ADVANCED DETERMINISTIC OPTIMIZATION

T. M. Liebling and **A. Prodon**
Ecole Polytechnique Federale de Lausanne, France

L. E. Trotter, Jr.
Cornell University, Ithaca, New York, USA

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Summary

This paper highlights the main ideas that shaped the field of linear and discrete optimization, which is a main pillar of deterministic operations research. The history is sketched starting with the early contributions of Fourier, Weyl, Minkowski, Farkas, continuing through the golden age of Dantzig, von Neumann, Fulkerson, Hoffman,

Edmonds (who laid the foundations of the field), and ending with its most recent developments. It describes the ideas that made this branch become one of the most lively and productive in discrete mathematics. It surveys the basic methods and the underlying theory used in solving linear and discrete optimization problems. These topics include: algebra and geometry of linear systems, fundamental algorithms, spanning trees, shortest paths, job scheduling, network flows, routing, matchings and extensions, matroids, packing and covering. It also discusses computational complexity, integral polyhedra, cutting planes, approximation schemes and heuristics.

1. Introduction

Real-world applications of optimization often concern economic efficiency: resource allocation, production and distribution of commodities, routing and sequencing of materials in networks underlying manufacturing, computation, or communication processes. Such applications are commonly formulated as linear programming models for managerial decision making. Linear programming is often cited as one of the most heavily used scientific computing tools, a claim whose validity is evidenced by the wealth of practical applications. Every organization of moderate size, whether private or public, faces routine planning problems either of maximizing profit under restrictions on resource availability for production, or of minimizing operational cost under constraints on levels of services provided.

The linear programming models formulated for these applications, however, usually neglect the inherently *discrete* nature of their basic commodities or components, and thus may represent only coarse approximations to the processes being modeled. In many cases, models of sufficient accuracy are obtained only through explicit inclusion of discrete-valued restrictions on problem variables. The resulting *integer programming* or *combinatorial optimization* models define the area of operations research and applied mathematics known as *discrete and combinatorial optimization*. In addition to natural discreteness conditions on problem components representing quantities of goods produced, stored, or shipped, integer-valued variables can also be used to model logical decisions. Thus, models involving design considerations often are formulated as integer programming problems. Some examples include: optimal location of product distribution centers or emergency public service centers, optimal layout of networks for computation or communication, and optimal product construction given component alternatives.

The fundamental mathematical model for such applications is

$$max\left\{\sum_j c_j x_j : \sum_j a_{ij} x_j \le b_i \forall\ i;\ x_j \ge 0,\ integral\ \forall\ j\right\} \tag{1}$$

or in matrix-vector notation, $max\{cx : Ax \le b;\ x \ge 0,\ integral\}$. (Transpose notation is suppressed, as it is clear from the context.) A standard interpretation of this model is that values x_j are sought for various production activities in order to maximize total profit, with activity j returning profit at rate c_j. Resource availability limits the production process: only b_i units of resource i are on hand over the planning horizon, and resource i is consumed by production activity j at rate a_{ij}. Note the explicit stipulation that production levels x_j are discrete, i.e., integer-valued. Straightforward variations on this model, e.g., models requiring minimization and models with equality constraints, also are considered.

2. Foundations

2.2 Algebra and Geometry of Linear Systems

The constraints of the integer programming model stipulate that its *feasible solutions* satisfy a finite system of linear inequalities/equalities with additional integrality restrictions. Directing attention first to the linear relations, systematic study of the structure of solutions to linear systems of equalities dates back to the mid-eighteenth century (see Schrijver (1986), with fundamental contributions to the theory of determinants and matrices by Cramer, Cauchy, Sylvester, and Caley, and results on solving linear equality systems by Lagrange, Legendre, and Gauss, which now are considered classical in linear algebra. Investigation of linear inequality systems began in the early nineteenth century with the work of Fourier, and continued into the 1900s with the fundamental results of Gordan, Farkas, Minkowski, Voronoï, and Carathéodory on finite cones, culminating with the contributions of Weyl and Motzkin in the mid-1930s. With respect to inhomogeneous inequality systems, i.e., systems that define polyhedra, there were also many early contributors, including Weyl, Motzkin, Helly, and Steinitz; development of this subject flourished with the advent of linear programming in the late 1940s.

A *subspace* of $\mathbb{R}^n$ (the real n-vectors) contains 0 and is closed under linear combinations. Geometrically, the origin is a subspace, as are the lines, planes, and hyperplanes in $\mathbb{R}^n$ containing the origin. Algebraically, the rows of any matrix $A \in \mathbb{R}^{m\times n}$ naturally give rise to two subspaces: the *row space* of A, $R = \{yA : y \in \mathbb{R}^m\}$, comprising all linear combinations of the rows; the *orthogonal complement* of R, denoted $S = \{x : Ax = 0\}$, consisting of all points *orthogonal* to each element of R. In fact, orthogonality provides a *duality relation* linking pairs of subspaces; this relation is completely symmetric, in the sense that R is also the orthogonal complement of S. Thus, any vector not in R must also fail to be in the orthogonal complement of S, a fact expressed by the following *theorem of the alternative*, whose origins can be traced to the work of Gauss.

Theorem 1 (Fredholm 1903)

For $A \in \mathbb{R}^{m\times n}$ and $c \in \mathbb{R}^n$, exactly one holds:

$$\begin{aligned} &(i)\ \exists y \in \mathbb{R}^m \text{ such that } yA = c; \\ &(ii)\ \exists x \in \mathbb{R}^n \text{ such that } Ax = 0,\ cx \neq 0 \end{aligned} \tag{2}$$

A further consequence of the duality relation is that there exists a second matrix B, whose rows are elements of S, for which $R = \{z : Bz = 0\}$. Any *basis* of S, i.e., any linearly independent generating set, gives such a matrix *B*. There are thus two ways to describe any subspace: either *internally*, as the set of vectors constructed via linear combinations of a (finite) basis, or *externally*, as the set of solutions to a finite linear system of homogeneous equations, i.e., a finite intersection of homogeneous hyperplanes.

A *cone* contains 0 and is closed under nonnegative linear combinations. Just as with subspaces, two cones arise naturally from the rows of $A \in \mathbb{R}^{m\times n}$: the *row cone* of *A*, $K = \{yA : y \geq 0\}$, generated by nonnegative combinations of its rows; the *dual* of *K*, given by $L = \{x : Ax \geq 0\}$, whose members make nonnegative inner product with each element of *K*. Like orthogonality, cone duality is symmetric and *K* is the dual of *L*, as is summarized in the following *theorem of the alternative*, analogous to Theorem 1.

Theorem 2 (Farkas 1894)

For $A \in \mathbb{R}^{m\times n}$ *and* $c \in \mathbb{R}^n$, *exactly one holds:*

$$\begin{array}{l}(i)\ \exists y \geq 0 \text{ such that } yA = c;\\ (ii)\ \exists x \in \mathbb{R}^n \text{ such that } Ax \geq 0,\ cx < 0.\end{array} \tag{3}$$

In geometric terms, the theorem assures that if *c* is not in the cone $K = \{yA : y \geq 0\}$ (when (*i*) fails), there exists a hyperplane $\{z : xz = 0\}$, defined by *x* obtained from alternative (*ii*), which *separates c* from *K*, i.e., for which $xc < 0$, but $xz \geq 0$ for all $z \in K$. The descriptive duality of constraints and generators for cones follows from the fact that every cone that can be expressed with finitely many constraints (as with *L* above), i.e., every *finitely constrained* or *polyhedral* cone, is also *finitely generated* (like *K*), and conversely. These fundamental results are due to Minkowski (1896) and Weyl (1935), respectively. Hence there exists a matrix *B* for which $L = \{uB : u \geq 0\}$ and $K = \{v : Bv \geq 0\}$, the dual of *L*, and any *finite* cone may be represented, just as with subspaces, either as constructed from a finite set of generators, or as determined by a finite intersection of homogeneous half-spaces.

Theorem 3 (Minkowski 1896; Weyl 1935)

A cone is finitely generated if and only if it is polyhedral.

Thus finite cones, arising from finite homogeneous systems of linear inequalities, provide an inequality counterpart to linear subspaces and much of the algebraic and geometric development for subspaces carries over directly to finite cones. One significant difference deserves mention, however. Recall that for subspaces all bases are equicardinal, i.e., that every linearly independent generating set has the same number of elements. This fails in the cone setting, although the following *local* version of this result does remain valid.

Theorem 4 (Carathéodory 1911)

Where $K = \{yA : y \geq 0\}$, *with* $A \in \mathbb{R}^{m\times n}$, *each element of K is a nonnegative* *combination of linearly independent rows of A.*

Inhomogeneous linear equality systems give rise to *affine spaces,* sets closed under affine combinations (linear combinations with unit coefficient sum). For $A \in \mathbb{R}^{m\times n}$ and $b \in \mathbb{R}^m$,

$S = \{x : Ax = b\}$ is an affine space, and conversely, every affine space can be expressed in this form. Geometrically, affine spaces are simply translates of subspaces. For inequality systems, the corresponding geometric object is a finite intersection of halfspaces, and is termed a *polyhedron*. Any polyhedron is *convex*, i.e., closed under convex combinations (nonnegative combinations with unit coefficient sum).

Theorem 2 implies the following characterization of inconsistent inequality systems, or equivalently, of empty polyhedra: for $A \in \mathbb{R}^{m\times n}$, $b \in \mathbb{R}^m$, system $\{Ax \geq b\}$ has no solution if and only if there exists a vector y satisfying $\{yA = 0, yb > 0, y \geq 0\}$. This restatement allows a direct adaptation of Carathéodory's Theorem to inequalities, a result that also follows from the work of Helly on general convex sets.

Theorem 5 (Helly 1923)

For $A \in \mathbb{R}^{m\times n}$ *and* $b \in \mathbb{R}^m$, $\{x : Ax \geq b\} = \emptyset$ *if and only if some subsystem of* $[A\ b]$ *comprising linearly independent rows is also inconsistent*.

A *polytope* is a convex set generated (via convex combinations) from a finite point set. Carathéodory (1911) established a geometric analogue of Theorem 2 for polytopes: when a point is not in a polytope, it is separated from the polytope by a hyperplane. The descriptive equivalence of constraints and generators also extends to the present setting. Steinitz (1916) established the equivalence of polytopes and bounded polyhedra. For unbounded polyhedra, the following *Double Description Theorem* shows that not only convex combinations due to polytopes, but also nonnegative combinations arising from finite cones must be considered.

Theorem 6 (Motzkin 1936)

P is a polyhedron if and only if P = K + Q, for K a finite cone and Q a polytope; i.e.,

$$\begin{aligned} & P = \{x : Ax \geq b\},\ \textit{for some}\ A \in \mathbb{R}^{m\times n},\ b \in R^m \\ \Leftrightarrow\ & P = \left\{ yB + zC : y \geq 0;\ z \geq 0;\ \textstyle\sum_{i=1}^{q} z_i = 1 \right\},\ \textit{for some}\ B \in \mathbb{R}^{p\times n},\ C \in \mathbb{R}^{q\times n} \end{aligned} \quad (4)$$

Turning now to the integer programming discreteness conditions, integer-valued solutions for linear systems is also a well-studied topic in classical mathematics. Gauss (1801) established that a single linear equation with integral coefficients has an integral solution if and only if the greatest common divisor of its coefficients divides the right-hand side element. Heger (1858) and Smith (1861) extended this result to linear equality systems.

Any subset of $\mathbb{R}^n$ containing 0 and closed under *integral* linear combinations is called a $\mathbb{Z}$*-module*; a $\mathbb{Z}$-module which has a *basis* (linearly independent generating set) is a *lattice*. In fact, any $\mathbb{Z}$-module of the form $\{yA : y \in \mathbb{Z}^m\}$ with A *rational*, i.e., $A \in \mathbb{Q}^{m\times n}$, is a lattice. This follows from the classical result of Hermite (1851) that when A is of full row rank, there exists a *unimodular* matrix U (i.e., $U \in \mathbb{Z}^{n\times n}, |\det(U)| = 1$), so that $H = AU$ satisfies $0 \le h_{ij} < h_{ii}\ \forall\ j < i$ and $h_{ij} = 0\ \forall\ j > i$ for each row i; the *Hermite normal form H* is the unique matrix with the indicated properties. There are many important applications of the Hermite normal form; it suggests, for example, a simple proof of the following membership characterization for rational lattices (compare Theorems 1 and 2).

Theorem 7 (Kronecker 1884)

For $A \in \mathbb{Q}^{m\times n}$ *and* $c \in \mathbb{Q}^n$, *exactly one holds:*

(i) $\exists y \in \mathbb{Z}^m$ *such that* $yA = c$ *;*

(ii). $\exists x \in \mathbb{Q}^n$ such that $Ax \in \mathbb{Z}^m, cx \notin \mathbb{Z}$.

A complete descriptive duality in the present setting holds for rational $\mathbb{Z}$-modules of the form $M = \{yA + zB : y \in \mathbb{Q}^m; z \in \mathbb{Z}^p\}$, i.e., allowing both rational and integral combinations, along with those of the form $N = \{x \in \mathbb{Q}^n : Cx = 0;\ Dx \in \mathbb{Z}^q\}$, in which both orthogonality and integrality restrictions are present. (The matrices A, B, C, D here are rational.)

Thus there has been significant classical development for various components of the integer programming model, with substantial structural similarity evidenced by results concerning nonnegative or integral solutions to linear systems. But, of course, the model stipulates *both nonnegativity and integrality*, directing geometric attention to the integral elements of polyhedra. Therein lies the complexity of integer programming. The classical treatment of combined nonnegativity and integrality restrictions is limited to a few special cases.

A single equation $a_1x_1 + \cdots + a_nx_n = b$, with relatively prime coefficients $a_i \in \mathbb{Z}_+$, has an integral solution for all $b \in \mathbb{Z}$; this is immediate from Gauss' divisibility criterion mentioned earlier. Moreover, the number of values of $b \in \mathbb{Z}_+$ for which there is no *nonnegative* integral solution, was shown by Sylvester (1884) and Curran Sharp (1884) to be $(a_1 - 1)(a_2 - 1)/2$, assuming the a_i are indexed in increasing order of magnitude. Frobenius posed the problem of determining the smallest b_0 so that nonnegative integral solutions exist for all $b \ge b_0$; Schur (1935) established that $b_0 \le (a_1 - 1)(a_n - 1)$.

For systems of equations, say $\{Ax = b\}$ with $A \in \mathbb{Z}_+^{m\times n}$, $b \in \mathbb{Z}_+^m$, Euler (1748) noted that the coefficient of $z_1^{b_1}\cdots z_m^{b_m}$ in the expansion of $\left(1-z_1^{a_{11}}\cdots z_m^{a_{m1}}\right)^{-1}\cdots\left(1-z_1^{a_{1n}}\cdots z_m^{a_{mn}}\right)^{-1}$ gives the number of nonnegative integral solutions. Other counting formulas were developed by Cayley, Sylvester, and Laguerre. One classical counting result is of particular interest. For $A\in\mathbb{Z}^{n\times n}$ of rank *n*, Hermite normal form uniqueness shows that any two bases of the lattice $M = \{yA : y\in\mathbb{Z}^n\}$ are related by a unimodular transformation; hence $|det(A)|$ is an invariant of *M*, denoted $\delta(M)$ and called the *determinant* of *M*. Moreover, $\delta(M)$ gives the number of integral elements in $\{yA : 0\le y_i < 1\ \forall\ i\}$, the (partially open) *parallelotope* spanned by the rows of *A*. Hermite showed that one can bound the size of a "small" basis for *M* in terms of $\delta(M)$. Specially, in the following theorem, he established the bound $\gamma_n = \left(\frac{4}{3}\right)^{n(n-1)/4}$. In his classic treatise on the *Geometry of Numbers*, Minkowski (1896) improved this to $\gamma_n = \frac{2^n}{V_n}\approx\left(\frac{2n}{\pi e}\right)^{n/2}$, where V_n is the volume of the unit *n*-sphere.

Theorem 8 (Hermite 1850)

Any n-dimensional lattice M has a basis $b_1, \dots, b_n$ *such that* $||b_1||\dots||b_n||\le\gamma_n\delta(M)$.

The classical development of convexity theory provides important background for all areas of mathematical programming. Indeed, many results indicated above for polyhedra, such as separation results and Helly's Theorem, remain valid for broader classes of convex sets. The fundamental issue of integer programming, understanding the structure of integral elements in polyhedra, again assumes central importance for general convex sets in the geometry of numbers, as in the following seminal result from this area.

Theorem 9 (Minkowski 1893)

If $C\cap\mathbb{Z}^n = \{0\}$ *for the* 0*-symmetric, convex set* $C\subset\mathbb{R}^n$, *then* $vol(C)\le 2^n$.

0-symmetry means $x\in C\Leftrightarrow -x\in C$. If this requirement is removed, it is obvious that the volume cannot be bounded, even for polyhedral sets. Nevertheless, this theorem has important counterparts for polyhedra. In particular, any polyhedron that fails to contain integral elements must be *thin* in some direction, and, furthermore, its *width* with respect to this direction is bounded by a function (ω_n below), which depends solely on dimension.

Theorem 10 (Khintchine 1948)

If $P\cap\mathbb{Z}^n = \emptyset$ *for polyhedron* $P\subset\mathbb{R}^n$, *then* $\max_{x\in P} dx - \min_{x\in P} dx = \le\omega_n$ *for some* $d\in\mathbb{R}^n$.

A further classical result on the integral elements of polyhedra was obtained for cones of the form $\{yA : y\ge 0\}$ with *A* rational. For such cones it is not difficult to show that *all* integral elements arise as nonnegative integral combinations of $\mathbb{Z}^n \cap \{yA : 0 \le y_i < 1 \ \forall \ i\}$, and thus have a finite generating set over the nonnegative integers, a so-called *Hilbert basis* (compare the *if* assertion of Theorem 3).

Theorem 11 (Gordan 1873; Hilbert 1890)

Let $K = \{x : Ax \ge 0\}$, *with* $A \in \mathbb{Q}^{m \times n}$.

Then there exists a matrix $B \in \mathbb{Z}^{p \times n}$ *such that* $K \cap \mathbb{Z}^n = \{zB : z \in \mathbb{Z}_+^p\}$.

When a finite cone is *pointed*, i.e., *x*, –*x* in the cone implies *x* = 0, then its minimal generating set is unique, up to positive scalar multiplication. Moreover, van der Corput (1931) has established the uniqueness of the minimal Hilbert basis in this case.

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Biographical Sketches

Leslie E. Trotter spent one year as a postdoctoral research associate with the Mathematics Research Center at the University of Wisconsin, and one and one-half years as an assistant professor in the School of Organization and Management at Yale University before joining the faculty of Cornell's School of Operations Research and Industrial Engineering in 1975. He was a visiting professor at the Institute for Econometrics and Operations Research at Bonn University in Germany (1977-79), and the Swiss Technical Institute in Lausanne, Switzerland (1984-85, 1991-92 and 2000). He received an Alexander von Humboldt U.S. Senior Scientist Award and spent the 1987-88 academic year at Augsburg University in Germany. Trotter is a member of the Institute for Operations Research and the Management Sciences, the Mathematical Programming Society, and the Society for Industrial and Applied Mathematics. He is codirector of the Advanced Computational Optimization Laboratory of the Cornell Theory Center. His general research interests are in the field of optimization with specific concentration in discrete optimization models.

Thomas M. Liebling is Professor at the Mathematics Institute of EPFL since 1980, he teaches operations research, algorithmics and engineering mathematics and leads ROSO, a group of a dozen researchers working on various aspects of modeling, simulation and optimization. Jury member of over 70 Ph.D. and habilitation theses, he directed some 30 of them, as well as over 100 diploma theses. Codirector of a european doctoral curriculum in discrete mathematics and optimization, he also codirects the AUSSOIS series of workshops in combinatorial optimization as well as the Oberwolfach Symposia on that topic. His main research interests are combinatorial optimization, models from discrete geometry, the development and analysis of algorithms and modeling of processes encountered in industrial logistics, as well as in technology and nature, in particular materials science and biology. He has published over 100 papers, book chapters and three books.

Alain Prodon is a senior Scientific Fellow and Lecturer at the Mathematics Institute of EPFL. He teaches graduate and undergraduate students in engineerings mathematics, discrete and combinatorial optimization and algorithmics. His main research interests are combinatoral optimization and algorithmics.