ADAPTIVE SYSTEMS

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Summary

In order to simulate living systems, and specifically computational intelligence, a general theory of Adaptive Systems, which change its behaviour through interaction with its environment, has to be developed.

The article begins introducing some general concepts: controllability (relative to the possibility to arrive to some state from another through some series of actions), fulfilment of goals (distinguishing between optimistic, pessimistic, statistical and fuzzy optimal decisions and satisfactory decisions) and strategy of decision with a certain depth of memory.

The article continues with a survey of a general theory of learning. After a reference to deterministic approaches, it focuses on a probabilistic approach, by using the general informational concept of entropy and developing the concepts of oriented and self-organised learning.

The article specifically develops some models of probabilistic learning, beginning with a general linear model (with a goal function and a function of accumulation of

memory), which will be the basis of the other models. After, a reciprocal linear model (in order to simulate the interaction between different systems with learning) is studied. And finally, an adaptive linear model (to choose between different strategies of decision) is designed. These models are applied to the problem of the dilemma of the prisoner, both single as iterative one.

Finally, the article studies the problem of the anticipatory adaptation, in order to simulate the capacity of forecasting and planning, showing a specific model of it. And a general model of social evolution through a probabilistic learning is introduced, by considering different processes of social repression, scientific communication, personal resignation and ecological adaptation.

1. Introduction

In order to simulate living systems, and specifically computational intelligence, a general theory of Adaptive Systems, which change its behavior through interaction with its environment, has to be developed.

Adaptive Systems will be a particular class of Dynamical Systems. A Dynamical System is characterized by some input, output and state variables, so that:

(a) Input variables represent the actions from the environment on the System, output variables represent the actions from the System on the environment, and state variable express the actual situation of the System, in which some relation between input and output is established.
(b) The evolution of the output variables from a temporal instant depends on the state of the System in this instant and the evolution of the input variables from this instant.
(c) The transition of the state from one to another instant depends on the initial state and the evolution of the input variables between these two states, so that the composition of two transitions of states is corresponding with the concatenation of the evolutions of the input variables.

We'll have to consider, first, the conditions in which Dynamical Systems can evolve to arrive to some final state or *goal*. And, second, the conditions in which the path to this goal can change to being adapted to the changes of the environment. We'll speak about Adaptive Systems when the same System can make autonomously this adaptation. (See Dynamical Systems)

2. Controllability

We will say that a Dynamical System is *completely controllable* is and only if from any state it can arrive to any other state, through some series of inputs.

Note that the complete controllability is a symmetric concept, and therefore it is only applicable in reversible processes. In irreversible processes we have to use other concepts:

We will say that a Dynamical System is *controllable from a state* if it is possible to arrive to any state from that state. Of course, this concept is compatible with irreversible processes in that would be not possible to return to such initial state.

On the contrary, we will say that a Dynamical System is *controllable toward a state* if it is possible to arrive to this state from any state. This concept emphasizes the search of a series of inputs in order to arrive to some goal.
Of course, if a Dynamical System were simultaneously controllable toward and from the same state, it would be completely controllable: it would be enough to search a path that passes through this state. But this is not a realistic situation relative to Adaptive Systems: on one hand, real complex processes are usually irreversible, according to the Second Principle of Thermodynamics; on the other hand, we are not really interested in arriving to any state, but only to some state or states, which define the goal or goals of the System. (See Second Principle of Thermodynamics)

3. Fulfillment of Goals

Now then, in many cases, the System won't be able to fully reach and persist in a goal state, and we have to evaluate the degree of *fulfillment* of a goal. Likewise, we can distinguish between controlled inputs (*decisions*) and uncontrolled inputs from the environment (perturbations).

We'll have to define an ordered set V of values of fulfillment of a goal, so that a minor value expresses a better fulfillment (if we introduced a metric, this value would express a possible distance to the goal), and a *goal function* **g** such that, for any decision **x** and any perturbation ω, give us a value $g(x,\omega)$ of fulfillment of a goal.

In absence of perturbations, o with constant perturbations, we can univocally define an *optimal* decision as such decision that make minimal the fulfillment of the goal. And, if the number of possible decisions is finite, there will be some optimal decision (although perhaps it weren't unique). But if there are perturbations we can realize different approaches:

We will say that a decision is *optimistically optimal* if and only if it carries, for some perturbation, to the better possible fulfillment of the goal.

We will say that a decision is *pessimistically optimal* if and only if its worst fulfillment of the goal is better or equal than the worst fulfillment with any other decision.

Of course, if both the number of decisions and the number of possible perturbations are finite, then there are some optimistically optimal decision and some pessimistically optimal decision.

We also can introduce a global approach if we have a measure μ on the set Ω of perturbations and the function **g** is M-sumable on this set, that is to say, we can calculate

$$\bar{g}(x) = \sum_{\omega\in\Omega} g(x,\omega)\mu(\omega).$$

Thus, the optimal decision relative to the measure μ will be the decision **x** that makes

$\overline{g}(x)$ minimal. This decision will exist if the number of decisions is finite.
If μ is a measure of probability we'll speak about a decision *statistically optimal*. If μ is a (fuzzy) measure of possibility we'll speak about a decision *fuzzily optimal*.

We can also define a decision as *regularly optimal* if it carries to the better possible fulfillment of the goal for any perturbation.

This criterion is stronger: if a decision is regularly optimal, this will also be optimistically, pessimistically, statistically and fuzzily optimal: that will be the best in the better situation, and not so bad in the worst situation (see Figure 1). But a regularly optimal decision not always exists.

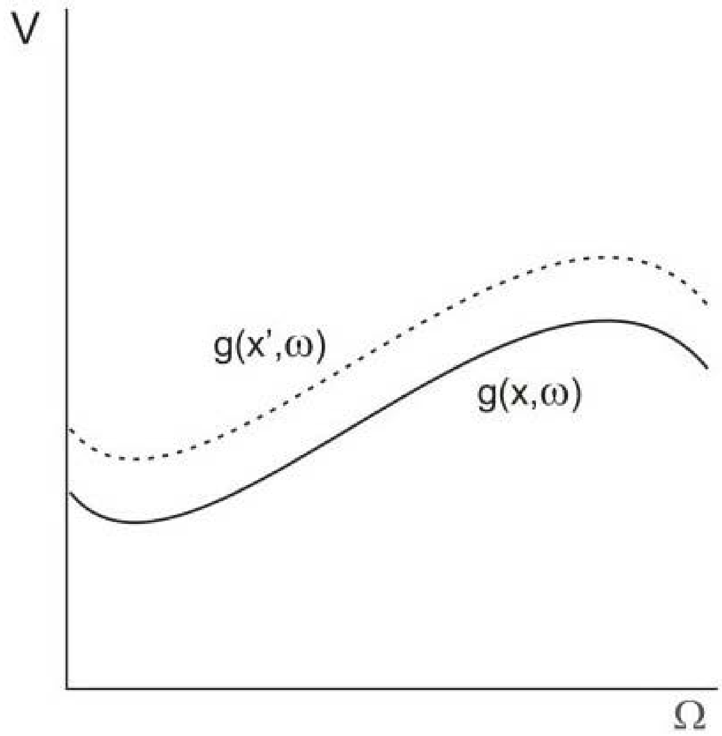

Figure 1. A regularly optimal decision *x*

If a regularly optimal decision does not exist, we can prefer an intermediate option, which we express through a function of *tolerance* τ, which gives as the level of admissible fulfillment of the goal for each perturbation. Thus, we will say that a decision is *satisfactory* if it carries, for any perturbation, to a fulfillment of the goal not worse than the corresponding level of tolerance.

Furthermore, note that a regularly optimal decision may be not satisfactory, if any decision does not guarantee a tolerable fulfillment of the goal: "optimal" not always means "good"!

Also, although there were any satisfactory decision, the optimistically optimal decision and, if so, the statistical or the fuzzily optimal decision may be not satisfactory: perhaps for some "bad" perturbation they do not carry to a tolerable fulfillment of the goal! (see

Figure 2.a). And only if the function of tolerance is constant we can state that the pessimistically optimal decision will be in the number of satisfactory decisions: with variable tolerance, perhaps the worst perturbation for the pessimistically optimal decision corresponds to a relatively "intolerant" perturbation! (see Figure 2.b).

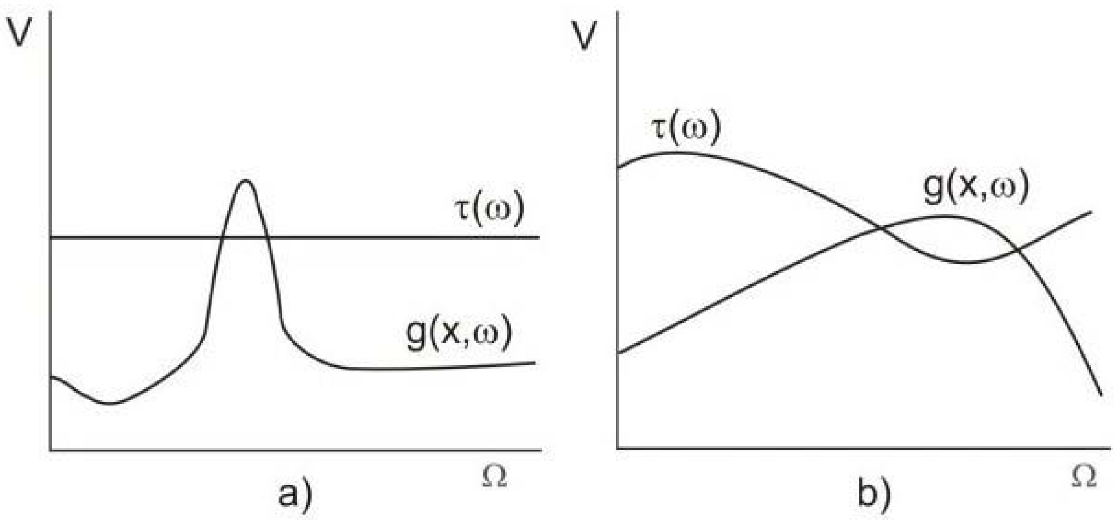

Figure 2. Possible functions of tolerance (τ) and fulfillment (*g*).

Now then, if there is any satisfactory decision and a regularly optimal decision, this **will be** satisfactory: this is better in any case! (See Fuzzy Set, Possibility)

4. Strategies of Decision

But an Adaptive System cannot be a System of one-only-decision. It will have to adopt different decisions according to different circumstances, and specially according its past experience. Particularly, with a discrete temporal transition, if U is the set of possible inputs to the System in each instant, X is the set of its possible outputs/actions/decisions, and **m** is a natural number, we can name *strategy of decision* with a depth of memory **m** to any function **i** which determines a decision **x** for each sequence of previous inputs **u**, that is to say, $i: U_m \to X$. Thus, the decision in the instant t will be given by $x(t) = i(u(t') : t' = t\text{-}1, t\text{-}2, ..., t\text{-}m)$.

Note that the present input **u(t)** can include part of the past perturbation ω or decision **x**. Nevertheless, we will suppose that the System doesn't "know" this present perturbation, and therefore it doesn't directly influence the present decision, although it can determine its consequences in the fulfillment $v = g(x,\omega)$ of its goal. On the other hand, the perturbation ω can include components which were unknown also in the past and weren't included in the input, and the fulfillment v can be later known and be included in the input **u**. We can represent these possible relations in Figure 3.

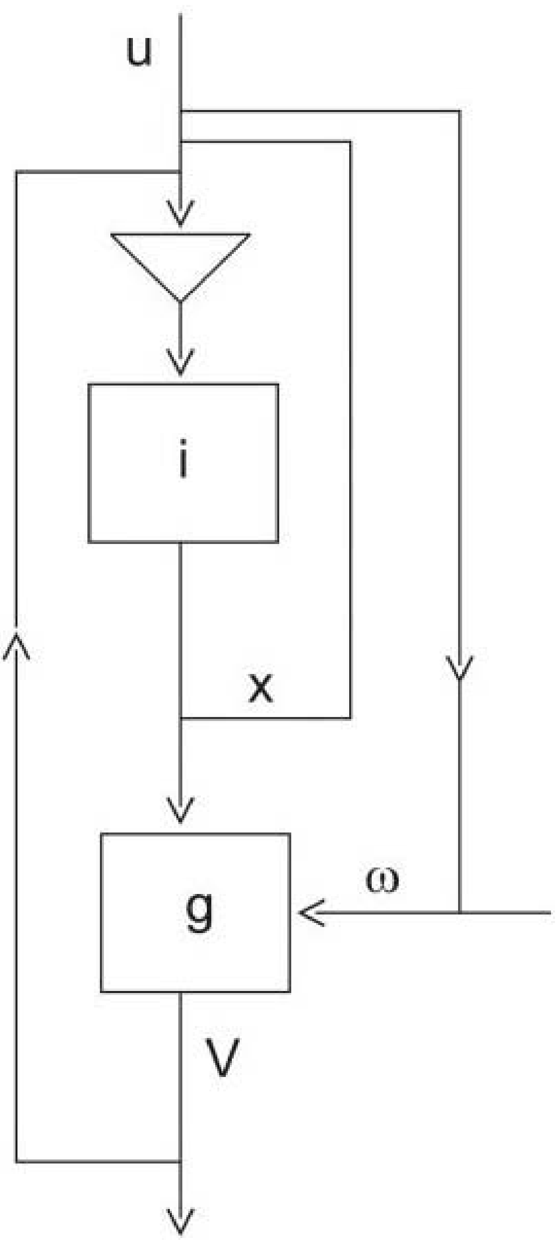

Figure 3. Strategy of decision and fulfillment of goal.

The particular case $U = \Omega = X$ is especially interesting for social or living Systems which are "patients" of the actions of similar Systems. In this case, a strategy of decision will be $i: X_m \to X,$ where **m** indicates the depth of memory.

If the System discriminates between **n** different similar Systems interacting with it, its actions can also discriminate between them, and so we can decompose its actions **x** in different components, $y_1, y_2, \ldots, y_n$, in front of its different partners. So, we would have to take $X=Y^n$, $U=X=Y^n$, and a strategy of decision will be $i: (Y^n)^m \to Y^n,$ where the input $\omega=(\omega_1, \omega_2, \ldots, \omega_n) \in Y^n$ indicates the "perturbations" from the actions of its different partners, and the output $x=(y_1, y_2, ..., y_n)\in Y^n$ indicates the actions toward its different partners.

Other possibility, as we have underlined, is $U=\Omega\times X$, that is to say, that input included past perturbation and decision. If the depth of memory is m=1, a strategy will be $i:\Omega\times X\rightarrow X$, and the future decision will be $x'=i(\omega,x)$. Some of these strategies can be equivalent to strategies of the type $i:X\times V\rightarrow X$, that is to say, with $x'=i(x,g(x,\omega))$, in which the future decision depends directly on the past decision and its corresponding fulfillment of the goal. Such strategy can be an Adaptive Strategy itself: for example, it can state that, if the fulfillment of the goal is "tolerable" $(g\leq\tau)$, the System repeats its decision, and change it on the contrary.

But, furthermore, in an Adaptive System its strategy of decision itself will have to change in order to being adapted to the changes of the environment through a process of learning from the experience. Note that the object of this learning is the strategy of decision, not the isolate decisions.

5. General Theory of Learning

Neural Networks provide a tool to model a deterministic learning through the change of coefficients. But we can do a more general probabilistic approach, by defining the *Learning* of a variable U in a System by means of is increase of Information, or, that is equivalent, its decrease of Shannon Entropy, $H(U) = -\Sigma_u P(u) \log_2 P(u)$, where P(u) is the probability of that the variable U valued u.

If I is a variable for the type of Systems, and for each value i of I there is a distribution of conditional probabilities P(x|i) of actions x, we'll define the corresponding *Conditioned Entropy* $H(X|I) = \Sigma_i H(X|i) P(i)$ of the population of Systems, where H(X|i) is the Shannon Entropy of this distribution of conditional probabilities and P(i) is the probability of the type i in this population.

Now, from the Bayes Theorem P(x|i)= P(x,i)/P(i), we arrive to the relation H(X,I) = H(X|I) + H(I). Note that if the distribution of types I remains constant in the population, only H(X|I) can change, through a change of the conditional probabilities for each individual System. But if these conditional probabilities remain constant, only H(I) can change, through a change of the distribution of types I in the population.

We'll name *Learning by Selection* to the decrease of the Shannon Entropy of the distribution of the types I of Systems in the population, H(I). And we'll name *Autonomous Learning* to the decrease of the Conditioned Entropy into the individual Systems, H(X|I).

We'll say that a *Learning is Oriented* toward a goal if the actions which carry to a better fulfillment of this goal increase its probability, and the actions which carry to a worst fulfillment of the goal decrease its probability.

Learning by Selection is tautologically oriented to the increase of the number of Systems of the same type: this increase is the same goal of the Learning, by promoting the survival and reproduction of Systems and avoiding its destruction. The corresponding increase of Information is accumulated in the whole of the population, through the distribution of the different types of Systems.

On the contrary, an Autonomous Oriented Learning requires a specific tool or memory accumulator to keep the learned information. In this case, we'll speak about *Self-organized Learning*.

Note that this General Theory of probabilistic Learning can be applied to processes of very different nature, like the biological evolution of Species, the psychological learning and the social evolution, including the evolution of the Science.
(See Neural Networks)

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Biographical Sketch

Rafael Pla-Lopez is a teacher of Applied Mathematics in the Universitat de València. His initial formation was in Physics, but its Ph.D. was in Mathematics. He has worked in Epistemology, Systems Theory and Learning Theory. He is also interested in the political evolution in his country and in the world, and had published many articles about this theme. In the present, he is focused on the development of mathematical models of Social Evolution and is a member of the Board of the Spanish Society of General Systems (SESGE). You can find full information about its works and activities in his web page in http://www.uv.es/~pla