MATHEMATICAL MODELS OF RESOURCE AND ENERGY ECONOMICS

Ngo Van Long
McGill University, Canada

Keywords: Hotelling's Rule, Herfindahl's Rule, Order of Exploitation, Faustmann's Rule, Tragedy of the Common, Energy Efficiency

Contents

Summary

This chapter reviews major economic models of natural resources and energy use. The center-piece of the theory of non-renewable resource, Hotelling's Rule, is explained and various versions of it are derived. Herfindahl's Rule on the order of exploitation of deposits is shown to have limitations. This is followed by an exposition of equilibrium concepts in dynamic games involving natural resources. The role of imperfect property rights in the inefficient exploitation of resources is highlighted, and supported by models that predict or explain the extinction of some renewable resources. We also review some models of investment in energy efficiency.

1. Introduction

With the publication of the Club of Rome's report titled Limits to Growth in 1972, which predicted catastrophic consequences of unrestrained economic growth, and the oil embargo in 1973 which resulted in a world-wide energy crisis, the economic profession began to have serious doubts about the optimistic beliefs in the functioning of the market mechanism. Does the finite-horizon Arrow-Debreu model which assumes perfect property rights and ignores the possibility of a doomsday fail to account for many problems that the world economy is facing? How well do we understand the behavior of economic agents in the face of uncertainty about reserves of natural resources, and under imperfect property rights? Can we rely on the market forces to provide sufficient research and development efforts to discover a backstop technology? What are the consequences of lack of international cooperation in the management of the earth's resources? Do we have a firm microeconomic foundation of the theory of the extractive firm under diverse market structures?

This chapter reviews major advances in economic sciences, especially those that have been made since 1973, that help us provide some partial answers to the questions that are posed above and related questions. It also explains the major methods that have been used to obtain those advances.

2. Non-Renewable Resources

Non-renewable resources are irreplaceable assets that deplete with use. They include oil, gas, mineral resources, and are sometimes referred to as exhaustible resources, or wasting assets. They do not include replaceable assets such as forests and stocks of fish.

2.1. Hotelling's Rule

An important question in the economics of non-renewable resources is: since the economy is by definition non-stationary, how do resource prices evolve over time? A key element to the answer to this question lies in a result called Hotelling's Rule, which was first discovered in 1931, but did not receive much attention until the 1970s. In fact, in the 1970s, economists rediscovered some results in the theory of the intertemporal market allocation of a non-renewable resource (such as oil) that had been obtained by Hotelling. Using the calculus of variations, Hotelling showed that *if the market is perfectly competitive, the net price (price minus marginal extraction cost) must rise at a rate equal to the rate of interest.* This is known as Hotelling's rule. It is at the heart of the economics of natural resources.

Let us derive Hotelling's Rule using optimal control theory. Let the state variable $X(t)$ denote the resource stock owned by a firm, and $q(t)$ the rate of extraction. The evolution of the stock obeys the differential equation:

$$\dot{X}(t) = -q(t)$$

The cost of extracting $q(t)$ is $C(q(t),t)$. The profit at time t is $p(t)q(t) - C(q(t),t)$. The firm takes the price path $p(t)$ as given, and chooses the extraction path $q(t)$ to maximize the integral of discounted profit, where r is the interest rate.

Let the planning horizon be T. At time $t=0$, the stock is $X(0)$. The terminal stock $X(T)$ must be non-negative. Let $w(t)$ be the co-state variable. The Hamiltonian function is

$$H = \exp(-rt)(p(t)q(t) - C(q(t),t)) - w(t)q(t)$$

The necessary conditions include the maximality condition and the adjoint equation. The former says that the control variable must be chosen to maximize the Hamiltonian, for given values of the state variable and the co-state variable. The latter states that the sum of the rate of change in the co-state variable and the partial derivative of the Hamiltonian with respect to the state variable must be zero.

$$\partial H / \partial q(t) \exp(-rt)(p(t) - C_q(q(t),t)) - w(t) = 0$$

$$\dot{w}(t) = -\partial H / \partial X(t) = 0$$

The transversality conditions are:

$$\exp(-rT)(w(T)) \geq 0$$

$$\exp(-rT)(w(T))X(T) = 0$$

Let $\psi(t)$ denote the current-value shadow price of the stock, that is,

$$\psi(t) = w(t)\exp(rt)$$

Then the maximality condition and the adjoint equation become, respectively,

$$\psi(t) = p(t) - C_q(q(t),t)$$

$$\dot{\psi} = r\psi$$

It follows that the following condition holds along the firm's extraction path:

$$\frac{d}{dt}\left(p - C_q\right) = r\left(p - C_q\right)$$

This equation is the standard form of Hotelling's Rule. The variable $\psi(t)$ is the *in situ* price of the resource, and Hotelling's Rule says *it must rise at a rate equal to the rate of interest*. When marginal cost is a constant, say c, Hotelling's Rule simplifies to:

$$\dot{p} = r(p - c)$$

In general, the cost function may contain the independent term τ that represents technological progress. Therefore Hotelling's Rule admits the possibility that a price path is at first decreasing and then increasing. When the Ricardian consideration that as an aggregate resource stock gets depleted, lower grades will be used, Hotelling's Rule must be modified, and net price must grow at a rate which is less than the rate of interest.

Hotelling's Rule was formulated under the assumption that firms operate under perfect competition, in the sense that each firm perceives the market price as its own marginal revenue. At the other extreme of the spectrum of possible industrial structures, one encounters the classic monopoly. To a monopolist, the marginal revenue is below the market price. In fact, it is well known that the ratio of marginal revenue to price, denoted by β, is equal to one minus the inverse of the elasticity of demand. Applying the calculus of variations to the problem of a resource monopoly, one obtains the modified Hotelling's Rule. This rule now says that the *net marginal revenue*, i.e., the difference between the monopolist's marginal revenue, R_{M}, and his marginal cost, C_{M}, must rise at a rate equal to the rate of interest:

$$\frac{d}{dt}\left(R_{\mathrm{M}} - C_{\mathrm{M}}\right) = r\left(R_{\mathrm{M}} - C_{\mathrm{M}}\right)$$

Now assume marginal cost is a constant, c. Then Hotelling's Rule for the monopolist becomes:

$$\frac{\dot{p}}{p} = r\left(1 - \frac{c}{\beta p}\right) - \frac{\dot{\beta}}{\beta}.$$

Here, the left-hand side is the rate of change in the price of the extracted resource. On the right-hand side, r is the interest rate, c is the marginal extraction cost, here assumed to be a constant, and β is the ratio of marginal revenue to price. In the special case where marginal cost is zero and the elasticity of demand is a constant, the rate of change in price under monopoly is the same as under perfect competition. This means that, under these assumptions, if the initial stock of the monopolist is the same as that of a competitive industry, the extraction path of the monopolist is identical to that of a competitive industry. (However, if we take into account that discovered stocks depend on search efforts, a monopoly may restrict search effort in order to restrict final output and raise prices.) On the other hand, if demand becomes more and more inelastic as one moves down the demand curve (e.g. if the demand function is linear) then the monopolist's optimal total accumulated output at any time is lower than that of a perfectly competitive industry. This is the case that justifies the saying that the monopolist is the conservationist's best friend.

If there is uncertainty of tenure, such as the possibility of nationalization, the rate of interest should be adjusted upward to reflect the fact that leaving the resource in the ground becomes a risky investment, as the owner has no guarantee that he will be able reap the rewards of that investment. Alternatively, uncertainty can take the form of productivity disturbances in the resource-extracting sector and in the final-good production sector. In this case, Hotelling's Rule must be modified as follows: *the expected rate of appreciation of the in situ price of the resource, ψ, should be equated* *to the sum of the riskless interest rate and the weighted covariance of the rate of change in the in situ resource price and the rate of change of consumption, where the weight is the coefficient of relative risk aversion*

$$\left(\frac{1}{dt}\right)\frac{\mathrm{E}d\psi}{\psi} = r + A(y)\mathrm{Cov}(y,\psi)$$

This modified rule can be derived using stochastic calculus. Generalizing to the case of many risky assets, let M be a portfolio that has a non-zero covariance with consumption. Then the rate of return on M , denoted by μ satisfies the equation

$$\mu = r + A(y)\mathrm{Cov}(y,M)$$

Thus when we take into account the desire to diversify risks, the Hotelling's Rule becomes:

$$\left(\frac{1}{dt}\right)\frac{\mathrm{E}d\psi}{\psi} = r + (\mu - r)\left(\frac{\mathrm{Cov}(y,\pi)}{\mathrm{Cov}(y,M)}\right)$$

This equation tells us about the expected rate of change in the price of the resource stock, not about the price p of the extracted resource. The latter is a weighted average of the former and the expected rate of change in marginal cost:

$$\left(\frac{1}{dt}\right)\frac{\mathrm{E}dp}{p(t)} = \left(1-\frac{c(t)}{p(t)}\right)\left(\frac{1}{dt}\right)\frac{\mathrm{E}d\psi}{\psi} + \left(\frac{c(t)}{p(t)}\right)\left(\frac{1}{dt}\right)\frac{\mathrm{E}dc}{c(t)}$$

Empirical tests of Hotelling's Rule that take into account seriously the possibility of diversification thus require data on asset prices. A few tests that do use asset prices seem to support the general version of Hotelling's Rule.

So far, we have assumed that the marginal cost of extraction is independent of the size of the remaining stock. However, there are situations where, as the stock dwindles, the marginal extraction cost rises. Under these conditions, Hotelling's Rule must be modified. For example, let X denote the stock that remains, and assume that the cost of extracting q is $c(X)q$. Then $c(X)$ is the marginal extraction cost. Let ψ denote net price, i.e. price net of marginal extraction cost. Application of optimal control theory shows that it must be true that efficiency implies that net price rises at a rate less than the rate of interest:

$$\dot{\psi} = r\psi + c'(X)q < r\psi$$

Hotelling's Rule must also be modified if as extraction proceeds the firms gain information about the true size of the reserves. c' is the first derivative of c with respect to X

2.2. Herfindahl's Rule

If the resource-extracting industry is perfectly competitive, and deposits differ only with respect to marginal extraction costs, which are assumed to be independent of extraction rates, it can be shown that the order of extraction must obey Herfindahl's Rule: *exhaust the least cost deposit before moving on to the second lowest cost one, and so on*. The equilibrium price path is continuous even at points of transition from one deposit to the next.

There are exceptions to this rule. If there are constraints on labor supply, and if the economy also has a renewable resource, with higher labor requirement per unit of extraction, the optimal order of extraction may not follow Herfindahl's Rule. Within limits, the order of extraction can be a matter of indifference.

Another exception is that if a set-up cost must be incurred before one can exploit a deposit, it is no longer the case that the price path is continuous. In fact, the price will jump down each time after the set-up cost is incurred. The general time path of price can thus display the saw-tooth pattern (Hartwick, Kemp and Long, 1986).

Herfindahl's Rule is based on the assumption that deposits differ only with respect to their marginal extraction costs. If they differ in other dimensions as well as in marginal costs, it is clear that the rule must be modified. One kind of cost which is not included in the standard definition of marginal cost is the potential of supply interruption. In general extraction costs must be defined to be net of the benefits that extraction might convey. One of these benefits is information. Consider the following simple model.

Assume we have two mines. The first mine consists of two layers, and one must exhaust the first layer before reaching the second one. The size of the first layer is A, a known positive number. The size of the second layer is a random variable X which can be zero or x, also a known positive number. (Here we use the convention that the capital letter X denotes a random variable, while the lower case x is an actual value.) The subjective probability that $X = x$ is $p > 0$ and the subjective probability that $X = 0$ is $1 - p > 0$. We assume that the uncertainty is resolved as soon as the first layer, A, is exhausted. Assume the resource is not storable after extraction. If the decision maker wants to advance the date at which information becomes available, he must extract at a faster rate. How fast should the planner exhaust A?

Mine 2 also has two layers, B and Y, where Y is a random variable that can take on one of two possible values y or 0 (with probabilities q and $1-q$ respectively.) Except in singular cases, $p \neq q$, $A \neq B$ and $x \neq y$. Assume extraction cost is $c = 0$. Under what condition would it be optimal to exhaust A before extracting B?

It turns out that in the case where the demand curve exhibits constant price elasticity of demand, if $A = B$ and $x = y$ then A should be exhausted first if $p > q$. This is because the value of information from the extraction of A is higher than from B. Similarly, if $A = B$ and $p = q$ then A should be exhausted first if $x > y$.

The situation becomes more complicated if some information arrives while the firm is in the process of extracting the first layer. A simple way of modeling this is as follows.

The owner's subjective probability numbers, p and q, are only ex-ante, or preliminary, probabilities. When he is in the process of extracting layer A, he receives news that allows him to update his p. To make things as simple as possible, suppose that, for mine 1, there exists a number α (where $0<\alpha<1$) such that after the fraction α of A is used up, he will be able to revise p upwards, to $p+\delta$, or downwards, to $p-\delta$. (We restrict δ so that $0<p-\delta<p+\delta<1$.) Before αA is exhausted, he does not know if the revision is going to be upwards, or downwards. He only knows that the probability of upward revision is π and that of downward revision is 1-π. A similar situation applies to mine 2, with corresponding parameters, e.g., β instead of α, $q-\delta$ instead of $p-\delta$, and so on.

How should the planner proceed? How fast should the fraction αA? And should he begin with βB instead of αA? It turns out that in this case there are a large number of extraction strategies. Numerical solutions to determine optimal strategies can be found, but analytical answers are hard to come by.

-
-
-

Bibliography

Amigues, J.P., p. Favard, G. Gaudet and M. Moreaux (1998). On the optimal order of natural resource use when the capacity of the inexhaustible substitute is limited. *Journal of Economic Theory* **80**(1), 153-170. [A model on the optimal order of resource exploitation].

Brander, J.A., and M.S. Taylor (1998). The simple economics of Easter Island: A Ricardo-Malthus model of renewable resource use. *American Economic Review* **88**(1), 119-138. [A model of resource exploitation and population dynamics that explains the rise and fall of civilization in Easter Island].

Clark, C. W. (1990). *Mathematical Bioeconomics: The Optimal Management of Renewable Resources*, 350 pp. Wiley, NY. USA: [A comprehensive treatment of the theory of management of renewable resources].

Clemhout, S. and H. Wan, Jr..(1985). Dynamic common property resources and environmental problems. *Journal of Optimization Theory and Applications* **46**, 471-481. [A model of renewable resource exploitation, with applications to fishery].

Dockner, E., S. Jorgensen, N.V. Long, and G. Sorger (2002). *Differential Games in Economics and Management*, 350 pp. Cambridge, UK: [A comprehensive treatment of dynamic games, with applications to the economics of resources and environment, as well as industrial organization and management].

Dockner E. and , N.V. Long (1993). International pollution control: cooperative versus non-cooperative strategies. *Journal of Environmental Economics and Management* **24,** 13-29.[This has a full discussion of non-linear Markov-perfect equilibria in a linear-quadratic differential game of transboundary pollution].

Gaudet, G..(2007) Natural Resource Economics under the Rule of Hotelling, Canadian Journal of Economics, to appear. [A comprehensive treatment of Hotelling's Rule].

Hartwick, J. M., M. C. Kemp, and N.V. Long (1986). Set-up costs and the theory of exhaustible resources. *Journal of Environmental Economics and Management*, **12** [A model on the discontinuities in the time path of the price of an exhaustible resources when set-up costs must be incurred before extraction].

Kemp, M.C. and N.V. Long, (1980). *Exhaustible Resources, Optimality and Trade,* 254pp. North Holland, Amsterdam [A collections of essays on the economics of resources its relationship to the theory of international trade].

Kemp, M.C. and N.V. Long, (1980). On two folk theorems concerning the extraction of exhaustible resources. *Econometrica* **48**(3), 663-674 [This presents shows that Herfindahl's order of exploitation may not hold under a general equilibrium formulation].

Levhari, D. and L. Mirman, (1980). The great fish war: an example using a dynamic Nash-Cournot solution. *Bell Journal of Economics* **11,** 322-334 [This is a classic paper on dynamic fish wars, using dynamic programming].

Long, N. V. (1975). Resource extraction under the uncertainty about possible nationalization. *Journal of Economic Theory* **10** , 42-53. [This is a model of a mining firm that faces the risk of future nationalization]

Long, N. V. and G. Sorger (2006). Insecure property rights and growth: the roles of appropriation costs, wealth effects, and heterogeneity. *Economic Theory* **28**(3) , 513-529. [This paper is a dynamic model of the tragedy of the common when agents can accumulate private assets]

Salo, S., and O. Tahvonen, (2001). Oligopoly equilibria in nonrenewable resource markets. *Journal of Economic Dynamics and Control* **25** , 671-702. [This paper analyzes a dynamic game between two resource-extracting firms]

Taylor, M. S.(2007). Buffalo hunt: international trade and the virtual extinction of the North American bison. *National Bureau of Economic Research NBER working paper 12969*. www.nber.org/papers/w12969. [This paper is dynamic model of the effect of international trade on the virtual extinction of the bison]

Biographical Sketch

Ngo Van Long obtained his Bachelor of Economics (First Class Honors) from LaTrobe University (Melbourne, Australia) in 1972, and his Ph.D. in economics from the Australian National University (Canberra, Australia) in 1975.

He is currently holder of the James McGill Chair in economics at McGill University, Montreal, Quebec, Canada. Before joining McGill, he was professor of economics at the Australian National University, and taught there for more than a decade. He has published over 120 articles in international journals, including Econometrica, Quarterly Journal of Economics, Review of Economic Studies, American Economic Review, Journal of Economic Theory, Economic Journal, Economic Theory, Journal of Environmental Economics and Management, Journal of Public Economics, Journal of International Economics, International Economic Review, Journal of Economic Dynamics and Control. He is the co-author of several books. His research interests include economics of natural resources and environment, international trade theory, economic dynamics, and dynamic games.

Dr. Long is member of the American Economic Association and the Canadian Economic Association. A former co-editor of the Canadian Journal of Economics, he is currently an associate editor of Review of International Economics and is on the Editorial Boards of several journals, including Mathematical Social Sciences and Journal of Environmental Economics and Management.