MODEL REFERENCE ADAPTIVE CONTROL

Anuradha M. Annaswamy
Massachusetts Institute of Technology, Cambridge, MA, USA

Keywords: adaptive control, adaptive laws, algebraic part, analytical part, convergence, direct control, error model, global stability, identification model, indirect control, parameter identification, persistent excitation, reference model, stability

Contents

Summary

This article presents the elements of model reference adaptive control, which refers to a particular control procedure for uncertain dynamic systems. The control problem as well as the adaptive control problem are described. The use of various models for control, including identification model and reference model is presented. The model-following in the presence of various inputs is discussed. Using these properties, the problem of model reference adaptive control is described. The error model approach for designing the requisite controllers is delineated. The solution to the model reference adaptive control for linear plants is presented. Its extension to nonlinear systems is briefly mentioned. The role of parameter identification and its relation to persistent excitation is described. Major developments in the field of model reference adaptive control have taken place in the eighties and nineties and have been applied in a number of practical control problems with success.

1. Introduction

The aim of control is to keep the relevant outputs of a given dynamic process within prescribed limits. Denoting the process to be controlled as a *plant*, its input and output as u and y, respectively, and the aim of control is to keep the error $e_1 = y - y_d$, between the plant output and a desired output y_d within prescribed values. If y_d is a constant, the control problem is referred to as *regulation* and if y_d is a function of time, the problem is referred to as *tracking*. In the former case, the value of y_d around which the system is to be regulated is also referred to as a *setpoint* or *operating point*. The goal of control, in both cases, is to ensure that the output error e_1 is as small as possible, in the presence of disturbances and modeling errors, for all time, and that the controlled system is stable. *Feedback control* is one procedure by which regulation and tracking can be accomplished in a number of dynamic processes. When the differential equations describing the behavior of the plant are linear and known a priori, powerful analytical techniques in both time-domain and frequency-domain have been developed. When the characteristics of the plant are unknown, both regulation and tracking can be viewed as adaptive control problems.

The field of adaptive control, in general, and model reference adaptive control in particular, has focused on problems where the uncertainties in the system are parametric. Such parametric uncertainties occur due to a variety of reasons on practical applications. Typically, system dynamics, which are invariably nonlinear, are often linearized to derive the requisite linear controller. The resulting linear model and its parameters are therefore dependent on and vary with the operating condition. Parameters, also, may vary due to aging, disturbances, or changes in the loading conditions. Parameters may be unknown due to approximations made in the modeling process.

In all these cases, a controller is called for that provides a uniformly satisfactory performance in the presence of the parametric uncertainties and variations. The adaptive approach to this problem is to design a controller with varying parameters, which are adjusted in such a way that they adapt to and accommodate the uncertainties and variations in the plant to be controlled. By providing such a time-varying solution, the exact nature of which is determined by the nature and magnitude of the parametric uncertainty, the closed-loop adaptive system seeks to enable a better performance. The results that have accrued in the field of adaptive control, over the past three decades, have provided a framework within which such time-varying, adaptive controllers can be designed to yield stability and robustness in various control tasks.

Model reference adaptive control refers to a particular class of adaptive systems. In this class, adaptive controllers are designed by using a reference model to describe the desired characteristics of the plant to be controlled. The use of such reference models facilitates the analysis of the adaptive system and provides a stability framework.

Two philosophically different approaches exist for synthesizing model reference adaptive controllers: *indirect control* and *direct control*. In the indirect approach, the unknown plant parameters are estimated using a model of the plant before a control input is chosen. In the direct approach, an appropriate controller structure is selected and its parameters are directly adjusted so that the output error is minimized. For the sake of mathematical tractability, the desired output y_d needs to be characterized in a suitable form, which is generally accomplished by the use of a reference model. Thus, in a model reference problem formulation, the indirect approach employs both an identification model and a reference model while the direct approach uses a reference model only. We describe these models in the following section.

2. Dynamic Models

2.4. Identification Model

Mathematical modeling is an indispensable part of all sciences, whether physical, biological, or social. One often seeks to characterize the cause and effect relations in an observed phenomenon using a model and tune the model parameters so that the behavior of the model approximates the observed behavior for all cases of interest. One form of quantitative models is mathematical and in the cases of dynamic systems, these take the forms of differential or difference equations.

These equations can be obtained either by using physical principles including conservation equations of mass, momentum, and energy, or by using input-output data that captures the relevant dynamics of a given process.

The model obtained in the latter case is often referred to as an *Identification Model*, since those from the first approach are either not available or too complex for control purposes. Often, especially for linear problems, frequency domain methods are used to identify the system parameters. When measurement noise is present, the identification methods include statistical criteria so as to determine the model that best fits the observed data.

Systems identification, which is based on such approaches, is a well-developed area of systems theory (see *Identification of Linear Systems in Time Domain*). The systems identification based modeling consists of using the input-output data and a black-box approach to derive the model structure and parameters. A typical system identification procedure includes:

i. model-structure selection;
ii. determination of the ‘best’ model in the structure as guided by the data; and
iii. selection of an appropriate excitation signal that includes a wide range of frequencies in order to accurately estimate the model parameters.

The most common of these model structures is linear, with constant coefficients. In this case, the excitation signal consists of sufficient number of sinusoids chosen so as to excite all of the modes of the system that represent its dominant dynamics. Typical linear model structures include ARMAX and N4SID, details of which can be found in *Identification of Linear Systems in Time Domain*.

2.5. Reference Model

The use of a reference model for controls can be traced to aircraft systems. Often, the situation therein is such that the controls designer is sufficiently familiar with the plant to be controlled and its desired properties. Thus, by choosing the structure and parameters of a reference model suitably, its outputs can be used as the desired plant response. While in principle such a model can be linear or nonlinear, considerations of analytical tractability have made linear reference models more common in practice.

2.5.1. Explicit and Implicit Model Following

Two methods that have been studied extensively in this context include explicit and implicit model-following methods, both of which include the use of a reference model described by the homogeneous differential equation

$$\dot{y}_m = A_m y_m \tag{1}$$

where the constant matrix $A_m \in \mathbb{R}^{m\times m}$ is chosen so that the desired dynamics in terms of transient behavior, decoupling of modes, bandwidth, and handling qualities are captured. Suppose that the plant to be controlled is described adequately by an *n*th order differential equation with $m(m << n)$ outputs as

$$\dot{x}_p = A_p x_p + B_p u \tag{2}$$

$$y_p = C_p x_p \,. \tag{3}$$

The reference model in Eq. (1) is chosen so that the output y_p follows y_m as closely as possible. The explicit and implicit model-following methods are based on different performance indices of the model-following error $y_p - y_m$. In explicit model-following, the performance index is of the form

$$I_e = \int_0^\infty \left[\left(y_p - y_m \right)^T Q_e \left(y_p - y_m \right) + u^T R u \right] dt \tag{4}$$

while in the latter, the performance index implicitly includes the reference model as

$$I_i = \int_0^\infty \left[\left(\dot{y}_p - A_m y_p \right)^T Q_i \left(\dot{y}_p - A_m y_p \right) + u^T R u \right] dt \quad Q_i > 0 \,. \tag{5}$$

In both cases, it can be shown that quadratic optimization theory can be used to determine the control input. In the former case, the optimal input has the form

$$u(t) = K_m y_m(t) + K_p x_p(t) \tag{6}$$

and in the latter case,

$$u(t) = K_p x_p(t) \,. \tag{7}$$

The structure of the controller can be used in an adaptive situation when the parameters of the plant are unknown, though the control parameters have to be estimated to compensate for parametric uncertainties.

2.6. Reference Model with Inputs

In Eq. (1), the output of the reference model was specified as the output of a homogeneous differential equation. A more general formulation of a reference model includes external inputs and is of the form

$$\dot{x}_m = A_m x_m + B_m r \quad , \tag{8}$$

$$y_m = C_m x_m \tag{9}$$

where A_m is a stable $n \times n$ matrix with constant elements, B_m, and C_m are constant matrices with appropriate dimensions, and r is an arbitrary continuous uniformly bounded input. The goal of the control input u into the plant in Eq. (3) so that the output $y_p(t)$ tracks the output $y_m(t)$ as closely as possible. In this case, the reference input r along with the model in Eqs. (8) and (9) with the parameters $\{A_m, B_m, C_m\}$ determines the output of the reference model.

The introduction of the reference inputs significantly increases the class of desired trajectories that can be represented by a reference model. For a perfect model following to occur, the differential equations governing y_p and y_m as well as the initial conditions $y_p(t)$ and $y_m(t)$ have to be identical. This imposes restrictive conditions on the matrices A_p, B_p, A_m, and B_m, in terms of their canonical forms. It has been shown by Erzberger that the requisite control input in this case is of the form

$$u(t) = K_p x_p(t) + K_m x_m(t) + K_r r(t) \quad . \tag{10}$$

In an adaptive situation, it is more reasonable to have the objective of asymptotic model following where $y_p(t)$ is desired to follow $y_m(t)$ as $t \to \infty$. The problem in this case is to determine the conditions under which this can be achieved amidst parametric uncertainties.

-
-
-

Bibliography

Aström K. and Wittenmark B. (1995). *Adaptive Control*, 574 pp. 2nd Edition. New York:Addison Wesley.

Erzberger, H. (1968). Analysis and design of model following control systems by state space techniques. (Proceedings of the JACC).

Isidori A.(1989). *Nonlinear Control Systems*: *An Introduction*, 479 pp. 2nd Edition. New York:Springer Verlag.

Kanellakopoulos I., Kokotovic P., Morse A. (1991). Systematic design of adaptive controllers for feedback linearizable systems. *IEEE Transactions on Automatic Control* **34**, 1241-1253.

Krstic M., Kanellakopoulos I., Kokotovic P.V. (1994). Nonlinear design of adaptive controllers for linear systems. *IEEE Transactions on Automatic Control* **39**(4), 738-752.

Ljung L.and Söderström T. (1983). *Theory and Practice of Recursive Identification,* 529 pp. Cambridge: M.I.T. Press.

Morse A. (1973). Structure and design of linear model following systems. *IEEE Transactions on Automatic Control* **18**, 346-354.

Mudgett D.R. and Morse A.S. (1985). Adaptive stabilization of linear systems with unknown high frequency gains. *IEEE Transactions on Automatic Control* **30**, 549-554.

Narendra K.S. and Annaswamy A.M. (1989). *Stable Adaptive Systems*, 494 pp. Englewood Cliffs, N.J.: Prentice-Hall, Inc.

Narendra K.S. and Balakrishnan J. (1994). Adaptive control using switching and tuning. (Proceedings of the Eighth Yale Workshop on Applications of Adaptive Systems Theory). New Haven, Ct.:Yale University.)

Seto D., Annaswamy A.M., Baillieul J. (1994). Adaptive control of nonlinear systems with a triangular structure. *IEEE Transactions on Automatic Control* **39**(7), 1411-1428.

Stein G. and Athans M. (1987). The procedure for multivariable feedback control design. *IEEE Transactions on Automatic Control* **32**, 105-114.

Biographical Sketch

Dr. Annaswamy received her Ph.D. in electrical engineering from Yale University in 1985. She has been a member of the faculty at Yale, Boston University, and MIT, where currently she is the director of the Active-Adaptive Control Laboratory and a Principal Research Scientist in the Department of Mechanical Engineering. Her research interests pertain to adaptive control, active control of resonant thermo-fluid systems including combustion processes and supersonic flows, and neural networks. She has authored numerous journal and conference papers and co-authored a graduate textbook on adaptive control. Dr. Annaswamy has received several awards including the Alfred Hay Medal from the Indian Institute of Science in 1977, the Stennard Fellowship from Yale University in 1980, the IBM post doctoral fellowship in 1985, the George Axelby Outstanding Paper award from IEEE Control Systems Society in 1988, and the Presidential Young Investigator award from the National Science Foundation in 1991. Dr. Annaswamy is a Fellow of the IEEE and a member of AIAA.