implement the first part of the optimal input** until the next recalculation instant
4. continue with 2.

Shortly the key characteristics and properties of NMPC are:

- NMPC allows the direct use of nonlinear state space models for prediction.
- NMPC allows the explicit consideration of state and input constraints.
- In NMPC a specified time domain performance criteria is minimized on-line.
- In NMPC the predicted behavior is in general different from the closed loop behavior.
- For the application of NMPC a real-time solution of an open-loop optimal control problem is necessary.
- To perform the prediction the system states must be measured or estimated.

Many of these properties can be seen as advantages as well as drawbacks of NMPC. The possibility to directly use a nonlinear model is advantageous if a detailed first principles model is available.

In this case often the performance of the closed-loop can be increased significantly without much tuning. Nowadays first principle models of a plant are often derived before a plant is build.

Especially in the process industry is a strong desire to use (rather) detailed models from the first design up to the operation of the plant for reasons of consistence and cost minimization.

On the other side, if no first principle model is available, it is often impossible to obtain a good nonlinear model based on identification techniques. In this case it is better to fall back to other control strategies like linear MPC.

Basing the applied input on the solution of an optimal control problem that must be solved on-line is advantageous and disadvantageous at the same time. First, and most important, this allows us to directly consider constraints on states and inputs which are often difficult to handle otherwise.

Furthermore, the desired cost objective, the constraints and even the system model can in principle be adjusted on-line without making a complete redesign of the controller necessary. However, solving the open-loop optimal control problem, if attacked blindly, can be difficult or even impossible for large systems.

In the remaining sections some theoretical and computational aspects of NMPC are discussed.

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Biographical Sketches

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