SPACE MANIFOLD DYNAMICS

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Contents

Summary

The term “Space Manifold Dynamics" (SMD) is used to describe the applications of Dynamical Systems methods to spacecraft mission analysis and design. Since the late 1980’s, the application of tools coming from the general field of Dynamical Systems has gone from a mathematical curiosity in the space community to become a serious methodology for the design and operation of real space missions. Missions such as Gaia, Genesis, GRAIL, Herschel, MAP, Plank, and many others, are all using Dynamical Systems concepts for their design.

The Space Manifold Dynamics approach to mission analysis problems allows the analysis of the natural dynamics of the problem in a systematic and efficient way, and can be used to solve questions such as: the description of the phase space in a large vicinity of the collinear Lagrangian points, the analytical computation of libration point orbits (LPO) using Lindstedt-Poincaré methods, the design of optimal station-keeping strategies for LPOs, the determination of low-energy and interplanetary transfers, the computation of transfers between libration point orbits, or the design of eclipse avoidance strategies; in all the cases fitting the required mission constraints.

In this paper some of the main tools of the Dynamical Systems theory used in Astrodynamics are presented, as well as their application to some particular problems of

the above list. Nevertheless, many technical details are not given and must be found in the references.

1. Introduction

For the design of space missions to libration point orbits, the Circular Restricted Three–Body Problem (CRTBP) is the natural and simplest model to start with. Dynamical Systems theory has been extensively used in the study of the CRTBP, for instance to get a detailed analysis of the dynamics in the vicinity of its equilibrium points, where some of the most dynamical complications occur. Its qualitative and quantitative procedures allow us to obtain an accurate picture of the evolution of the states of the system. Next we briefly introduce and discuss the main features of the problem.

The CRTBP describes the motion of a massless particle under the gravitational influence of two point masses m_1 and m_2, called primaries, in circular motion around their common center of mass. It is usual to consider a *synodic* reference system, with origin at the center of mass and rotating with the same angular velocity than the primaries, so that they are fixed in this system. The CRTBP has a Hamiltonian structure, with Hamiltonian function H, that in terms of the synodic position (x, y, z) and momentum (p_x, p_y, p_z) of the massless particle is given by

$$H = \frac{1}{2}\left(p_x^2 + p_y^2 + p_z^2\right) - xp_y + yp_x - \frac{1-\mu}{r_1} - \frac{\mu}{r_2},$$

where $\mu = m_2 / \left(m_1 + m_2\right)$, and r_1 and r_2 the distances from the massless particle to both primaries. The constant value of the Hamiltonian over each solution, h, is called *the energy* of the orbit.

In the synodical reference system there exist five equilibrium (or libration) points (see Figure 1). Three of them, the *collinear* ones, are on the line joining the primaries and are usually denoted by L_1, L_2 and L_3, where L_1 is between the two primaries, L_2 is at the left-hand side of the small one (which is assumed to be on the negative x-axis), and L_3 is at the right-hand side of the big one (on the positive x-axis). The last two equilibrium points, L_4 and L_5, called *triangular* points, form equilateral triangles with the primaries. Around the triangular equilibrium points, there are large regions with good stability properties that could be used as parking regions at which almost no station keeping is needed.

From a dynamical point of view, the collinear libration points behave as the product of two centers by a saddle. According to Lyapunov's center theorem, each equilibrium point gives rise to two one–parametric families of periodic orbits, spanning a 2D manifold tangent at the equilibrium point to the real and imaginary parts of the eigenvectors with eigenvalues $\pm\left(\sqrt{-1}\right)\omega_i$. These two families are known as the planar and vertical Lyapunov family, respectively, of periodic orbits.

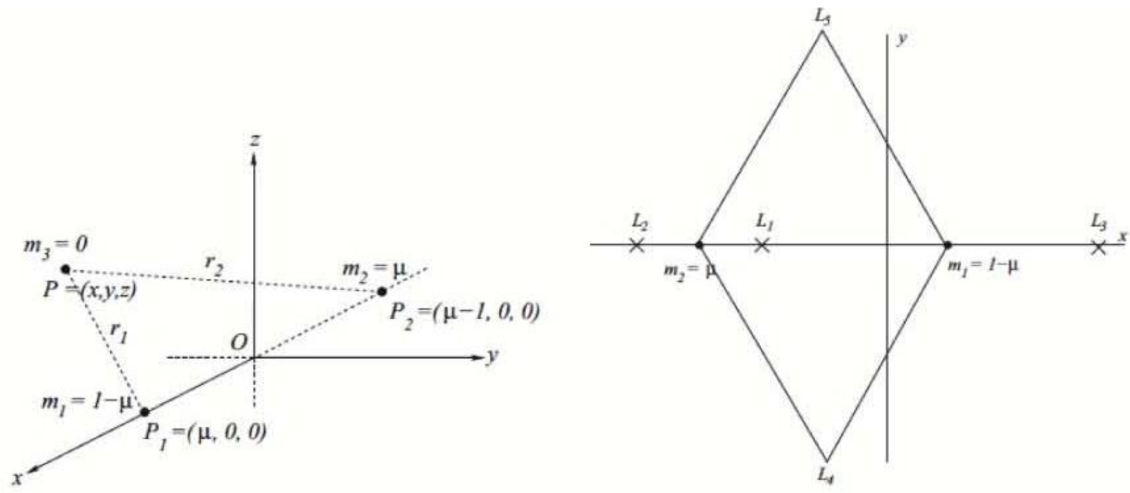

Figure 1. Left: The synodic reference frame of the CRTBP using adimensional units. Right: The five equilibrium points associated with the problem.

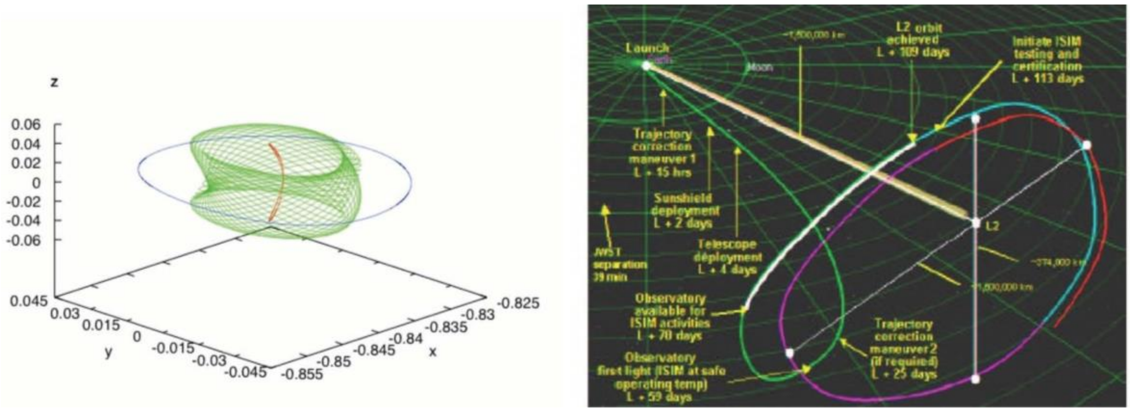

Figure 2. Left: A planar (blue) and a vertical (red) Lyapunov orbit, and a Lissajous orbit (green) for a fixed energy value around the equilibrium point L_1. Right: Nominal halo orbit (pink) and foreseen transfer orbit for the James Webb Space Telescope (from http:// ngsc.gsfc.nasa.gov /).

When we consider all the energy levels, the center $\times$ center part gives rise to four-dimensional central manifolds around these equilibria. Among the solutions in the central manifold, the quasi-periodic Lissajous orbits are those associated with two-dimensional tori. For a fixed energy level, these solutions can be viewed as families of quasi-periodic solutions that "connect" the planar and the vertical Lyapunov orbit at the same energy level (see Figure 2, left).

Following the families of Lyapunov periodic orbits, as the energy h increases, the linear stability of the orbits change and there appear bifurcating orbits where other families of periodic orbits are born. At the first bifurcation orbit of the family of planar Lyapunov orbits, there appear two families of 3-dimensional periodic orbits, symmetric with respect the $y = 0$ plane, that are called *Halo* orbits (see Figure 2 right).

Due to the hyperbolic character of the collinear equilibrium points, the invariant objects around them inherit the hyperbolicity, at least for values of the energy close to that of each equilibrium. This means that the orbits (periodic and quasi-periodic) in the central manifold are unstable and have a stable and an unstable invariant manifold associated. For the periodic orbits, the invariant manifolds look like 2D tubes filled with trajectories tending forwards (for the unstable) and backwards (for the stable) in time to the corresponding orbit. In the case of the Lissajous orbits, these invariant manifolds increase in one unit their dimension.

The stable invariant manifolds allow an efficient determination of transfer trajectories from the Earth to the libration point orbits of the Sun–Earth system, as well as the emergence of other trajectory and mission options. Furthermore, the intersections between the invariant manifolds give rise to homoclinic or heteroclinic connections that, in principle, allow to construct complicated itineraries between neighborhoods of two equilibrium points.

In connection with the computation of transfer orbits, it often appears in the literature the so called *weak stability boundary* (WSB), introduced by E. Belbruno after the rescue of the Hiten spacecraft. Although the WSB has not a precise definition, it can be seen as a boundary set in the phase space between stable and unstable motion relative to the second primary. After the work in the last decade of Koon, Gómez and Belbruno, it has been shown that the WSB, as well as its "rescue" role in missions like Hiten, can be completely explained in terms of the invariant hyperbolic manifolds associated to the central manifolds of the L_1 and L_2 libration points.

2. Spacecraft Missions to Libration Point Orbits

The orbits around the libration points, called *libration point orbits*, *LPO*, have unique characteristics suitable for performing different kinds of spacecraft missions. Among the most relevant characteristics, one can mention:

- In the Earth–Sun system, they are easy and inexpensive to reach from Earth.
- In the Earth–Sun system, they provide good observation sites, mainly solar observatories at L_1 and astronomy observatories at L_2. Near L_2 more than half of the entire celestial sphere is available at all times.
- Since the libration orbits around the L_1 and L_2 points of the Sun–Earth system always remain close to the Earth, at a distance of roughly 1.5 million km, and have a near-constant geometry as seen from the Earth, the communications system is simple.
- The L_2 environment of the Sun–Earth system is highly favorable for non-cryogenic missions requiring great thermal stability, suitable for highly precise visible light telescopes.
- The libration orbits around the L_2 point of the Earth–Moon system, can be used to establish a permanent communications link between the Earth and the hidden part of the Moon, as was suggested by A.C. Clark in 1950 and Farquhar in 1968.
- The LPO's can provide ballistic planetary captures, such as for the one used by the

Hiten spacecraft.

- The heteroclinic connections between libration point orbits provide Earth transfer and return trajectories, such as the one used for the Genesis mission or by the Artemis-P1 spacecraft.
- The libration point orbits provide interplanetary transport which can be exploited in the Jovian and Saturn systems to design a low energy cost mission to tour several of their moons (Petit Grand Tour mission).
- Formation flight, with a rigid shape, is possible using libration point orbits.

An example of a mission visiting libration points' neighborhoods is Genesis, launched in 2001 by NASA to study the solar wind and bringing back a sample to the Earth. The trajectory started travelling to the L_1 Sun-Earth point, resembled several times a halo orbit, and finally was inserted in a trajectory with a loop around L_2 before being captured back to Earth (see Figure 3 left). Another example is the trajectory of the Artemis-P1 spacecraft, devoted to study magnetism and how the solar wind flows past the Moon and tries to fill in the vacuum on the other side. This spacecraft follows a heteroclinic connection between orbits around the two Lagrangian points L_1 and L_2 of the Earth–Moon system (see Figure 3 right).

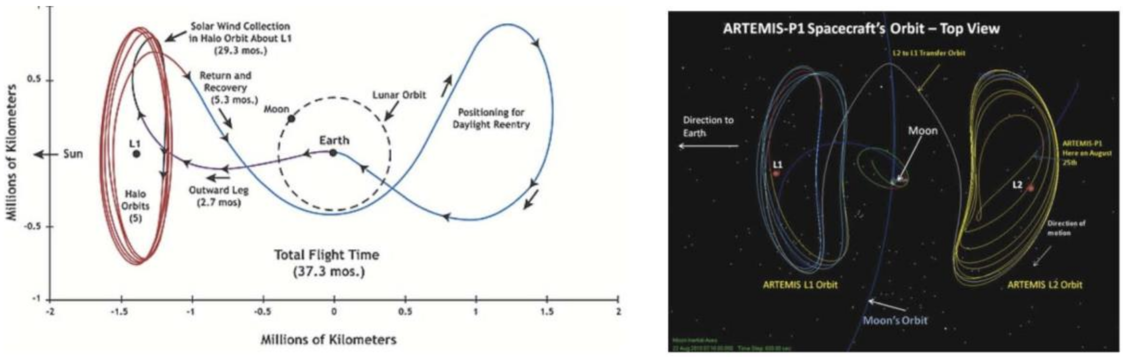

Figure 3. Left: Trajectory of the Genesis spacecraft. Right: Trajectory of the Artemis-P1 spacecraft following a heteroclinic connection in the Earth-Moon system. (From NASA's official web page).

Many more missions (past, current or future) use the above mentioned properties. Among the most relevant ones we can mention: ISEE-3 (1978), WIND (1994), SOHO (1996), ACE (1997), Herschel (2008), Plank (2008), Chang'e 2 (2010), GRAIL (2011), GAIA (2012), DARWIN, Constellation X, LISA Pathfinder, SAFIR, TPF, Triana, JWST (previously known as NGST), ...

2.1. LPO In Lunar and Exploration Missions

In the past few years there has been a renewed interest in the exploration of the Moon and, in particular, in its far side. Among the current missions to the Moon there is the previously mentioned Artemis, an extended mission of a constellation of five spacecrafts, two of which were moved into a lunar orbit, and GRAIL that will produce a high-resolution map of the Moon's gravitational field. GRAIL is composed by two small probes orbiting the Moon, which made use of a low-energy lunar transfer via the

Sun-Earth Lagrange point L_1 in order to reduce the fuel requirements and to slow down the velocity at lunar arrival.

Furthermore, the possibility of performing a temporary ballistic capture allowed us to keep the 40N engines available to the spacecraft low-cost bus: such a moderate thrust would have not allowed us to perform the classical one-shot Lunar Orbit Insertion (LOI) maneuver foreseen by a Hohmann-like transfer; thus the space manifold dynamics transfer removes the “single-point failure” character of the classical LOI.

Space manifold dynamics tools are currently used to design lunar missions, such as the preceding ones, with a significant energy (Δv) saving factor with respect to classical two-body problem approach. Departing from the Earth, it is possible to perform a ballistic capture in an elliptic orbit around the Moon using the manifolds associated to some particular libration point orbits.

The resulting transfer has an important saving at the lunar injection maneuver (up to 40% in missions like *LunarSat*, but with an additional mission duration. It must be also said that this gain vanishes when a low-altitude circular orbit (such as those used for manned missions, remote sensing or gravimetry) must be eventually achieved. Another example using these tools is a study of how to launch three small spacecraft on-board the same launch vehicle and send them to different orbits around the Moon with no significant difference in their Δv budgets (Marson et al, 2010).

It is known that the design of interplanetary transfers from the Earth to the planets can be optimised, from the energy point of view, by incorporating lunar swing-byes at the departure from the Earth sphere of influence (see Figure 4). Those transfers can also incorporate trajectory paths through the WSB region and in this way save up to 150 kg of propellant to missions like Mars Express, but again with the penalty of a larger transfer duration.

The use of libration point dynamics has been also considered in the design to inner planet capture missions, like *Bepi Colombo* to Mercury, *Venus Express* to Venus and *Mars Express* to Mars. In this case, the energy saving is low, but the mission design is highly *flexible* compared with the classical patched conics approach. In particular, the use of classical procedures imposes a given argument of pericenter and right ascension of ascending node of the resulting planetary orbit, while the use of LPO techniques give practically a full freedom to select above parameters, with a not too large penalty in the mission duration. From a scientific point of view, the capacity to choose freely the orbital plane orientation gives an extraordinary increase in the final outcome of the mission.

A similar conclusion can be obtained for the application of SMD techniques to the outer planet capture (Jupiter, Saturn, Uranus, Neptune). However, if a tour of giant planet natural moons (Jupiter tour) is designed, the use of SMD techniques gives again an important energy saving factor in addition to the high flexibility.

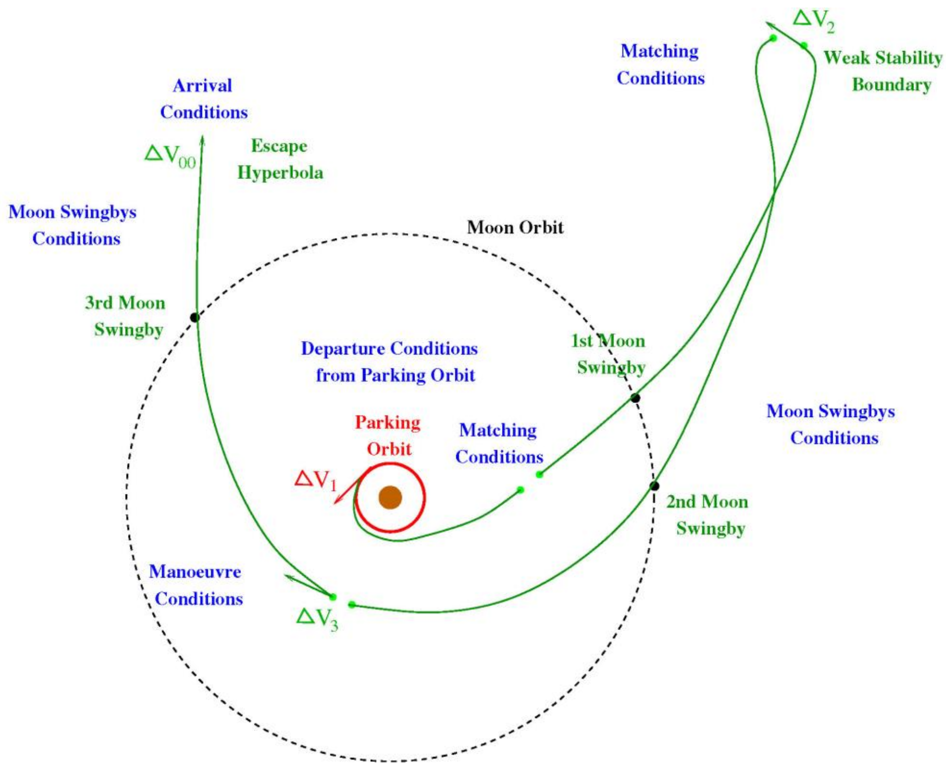

Figure 4. Example of a triple lunar swing-by Earth departure technique (obtained from Space Manifold Dynamics. Novel Spaceways for Science and Exploration)

2.2. Mission Design around Libration Points

The mission design of satellite flying orbits around libration points includes the consideration of the following aspects:

1. Definition of a *nominal trajectory*: the first step is the selection of the environment (the two- body system: Earth-Sun, Earth-Moon), the libration point (collinear L1, L2, L3 or triangular points L4 or L5) and the type of trajectory (Halo, Lissajous,...)
2. *Transfer trajectories* to the selected nominal orbit from initial launch conditions or parking orbits.
3. *Launch window* calculations taking into account the main mission constraints imposed for scientific or technical reasons.
4. *Navigation* of transfer and nominal trajectories: computation of the required trajectory correction maneuvers to correct launch injection dispersion, orbit determination errors and maneuvers mechanisation errors.
5. *Orbit maintenance*: strategies to keep the spacecraft in a neighborhood of the selected nominal path.
6. *Formation flying* techniques: new astronomy missions to LPO imposes the formation flying of several probes to implement interferometric techniques, the design of the formation architecture, the deployment, the tight control and the collision avoidance techniques must be defined.
7. *Eclipse avoidance*: most of the missions flying LPO orbits must avoid eclipses in order to continue nominal operations.
8. *Transfer between libration point orbits*: in some cases there is a need to transfer the

probe from one initial LPO orbit to another larger or smaller amplitude trajectory.

The dynamical systems approach provides solutions to all the above items as will be shown in the sections that follow.

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