

## QUANTUM PHENOMENA IN LOW-DIMENSIONAL SYSTEMS

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## Summary

A brief introduction to the physics of low-dimensional quantum systems is given. The material should be accessible to advanced physics undergraduate students. References to recent review articles and books are provided when possible.

## 1. Introduction

A low-dimensional system is one where the motion of microscopic degrees-of-freedom, such as electrons, phonons, or photons, is restricted from exploring the full three dimensions of our world. There has been tremendous interest in low-dimensional quantum systems during the past twenty years, fueled by a constant stream of striking discoveries and also by the potential for, and realization of, new state-of-the-art electronic device architectures.

The paradigm and workhorse of low-dimensional systems is the nanometer-scale semiconductor structure, or semiconductor “nanostructure,” which consists of a compositionally varying semiconductor alloy engineered at the atomic scale. Traditionally one would not include naturally occurring low-dimensional entities such as atoms and molecules in the subject of this chapter, but some of the most exciting recent developments in the field have involved the use of molecules and even biologically important materials and has blurred the boundaries between the subject and other physical and life sciences. In addition, there are systems of great interest in physics, such as high-temperature superconductors, where the effects of reduced dimensionality are believed to be essential, and these too will be regarded as low dimensional. Many of the subjects covered here are central to the currently fashionable fields of nanoscience and nanotechnology.

The study of low-dimensional quantum phenomena has led to entirely new fields of research, such as the physics of mesoscopic systems, which will be discussed below. And low-dimensional systems have shed new light on the difficult questions of how disorder (impurities, for example) and electron-electron interaction affect a quantum system. In fact, understanding the combined effects of disorder and interactions in condensed matter systems is currently a problem of enormous interest.

How are electrons, say, restricted from moving in three dimensions? The answer is *confinement*. Take, for example, an electron inside a long wire: The positively charged ions in the wire produce an electric field that prevents the electrons from escaping. Often, in fact, one can regard the electrons as being subjected to a hard-wall potential at the wire’s surface. The electronic eigenstates are given by a plane wave running along

the wire multiplied by a localized function in the transverse directions. For a range of low energies the eigenstates have the same transverse eigenfunction and only the plane wave factor changes. This means that motion in those transverse directions is “frozen out,” leaving only motion along the wire.

This chapter will provide a very brief introduction to the physics of low-dimensional quantum systems. The material should be accessible to advanced physics undergraduate students. References to recent review articles and books are provided when possible. The fabrication of low-dimensional structures is introduced in Section 2. In Section 3 some general features of quantum phenomena in low dimensions are discussed. The remainder of the chapter is devoted to particular low-dimensional quantum systems, organized by their “dimension.”

## 2. Making Low-Dimensional Quantum Structures

The most common method of fabricating low-dimensional structures is by “growing” compositionally graded semiconductor alloys in high-vacuum molecular-beam epitaxy (MBE) machines. Take, for example, the homogeneous alloy  $\text{Al}_c\text{Ga}_{1-c}\text{As}$  consisting of a periodic array of arsenic atoms together with a fraction  $c$  of aluminum and  $1-c$  of gallium. The special cases of  $c=0$  and  $c=1$  correspond to the crystalline semiconductors GaAs and AlAs, each with a distinct band structure. The microscopic potential produced by the alloy  $\text{Al}_c\text{Ga}_{1-c}\text{As}$ , although not strictly periodic, may be regarded as producing a band structure interpolating between that of GaAs and AlAs. In particular, the energy gap between the valence and conduction bands varies with  $c$ .

Structures that confine electrons are made by changing the aluminum fraction  $c$  during crystal growth, leading to a compositionally graded alloy of the form  $\text{Al}_{c(\mathbf{r})}\text{Ga}_{1-c(\mathbf{r})}\text{As}$ , where  $c$  varies spatially. The resulting band structure variation produces a spatially varying conduction band minimum. Hence, an electron added to the conduction band through doping, optical excitation, or electrical injection, sees a position-dependent potential. By varying  $c$  appropriately, one can engineer confining potentials that restrict electron motion to fewer than three dimensions.

In practice, however, it is possible to vary  $c$  in one direction only, resulting in, at best, a two-dimensional system. To complete the construction of a semiconductor nanostructure it is often necessary to follow growth with lithography, the selective etching of prepared surfaces. After coating a surface with a protective material, the “resist,” patterns are imprinted on the resist in a photographic process with focused light, electron beams, or even atoms. After the imprinted pattern is removed chemically, the underlying semiconductor is etched away, leaving an environment that confines electrons both in the MBE-growth direction and laterally (perpendicular to the growth direction). Another common way to produce lateral confinement is to use lithographic techniques to pattern metallic electrodes, or “gates,” on the surface of a crystal (grown by MBE, for example) that has electrons confined in a buried two-dimensional layer parallel to that surface. By applying voltages to these electrodes, electrons in the layer can be depleted from or attracted to the regions below the electrodes.

Finally, it is in many cases necessary to attach electrical contacts to the electron gases inside these nanostructures and to the metal gates on their surfaces.

The lower limit to the size of the structures one can make is usually determined by the size of the patterns one can make with lithography, which, in turn, is usually determined by the quality of the image formed during the exposure stage. At the time of this writing the semiconductor industry can produce, in an integrated circuit, wires with a thickness of only 10 nm, more than 10 times smaller than the wavelength of visible light.

To make even smaller structures naturally occurring nanometer-scale systems have been used. Examples include self-assembled nanostructures and structures incorporating molecules or biological materials. A self-assembly technique to make arrays of few-nanometer quantum dots (small “artificial atoms” where electrons are confined in all three dimensions) by growing a thin layer of highly strained material on top of another crystal has been particularly successful. The strained layer relaxes by breaking into small islands, which form the quantum dots. An excellent example of the use of molecules to make low-dimensional systems is provided by large carbon sheets that can be rolled into stable hollow spheres, known as Bucky balls, or into carbon nanotubes. It is even possible to put other molecules inside a Bucky ball, to arrange Bucky balls into a molecular crystal, and to electrically contact individual nanotubes. Electrical contacts have also been attached to other molecules, turning them into “transistors.” Polymers and even DNA strands have been used to engineer quantum structures at the nanometer scale.

### **3. Physics in Quantum Systems of Reduced Dimensions**

Physics in low-dimensional systems is often different than that in three dimensions. We now discuss some of these differences and the theoretical tools used to understand them.

#### **3.1. Effective Mass Theory**

Electrons in semiconductor nanostructures move in the presence of as many as three fields; the periodic or nearly periodic potentials produced by the atoms in the crystal, fields applied externally, and the electron-electron interaction potential. The atomic potentials, which vary at the few Angstrom scale, are usually varying much more rapidly than the others. In this case, assuming the electronic states have energies near the bottom of the conduction band or near the top of the valence band, there is an extremely useful description whereby the original problem of an electron moving in the presence of the atomic as well as other potentials is replaced by the much simpler problem of an electron, now with a different mass, moving in the presence of the slowly varying fields only. For example, the effective mass of an electron in a GaAs conduction band is about 0.067 times the ordinary bare mass. In general, the effective mass depends on the electron’s propagation direction, and can even vary with position.

#### **3.2. Density of States**

An important distinction between systems with different dimensionality is their density of states  $N(\varepsilon)$ , which is the number of states per unit “volume”  $L^D$  in an energy range  $\varepsilon$  to  $\varepsilon + d\varepsilon$ , divided by  $d\varepsilon$ .  $L$  is the linear size of the system, and  $D$  can be either 1, 2, or 3. (The zero-dimensional case has to be treated separately). A simple general formula can be derived to determine the energy dependence of  $N(\varepsilon)$ : Given an excitation (electron, phonon, photon) with dispersion relation  $\varepsilon(k) \propto |k|^\alpha$ , the number of states contained within a  $D$ -dimensional sphere of radius  $k$  in momentum space is proportional to  $(L/2\pi)^D k^D$  and the number per unit volume is therefore proportional to  $k^D$  or  $\varepsilon^{D/\alpha}$ . The density of states is evidently the derivative of the latter with respect to energy, so  $N \propto \varepsilon^{\frac{D}{\alpha}-1}$ . Phonons and photons ( $\alpha = 1$ ) in three dimensions have  $N \propto \varepsilon^2$ , and in lower dimensions would have  $N \propto \varepsilon^{D-1}$  as long as the dispersion remains linear. Electrons ( $\alpha = 2$ ) have a density of states proportional to  $\varepsilon^{\frac{1}{2}}$ ,  $\varepsilon^0$  (i.e., energy independent), and  $\varepsilon^{-\frac{1}{2}}$  in 3, 2, and 1 dimensions, respectively. The expressions for  $D = 2$  and  $D = 1$  assume only one transverse mode or sub-band present. When more are present, these additional branches simply add to  $N(\varepsilon)$ .

### 3.3. Mesoscopic Physics

Mesoscopic physics is an exciting new field of science made possible by nanostructures. A mesoscopic system is one that is in some sense between a microscopic and macroscopic system. It is typically much larger than a few atoms or molecules, yet it is small enough that the degrees-of-freedom (usually electrons) have to be regarded as fully quantum-mechanical. More precisely, a mesoscopic system has a size  $L$  that is larger than a microscopic length scale  $a$  (for example the Bohr radius), yet smaller than the phase-coherence length  $L_\phi$ , which is the characteristic length beyond which a particle loses phase coherence.  $L_\phi$  generally depends on the dimension and the temperature of the system, as well as on microscopic details. Thus, in simple terms, a mesoscopic system is one that is larger than microscopic and in which quantum mechanics manifests itself fully. In what follows we shall discuss a few examples and concepts from mesoscopic physics.

#### 3.3.1. Aharonov-Bohm Effect

In 1959 Yakir Aharonov and David Bohm predicted that a magnetic field can influence the quantum-mechanical phase of charged particles, even if the particles are prevented from entering the region containing the magnetic field. This proposal, although a direct consequence of quantum mechanics, violated everyone’s intuition and was extremely controversial at the time. The Aharonov-Bohm effect has been observed hundreds of times and shows up everywhere in the study of mesoscopic systems. Excellent accounts of it are given in quantum mechanics textbooks.

The basic idea is that a charged particle moving around a closed loop of circumference  $L$  accumulates, along with its kinematical phase  $2\pi L/\lambda$ , where  $\lambda$  is the deBroglie

wavelength of the particle, an additional Aharonov-Bohm phase given by  $2\pi\Phi/\Phi_0$ , where  $\Phi$  is the magnetic flux enclosed by the ring and  $\Phi_0 \equiv hc/e$  is the quantum of magnetic flux. The Aharonov-Bohm phase changes the energies of charged particles and also can shift quantum interference patterns. But the effect can only be observed if the particles are sufficiently phase coherent.

### 3.3.2. Persistent Currents

A simple example of a mesoscopic effect is as follows: Take a small metal or semiconductor ring of circumference  $L$ . The mesoscopic ring can be made using the lithographic techniques described above. The ring is an ordinary conductor, not a superconductor. It seems reasonable and is usually the case that in the absence of a magnetic field the ground state of the ring is non-current-carrying. However, if the phase-coherence length  $L_\phi$  is larger than  $L$  then by threading the ring with a magnetic flux  $\Phi$  the ground state (or finite-temperature equilibrium state) becomes a current-carrying state, and current will flow around the ring without ever dissipating, even in the presence of disorder. The electrons in the ring are affected by the magnetic field even though they are never in contact with it, a consequence of the Aharonov-Bohm effect.

Nondissipative current-carrying states can occur in a superconducting ring too, but in the superconducting case the current-carrying state is not, strictly speaking, an equilibrium state, but is instead an extremely long-lived metastable nonequilibrium state.

As the ring becomes larger, the magnitude of the persistent current decreases. One reason is that when  $L$  exceeds  $L_\phi$  the electrons no longer are moving coherently around the ring and the Aharonov-Bohm effect is no longer operative. The second reason is that even if  $L_\phi$  was always larger than  $L$ , the magnitude of the maximum persistent current decreases as  $1/L$ .

### 3.3.3. Phase-Coherent Transport

Electrons in a mesoscopic conductor (a conductor whose dimensions are smaller than  $L_\phi$ ) move as a wave, not as a particle. The behavior is similar, in fact, to an electromagnetic wave propagating inside a wave guide. This wave-like nature leads to many unusual physical properties.

Perhaps the most profound is the origin of resistance in mesoscopic conductors. Ordinarily resistance is caused by *inelastic* collisions of the current-carrying electrons with disorder (impurities and other imperfections), other electrons, and with lattice vibrations (phonons). In a mesoscopic conductor, however, an electron typically travels the entire length of the system without undergoing an inelastic collision. Thus it might seem that there would be no resistance at all. But to measure the resistance in a mesoscopic system one has to attach electrical contacts or leads to it, which are macroscopic. What occurs, then, in a mesoscopic conductor, is that the resistance is caused by a combination of *elastic* scattering of electron waves from disorder and

inelastic scattering in the macroscopic leads, although the magnitude of the resistance is determined exclusively by the former.

These phenomena are succinctly described by a formula, originally proposed by Rolf Landauer in 1957, for the conductance  $G$  (reciprocal of the resistance) of a mesoscopic system. Landauer's formula is simply  $G = |t|^2 e^2/h$ , where  $t$  is the quantum transmission amplitude for an electron to propagate through the system ( $|t|^2$  is the transmission probability). The ratio  $e^2/h$  of fundamental constants has dimensions of conductance and is about  $1/26k\Omega$ . The Landauer formula shows that the conductance of a mesoscopic conductor is directly related to the quantum mechanical probability that an electron can propagate through the system without scattering elastically. It makes no reference to the strength of inelastic scattering that actually causes the energy dissipation normally associated with a resistor.

Another important consequence of the phase-coherent nature of electron transport in a mesoscopic system is a phenomenon called weak localization. To understand weak localization it is first necessary to understand "ordinary" Anderson localization (sometimes called strong localization), named after Phillip Anderson. Anderson localization is the process whereby the eigenstates of an electron gas in a disordered environment become spatially localized around impurities, causing the system to behave as an insulator instead of a conductor. Weak localization is a very different process that also increases the resistance of a disordered conductor (although it is not strong enough to turn a conductor into an insulator).

It comes from a very special quantum interference effect that occurs in systems with time-reversal symmetry: To find the probability  $P$  for an electron to propagate from point  $\mathbf{r}$  to point  $\mathbf{r}'$ , one has to add the quantum amplitudes  $A_i$  for the electron to take all possible paths  $i$ , and then calculate the modulus squared,  $P = |\sum_i A_i|^2$ . (This expression is a consequence of Feynman's path integral formulation of quantum mechanics, but one can also view it as a generalization of the double-slit interference formula to an infinite number of "slits.") The cross-terms in this expression are responsible for interference.

Usually when open paths with  $\mathbf{r}$  different from  $\mathbf{r}'$  are considered, the randomness in the  $A_i$  wash out any quantum interference effects. However, there is a special class of paths, closed paths with  $\mathbf{r} = \mathbf{r}'$ , where interference effects can be important. In systems with time-reversal symmetry (which basically means that there can be no applied magnetic field) there will always be pairs of closed paths and their time-reversed counterparts in the above summation that have the same amplitude. The result is that the probability to go from  $\mathbf{r}$  to  $\mathbf{r}$ , in other words the probability to *go nowhere* is enhanced by quantum interference effects, and this amounts to a measurable increase in resistance.

### 3.3.4. Dephasing by Electron-Electron Interaction

As explained above, weak localization (and more generally, the Aharonov-Bohm effect)

occurs when the electron motion is sufficiently phase-coherent. This fact can be exploited to actually *measure* the phase-coherence length  $L_\phi$  or phase-coherence time  $\tau_\phi$ , the characteristic time beyond which the electron becomes decoherent, in an electron system.  $L_\phi$  and  $\tau_\phi$  are simply related to each other and only one needs to be measured. The resistance increase due to weak localization depends on the number of closed paths that contribute to the summation in  $P = |\sum_i A_i|^2$ , and a given closed path contributes as long as its length  $L$  is less than  $L_\phi$  (if  $L > L_\phi$  the electron would not have the phase coherence necessary to exhibit interference). Thus, the magnitude of the weak localization effect—which can be determined by “turning the effect off” by applying a magnetic field—can be used to infer the value of  $L_\phi$  or  $\tau_\phi$ .

At low temperatures the dominant mechanism for dephasing is electron-electron scattering: A given electron feels a fluctuating electric field produced by the other electrons that scrambles its phase after some time  $\tau_\phi$ . Theoretically it is predicted that the dephasing *rate* vanishes at low temperature as  $\tau_\phi^{-1} \propto T^\beta$ , where  $\beta$  is a positive exponent, but recently Richard Webb and others have given experimental evidence for a *saturation* of  $\tau_\phi^{-1}$  in the  $T \rightarrow 0$  limit. The physics of low-temperature dephasing is currently a problem of great controversy and interest.

### 3.3.5. Thouless Energy

Mesoscopic physics research has also led to a profound new discovery about quantum systems in general. In the 1970's David Thouless and collaborators showed that any quantum system possesses an important fundamental energy scale, now called the Thouless energy  $E_T$ . The Thouless energy is a measure of how sensitive the eigenstates in a quantum system are to a change in boundary conditions. Specifically,  $E_T$  is defined as the energy change of a state at the Fermi energy caused by a change from periodic to antiperiodic boundary conditions.  $E_T$  would be zero in a system—for example an insulator—with localized eigenstates, because if the wave functions do not extend to the boundaries their energies will be independent of boundary conditions. Thouless showed that the dimensionless ratio  $g \equiv E_T/\Delta\varepsilon$  of  $E_T$  with the energy level spacing at the Fermi energy,  $\Delta\varepsilon$ , determines whether the system is a conductor ( $g > 1$ ) or an insulator ( $g < 1$ ). In fact,  $g$  is equal to the conductance of the system in units of  $e^2/h$ .

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### Biographical Sketch

**Michael R. Geller**, received his Ph.D. in physics from the University of California, Santa Barbara, in 1994. After postdoctoral fellowships at the University of Missouri, Columbia, and at Simon Fraser University, Vancouver, he joined, in 1997, the faculty in the Department of Physics and Astronomy at the University of Georgia in Athens. His area of research is theoretical condensed-matter physics, and his interests include strongly correlated quantum systems, electron and phonon dynamics in low-dimensional and nanoscale systems, quantum computing, vortex dynamics in superfluids and superconductors, and nonperturbative quantum field theory.