GENERAL EQUILIBRIUM

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Summary

General equilibrium is a central concept of economic theory. Unlike partial equilibrium analysis which studies the equilibrium of a particular market under the clause "ceteris paribus" that revenues and prices on the other markets stay approximately unaffected, the ambition of a general equilibrium model is to analyze the simultaneous equilibrium in all markets of a competitive economy. Definition of the abstract model, some of its basic results and insights are presented. The important issues of uniqueness and local uniqueness of equilibrium are sketched; they are the condition for a predictive power of the theory and its ability to allow for statics comparisons. Finally, we review the main extensions of the general equilibrium model. Besides the natural extensions to infinitely many commodities and to a continuum of agents, some examples show how economic theory can accommodate the main ideas in order to study some contexts which were not thought of by the initial model.

1. Introduction

Since the second half of the twentieth century, the Walrasian model of general equilibrium is formulated in the concept of equilibrium of a so-called *private ownership economy*. In such an economy, finitely many price-taking consumers (who are endowed with initial holdings of the different commodities and who collect given shares of the profits from production) consume the commodities available on the market, optimizing their preferences among all possible consumption plans that satisfy their budget constraint. Finitely many producers, who also take prices as given, maximize their profit on their individual set of possible production plans; they produce the commodities which satisfy consumers' demand in competition with the initial resources not used by production. Market clearing determines equilibrium prices and the quantities actually consumed and produced at a state of equilibrium. Market clearing may result in the strict equality of supply and demand; we will then speak of (strict) *equilibrium*. But, conceivably, the conditions for equilibrium may require that excess demand be non-positive and that, for any commodity for which it is negative, the price be zero; we will then speak of *free-disposal equilibrium*.

Existence of equilibrium prices is the necessary test of consistency for a model which bases the coordination of plans of diverse economic agents on the fact that prices faced by all agents provide the common flow of information needed to coordinate the system. And, indeed, equilibrium exists under reasonable assumptions. Just as important is the relation between the existence of solutions and the problems of normative economics. The Pareto optimality study considers the problem of efficient organization of an economy with an unspecified distribution of resources, and shows that for any Pareto optimal feasible allocation, there exists a price system to which consumers and producers are adapted. The Debreu-Scarf theorem shows that the same is true at the limit for an allocation that no coalition can block, in an economy with a specified distribution of resources when the number of consumers tends to infinity. These results have been first obtained in the 1950s and 1960s simultaneously by the three founders of the general equilibrium theory: K.J. Arrow, G. Debreu and, in a slightly different setting, L.W. McKenzie. Extended since then, these achievements strongly rely on convexity assumptions on the characteristics of the agents and on the use of convex analysis and fixed point theory. They confirm the new dependence of Mathematical Economics on an increasing list of mathematical tools such as functional analysis, measure theory, differential topology, non-smooth analysis, ordered sets theory etc.

After 1970, the general equilibrium theory developed in several directions. First, in what we will call the classical model (finite number of commodities, finite number of agents), the conditions for equilibrium existence and optimality results have been strongly weakened. Besides, the model itself has been generalized in order to allow for an infinity of time periods and states of the world in the specification of commodities, and for an infinity of agents. The abstract model has also been modified in order to accommodate the analysis of a great variety of economic settings where properties of equilibrium may fail to hold. The different contexts share a same methodology with the classical model of general equilibrium: the assumption that individuals, who have perfect (imperfect in case of differential information) foresight for each future state of the world, make their transactions in one initial market for the whole future and all states of the world. This common methodology explains the central role of general equilibrium in economic theory and maybe some of their common drawbacks.

In what follows, Sections 2, 3, and 4 present the current state-of-the-art for the classical model. Section 5 sketches the issues of uniqueness of the equilibrium solution. Section 6 discusses some extensions.

2. The Classical Model

Commodities

In the classical model, only a finite number of commodities are exchanged, produced or consumed. The *commodity space* is thus R^{ℓ}, the real vector space of dimension ℓ. The vector $z = (z_k)_{k=1}^{\ell} \in R^{\ell}$ denotes a *commodity bundle* with sign conventions explained below.

Private ownership economies and their agents

On this commodity space, a *private ownership economy* is completely specified by:

$$E = \left(\left(X^i, P^i, \omega^i \right)_{i \in I}, \left(Y^j \right)_{j \in J}, \left(\theta_{ij} \right)_{i \in I, j \in J} \right)$$

Where

- I is a finite set of consumers. Typically, a *consumer* is an individual but may be a household or a larger group with a common purpose. A consumer may even be a country in a model of international trade. For each consumer $i \in I$, a *consumption set* $X^i \subset R^{\ell}$ is the set of all consumption plans physically (or socially) possible for him. The consumption plan $x^i \in X^i$ is a list $\left(x_k^i\right)_{k=1}^{\ell}$ of the quantities of the various commodities that the consumer i consumes (positive numbers) or delivers (negative numbers). The *preference correspondence* $P^i : \prod_{h \in I} X^h \to X^i$ describes the tastes of consumer i. Under the condition that $x^i \notin P^i(x)$, the set $P^i(x)$ is interpreted as the set of consumption vectors x'^i strictly preferred to x^i when the consumption vectors of all consumers $h \neq i$ are x^h. Such a dependence may be justified by imitation or other psychological effects. It encompasses the case when consumer's preferences depend only on his own consumption vector and, a fortiori, the case when consumer i has a complete preference preorder $\succeq^i$ on his consumption set and $P^i\left(x^i\right) = \left\{ x'^i \in X^i \middle| x'^i \succ^i x^i \right\}$. The vector ω^i is the i-th consumer's *initial* *endowment* in each one of the commodities.

- J is a finite set of producers. For each *producer* $j \in J$, the *production set* Y^j is the set of all production plans technically possible for the j-th producer. A production plan $y^j \in Y^j$ is a list $\left(y_k^j\right)_{k=1}^{\ell}$ of the quantities of the various commodities that the producer j consumes as *inputs* (negative numbers) or produces as *outputs* (positive numbers).
- For all $i \in I$ and $j \in J$, the *profit-share* θ_{ij} is the contractual claim of consumer i on the profit of producer j. By definition, the θ_{ij} are nonnegative and for every j, $\sum_{i \in I} \theta_{ij} = 1$.

A particular case of private ownership economy is the *exchange economy,* where there is no production, specified by:

$$E = \left(X^i, P^i, \omega^i\right)_{i \in I}$$

Feasibility and market clearing

An *allocation*

$$(x, y) \in \prod_{i \in I} X^i \times \prod_{j \in J} Y^j$$

is *feasible* (or *attainable*) if

$$\sum_{i \in I} x^i - \sum_{i \in I} \omega^i - \sum_{j \in J} y^j = 0$$

feasible (or *attainable*) *with free-disposal* if

$$\sum_{i \in I} x^i - \sum_{i \in I} \omega^i - \sum_{j \in J} y^j \leq 0$$

Equilibrium

Let $S = \{p \in R^{\ell} : \|p\| = 1\}$ be the sphere with center 0 and radius 1. If each $p \in S$ represents a possible list of the (normalized) prices of each commodity, an *equilibrium* of E is a point

$$\left(\bar{p}, \bar{x}, \bar{y}\right) \in S \times \prod_{i \in I} X^i \times \prod_{j \in J} Y^j$$

satisfying the following conditions:

- For each $i \in I$, $\bar{p}.\bar{x}^i \leq \bar{p}.\omega^i + \sum_{j \in J} \theta_{ij} \bar{p}.\bar{y}^j$ and $x^i \in P^i(\bar{x}) \Rightarrow \bar{p}.x^i > \bar{p}.\bar{x}^i$
- For each $j \in J$, for all $y^j \in Y^j$, $\bar{p}.y^j \leq \bar{p}.\bar{y}^j$
- The allocation $(\bar{x}, \bar{y})$ is feasible.

The point $(\bar{p}, \bar{x}, \bar{y}) \in S \times \prod_{i \in I} X^i \times \prod_{j \in J} Y^j$ is a *free-disposal equilibrium* if, in the previous definition, the third condition is replaced by:

3'. The allocation $(\bar{x}, \bar{y})$ is attainable with free-disposal and $\bar{p}.\left(\sum_{j \in J} \bar{y}^j + \sum_{i \in I} \omega^i - \sum_{i \in I} \bar{x}^i \right) = 0$.

Condition 1 states that at equilibrium every consumer has chosen in his consumption set a consumption plan which best satisfies his preferences under his budget constraint. Summing over i the budget constraints, it follows then from Condition 3) or 3') that, at equilibrium, every consumer spends his equilibrium revenue: $\forall i \in I$, $\bar{p}.\bar{x}^i = \bar{p}.\omega^i + \sum_{j \in J} \theta_{ij} \bar{p}.y^j$.

Condition 2 states that every producer chooses a production plan so as to maximize his profit in his production set.

Condition 3 states at equilibrium the equality between the total supply and the total demand. In case of free-disposal equilibrium, Condition 3' ensures that the cost of the disposal needed for achieving equilibrium is minimized and equal to zero.

A clever reader will have observed that the free-disposal equilibrium of an economy is thus the equilibrium of a fictitious economy identical to the original one but where an additional price-taker and profit-maximizing producer has the negative orthant $-R^\ell$ as his production set. It is easily deduced from this remark that, at a free-disposal equilibrium, the equilibrium vector price is non-negative. This need not be the case if the equilibrium is strict, that is, without free-disposal. Some commodities may be noxious or at least non-desired at equilibrium; the disposal needed for achieving equilibrium is not free anymore and may involve costly activities.

Quasi-equilibrium

A point $(\bar{p}, \bar{x}, \bar{y}) \in S \times \prod_{i \in I} X^i \times \prod_{j \in J} Y^j$ is a *quasi-equilibrium* (resp. *free-disposal quasi equilibrium*) if it satisfies the profit-maximization and feasibility conditions of the corresponding equilibrium definition and if Condition 1) is replaced by the weaker condition:

1'. For each $i \in I$, $\bar{p}.\bar{x}^i \leq \bar{p}.\omega^i + \sum_{j \in J} \theta_{ij} \bar{p}.\bar{y}^j$ and $x^i \in P^i(\bar{x}) \Rightarrow \bar{p}.x^i \geq \bar{p}.\bar{x}^i$.

The interpretation of Condition 1' is not very appealing since it means that, at a quasi-equilibrium, no consumer could be strictly better spending strictly less than his budget constraint. The interest of the quasi-equilibrium concept is purely mathematical. As we will see in the next section, the equilibrium existence proofs actually establish the existence of a quasi-equilibrium, allowing for a clear distinction between the quasi-equilibrium existence problem and the investigation of the conditions which guarantee that a quasi-equilibrium is an equilibrium.

3. Existence of Equilibrium

Several equilibrium existence proofs are available in the literature. Among the three main approaches, the *excess demand approach* obtains the equilibrium price as a zero of the excess demand correspondence (a zero of the excess quasi-demand correspondence for the quasi-equilibrium price). This approach requires the preferences of each consumer to be formalized by a complete preorder on their consumption set.

The so-called *Negishi approach* bases the equilibrium existence on a fixed-point theorem applied in the *utility space*, that is in the vector space R^I whose dimension is equal to the finite number of consumers. This approaches requires the preferences of each consumer to be represented by a utility function but, since the first writings of Debreu, economists know that, at least when the commodity space is finite dimensional and under the usual assumptions of the equilibrium existence theorem, there is no loss of generality to assume that complete preference preorders on the consumption sets are represented by utility functions.

The approach presented below is the *simultaneous optimization approach*. The interest of this approach and of the theorems which will be presented is to be not too much demanding in terms of the rationality of consumers' choices (consumers' preferences need not be complete or even transitive) and to allow for some dependence of the individual preferences on the actions of the other agents.

3.1. Equilibrium and Quasi-equilibrium of Abstract Economies

An *abstract economy* (*generalized qualitative game*, *social system*) is completely specified by

$$\Gamma = \left(X^i, \alpha^i, P^i\right)_{i \in N}$$

where N is a finite set of *agents* (*players*) and for each $i \in N$,

- X^i is a *choice set* (or *strategy set*) that we will assume to be a subset of some finite dimensional vector space,

- The correspondence $\alpha^i : \prod_{h\in N} X^h \to X^i$ is called *constraint correspondence*. For $x \in X := \prod_{h\in N} X^h$, the set $\alpha^i(x)$ is interpreted as the set of the possible strategies for i given the choices $\left(x^h\right)_{h\neq i}$ of the other agents,
- The correspondence $P^i : \prod_{h\in N} X^h \to X^i$ is a *preference correspondence*. For each $x \in X$, under the condition that $x^i \notin P^i(x)$, the set $P^i(x)$ is interpreted as the set of elements of X^i strictly preferred by i when the choice of the other agents is $\left(x^h\right)_{h\neq i}$.

An *equilibrium* of Γ is a point $\bar{x} \in X$ such that for each $i \in N$,

1. $\bar{x}^i \in \alpha^i(\bar{x})$
2. $P^i(\bar{x}) \cap \alpha^i(\bar{x}) = \varnothing$

The interpretation is that at each component of the equilibrium point, the corresponding agent best satisfies his preferences in his constraint correspondence.

As a special case, if for every $i \in N$ and for every $x \in X$, $\alpha^i(x) = X^i$, in other words if $\Gamma = \left(X^i, P^i\right)_{i\in N}$ is a *qualitative game*, an equilibrium of Γ is nothing other than a *Nash equilibrium* of the qualitative game Γ.

Now, let $\beta^i : X \to X^i$ be correspondences satisfying for every $i \in N$ and for every $x \in X$,

3. $\beta^i(x) \subset \alpha^i(x)$
4. $\beta^i(x) \neq \varnothing \Rightarrow \overline{\beta^i(x)} = \overline{\alpha^i(x)}$

where for a set A, $\bar{A}$ denotes the closure of A.
Given $\beta = \left(\beta^i\right)_{i\in N}$, a β*-quasi-equilibrium* is a point $\bar{x} \in X$ such that for each $i \in N$,

5. $\bar{x}^i \in \alpha^i(\bar{x})$
6. $P^i(\bar{x}) \cap \beta^i(\bar{x}) = \varnothing$.

β-quasi-equilibrium existence

The existence of a *β-quasi-equilibrium* can be deduced from the Gale–Mas-Colell lemma, lemma itself proved using Kakutani's theorem and a powerful selection theorem due to Michael.

Lemma (Gale–Mas-Colell) *Let N be a finite set of indices. Given $X = \prod_{i \in N} X^i$, where for each $i \in N$, X^i is a nonempty compact convex subset of some finite dimensional Euclidean vector space, let for each i, $\varphi^i : X \to X^i$ be a lower semi-continuous correspondence with convex (possibly empty) values. Then there exists $\bar{x} \in X$ such that for each $i \in N$, either $\varphi^i(\bar{x}) = \emptyset$ or $\bar{x}^i \in \varphi^i(\bar{x})$.*

The existence of a β- quasi-equilibrium for an abstract economy is proved under the following assumptions for each $i \in N$:

- (a) X^i is a nonempty compact convex subset of some finite dimensional Euclidean vector space,
- (b) α^i is an upper semi-continuous and non empty closed convex-valued correspondence,
- (c) β^i is convex-valued,
- (d) the correspondence $x \to \beta^i(x) \cap P^i(x)$ is lower semi-continuous,
- (e) the correspondence P^i is convex-valued and for all $x \in X$, $x^i \notin P^i(x)$.

Proposition 1 *Under the conditions (a) – (e) and for β defined as above, the abstract economy $\Gamma = \left(X^i, \alpha^i, P^i\right)_{i \in N}$ has a β-quasi-equilibrium. It is an equilibrium provided that for every i, $\beta^i(\bar{x}) \neq \emptyset$.*

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Bibliography

Aliprantis C.D., Brown D.J., Burkinshaw O. (1987). Edgeworth equilibria. *Econometrica* **55**, 1109–1137. [Seminal paper on Edgeworth equilibrium existence based on Scarf's theorem]

Aliprantis C.D., Florenzano M., Tourky R. (2005). Linear and non-linear price decentralization, *Journal of Economic Theory* **121**, 51–74. [On equilibrium existence in commodity spaces without lattice structure]

Arrow K.J. (1973). General economic equilibrium: purpose, analytic techniques, collective choice. *Nobel Memoriam Lecture December 12, 1972,* The Nobel Foundation 209–231. [Also available on the web. An interesting introduction to motivations and results of the author]

Arrow K.J., Debreu G. (1954). Existence of an equilibrium for a competitive economy. *Econometrica* **22**, 265–290. [Seminal paper for general equilibrium existence]

Arrow K.J., Hahn F.H. (1971). *General Competitive Analysis*, Holden-Day, San Francisco, California.[A very complete bible in its time]

Aumann R.J. (1964). Markets with a continuum of traders. *Econometrica* **32**, 39–50.[Seminal paper for core-equilibrium equivalence theorem in continuum economies]

Aumann R.J. (1965). Integrals of set-valued functions. Journal of Mathematical Analysis and Applications **12**, 1–12. [Useful mathematical paper]

Aumann R.J. (1966). Existence of competitive equilibria in markets with a continuum of traders. *Econometrica* **34**, 1–17. [Seminal paper for equilibrium existence in continuum economies]

Aubin J.P. (1979). Mathematical Methods of Games and Economic Theory, North-Holland, Amsterdam.[Useful textbook for games and general equilibrium theory]

Balasko Y. (1975). The graph of the Walras Correspondence. *Econometrica* **43**, 907–912. [Explicit introduction of differential topology methods in General equilibrium].

Beato P. (1982). The existence of marginal cost pricing equilibria with increasing returns, *Quarterly Journal of Economics.* **389**, 668–688. [A first paper on definition and existence of equilibrium in economies with increasing returns].

Bergstrom T.C. (1976). How to discard free-disposability – At no cost. *Journal of Mathematical Economics* **3**, 131–134.[Of historical importance for its approach to no disposal equilibrium]

Borglin A., Keiding H. (1976). Existence of equilibrium actions and of equilibrium. *Journal of Mathematical Economics* **3**, 313–316.[Important for its fixed point approach to equilibrium existence]

Cass D. (2006). Competitive equilibrium with financial markets. *Journal of Mathematical Economics* **42**, 384–405: [First circulated in 1984. Seminal paper on equilibrium existence in financial markets with nominal assets]

Cornet B. (1990) Existence of equilibria of economies with increasing returns. *Contributions to Operations Research and Economies. The Twentieth Anniversary of CORE*, MIT Press, Cambridge, MA: [First circulated in 1982. Introduces Clarke's normal cone in the equilibrium existence problem for non-convex production economies]

Debreu G. (1959). *Theory of Value – An axiomatic Analysis of Economic Equilibrium*, Cowles Foundation Monograph 17, Wiley, New York.[A great book]

Debreu G. (1962). New concepts and techniques for equilibrium analysis. *International Economics Review* **3**, 267–272. [A paper of historical importance]

Debreu G. (1970). Economies with a finite set of equilibria. *Econometrica* **38**, 387–392. [Introduces the idea of generic finiteness of the set of equilibria]

Debreu G. (1981). Existence of competitive equilibrium. In Arrow K.J., Intriligator M.D. (eds.) *Handbook of mathematical Economics*. North-Holland, Amsterdam. [Synthetic presentation of the equilibrium existence problem]

Debreu G. (1983). Economic theory in the mathematical mode. *Nobel Memoriam Lecture December 8, 1983*, 18 pp. [Available on the web. An interesting introduction to motivations and results of the author]

Debreu G., Scarf H. (1963). A limit theorem on the core of an economy. *International Economics Review* **4**, 235–246. [First mathematical translation and proof of Edgeworth's conjecture]

Duffie D., Shafer W. (1985). Equilibrium in incomplete markets: I A basic model of generic existence. *Journal of Mathematical Economics* **14**, 285–300 [Seminal paper on equilibrium existence in financial markets with real assets]

Edgeworth F.H. (1881). *Mathematical psychics; an essay on the application of mathematics to the moral sciences*, London: Kegan Paul. [With Walras, one of the main sources of inspiration for general equilibrium]

Florenzano M. (1990). Edgeworth equilibria, fuzzy core and equilibria of a production economy without ordered preferences, Journal of Mathematical Analysis and Applications **153** 18–36**.[**First result on Edgeworth equilibrium existence in economies with non-ordered preferences]

Florig M. (2001). On irreducible economies. *Annales d'Economie et de Statistiques*, **61** 183–199. [A survey paper which completes and extends Hammond (1993)]

Foley D.K. (1970). Lindahl's solution and the core of an economy with public goods. *Econometrica* **38**, 66–72. [A paper of historical importance]

Gale D., Mas-Colell A. (1975-1979). An equilibrium theorem for general model without ordered preferences. *Journal of Mathematical Economics* **2**, 9–15. Corrections. *Journal of Mathematical Economics* **6**, 297–298.[Important paper for its introduction of Michael's theorem in Mathematical Economics]

Geanakoplos J., Polemarchakis H. (1986) Existence, regularity and constraint suboptimality of competitive allocations when markets are incomplete. In W. Haller, R. Starr, D. Starett, eds. *Essays in honor of Kenneth Arrow*, Vol. 3, Cambridge University Press, Cambridge MA [A study of financial markets with numeraire assets]

Guesnerie R. (1975). Pareto optimality in non-convex economies. *Econometrica* **43**, 1–29.[Seminal paper often quoted]

Hammond P.J. (1993). Irreducibility, resource relatedness and survival in equilibrium in equilibrium with individual nonconvexities. In Becker R., Boldrin M. and Thomson W. (eds) *General Equilibrium, Growth and Trade*, 73–115, Academic Press, San Diego.[Useful survey on its subject]

Hildenbrand W. (1974). *Core and equilibria of a large economy*, Princeton University Press, Princeton N.J. [A master piece textbook for studying economies with a measure space of agents]

Hildenbrand W. (1994). *Market Demand. Theory and Empirical Evidence*, Princeton University Press, Princeton N.J.[One could not find better description of the content of this book than its title]

Kakutani S. (1941). A generalization of Brouwer's fixed point theorem. *Duke Mathematical Journal.* **8**, 457–459. [A theorem of tremendous importance for general equilibrium existence]

Mas-Colell A. (1974). An equilibrium existence theorem without complete or transitive preferences. *Journal of Mathematical Economics* **1**, 237–246. [The first paper on equilibrium existence in economies with non-ordered preferences]

Mas-Colell A., Whinston M.D., Green J. (1995). *Microeconomic Theory*. Oxford University Press, New York: [Masterful textbook whose different chapters give an idea of the diversity of themes in Economic Theory studied from a General Equilibrium point of view]

McKenzie L.W. (1954). On equilibrium in Graham's model of world trade and other competitive systems. *Econometrica* **22**, 147–161. [Historical paper by one of the three founders of general equilibrium theory]

McKenzie L.W. (1959-1961). On the existence of general equilibrium for a competitive market. *Econometrica* **27**, 54–71. Some corrections. *Econometrica* **29**, 247–248. [Historical paper by one of the three founders of general equilibrium theory]

McKenzie L.W. (2002). *Classical General Equilibrium Theory*, The MIT Press, Cambridge, Massachusetts. [An important monograph on the scientific interests of the author and his scholars]

Michael E. (1956). Continuous selections I. *Annals of Mathematics.* **63**, 361–382. [A deeply mathematical paper. One theorem in this paper is together with Kakutani's theorem a key argument for equilibrium existence]

Negishi T. (1960). Welfare economics and existence of an equilibrium for a competitive economy. *Metroeconomica* **12**, 92–97. [At the origin of the so- called Negishi approach to equilibrium existence]

Pareto V. (1909). *Manuel d'Economie politique*, Droz, Paris. [For the definition of Pareto optimality in Chap.VI Sections 32–38]

Shafer W., Sonnenschein H. (1975) Equilibrium in abstract economies without ordered preferences *Journal of Mathematical Economics* **2**, 345–348. [Paper of historical importance]

Villanacci, A. and U. Zenginobuz (2005), Existence and regularity of equilibria in a general equilibrium model with private provision of a public good. *Journal of Mathematical Economics* **41,** 617 – 635. [A general equilibrium analysis of public good provision]

Vind K. (1964) Edgeworth allocations in an exchange economy with many traders. *International Economics Review* **5**, 165–177.[An original appravh to continuum economies]

Walras L. (1874–1877) *Elements d'Economie Politique Pure*. L. Corbaz and Company, Lausanne.[With Edgeworth, one of the main sources of inspiration for general equilibrium]

Wilson C.A. (1981). Equilibrium in dynamic model with an infinity of agents. *Journal of Economic Theory* **24**, 95–111. [Equilibrium existence in overlapping generations of economies]

Biographical Sketch

Monique Florenzano received the Master degree in Mathematics from the Universities of Aix–Marseille and got the Aggregation of Mathematics in 1959. She also obtained the MA degree in Philosophy from the Universities Aix-Marseille in 1960, the Master degree in Economics from Paris University in 1963 and received the Habilitation from CNRS in 1984. Assistant Professor of Mathematics from 1959 to 1967 at the universities of Aix–Marseille and Paris, she entered in CNRS (National Center for Scientific Research) in 1967, was nominated MR in 1982 and DR (Research Director) in 1984. First affiliated in CEPREMAP, she moved in 1999 to CERMSEM, today a part of CES (Centre d'Economie de la Sorbonne), a research unit jointly managed by CNRS and University of Paris 1. Since September 2002, she is Emeritus Research Director at CNRS. In 1986, she participated as Visiting Professor of the University of California at Berkeley in the *Special Year in Mathematical Economics* organized by G. Debreu. In 1994–1995, she visited the Universidad Carlos III de Madrid as *Senior Researcher of the Human Capital Mobility European Program*. She is currently invited for research stays in Australia, Brazil, Chile, Spain, USA and has in France a large experience of teaching at the Master and PhD levels in Optimization and in Economics. Member of the Editorial Board for Springer Verlag *Lecture Notes in Economics and Mathematical Systems* from 2001 to 2003, she is, since 2003, member of the Editorial Board of *Economic Theory*, Since 2006, she is Chair of the Steering Committee of the ESF–Research Networking Programme *Public Goods*, *Public Projects*, *Externalities* (PGPPE). Her main research interests are fixed point theory and mathematical economics. In this last area, she has published many articles devoted in particular to general equilibrium, its relations with game theory, and its extensions to infinite dimensional (convex and non-convex) economies, with applications to intertemporal equilibrium of Leontief economies, overlapping generations models, incomplete financial markets, arbitrage–free exchange economies, linear economies with a measure space of agents, economies defined on an ordered topological vector space without lattice structure, economies with public goods. She is the author of: *General Equilibrium Analysis – Existence and Optimality Properties of Equilibria*, Boston MA, Kluwer Academic Publishers (2003).