THERMAL STRESSES IN VESSELS, PIPING, AND COMPONENTS

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Keywords: Thermoelastic Stresses, Thermal Shock, Temperature Gradients, Temperature Curvature, Internal Restraints, Coefficient of Thermal Expansion

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Summary

Tubular components such as pipes, radiant burners, and pressure vessels are subjected to a wide-range of severe thermal- and mechanical-conditions that involve excessive temperatures and pressures that can be transient or steady-state in nature. The ability of the designer to avoid failures from these conditions requires a detailed understanding of the underlying thermal-states and the resulting thermoelastic stresses. Although not widely known, there are actually two primary mechanisms for generating thermal-stresses in pipes, pressure vessels, and any other tubular components. The first and universally known mechanism involves internal or external restraints that generate forces when coupled with the thermal expansion or contraction associated with a temperature change. Although other possibilities exist, the most common form is seen from radial (through the tube wall) temperature gradients. The second and certainly lesser known mechanism for generating thermal-stresses is related to the curvature from an axial temperature variation along the length of a tube. Since the curvature of the axial temperature profile is proportional to the second derivative of the temperature profile, this implies that constant or linear temperature distributions will not induce such stresses. While thermal stresses and their underlying causes can be very complex in nature, the ultimate aim of this contribution is to give a useful introduction of thermal stresses in piping and vessels from an engineering point of view. For a more thorough analysis, it is recommended that the reader consult the references listed in bibliography

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at the end of this chapter.

1. Introduction

Throughout industry, tubular components such as pipes, radiant burners, and pressure vessels are subjected to a wide-range of severe thermal and mechanical conditions. More often than not, these conditions involve excessive temperatures and pressures that can be rapidly changing or transient in nature. Eventually, these conditions can become unchanging with time or steady-state. In either case, the stresses may be sufficient to induce fatigue or burst-type failures when alloys are used. When brittle materials such as ceramics are in service, the principal threat is usually from a catastrophic fast-fracture, especially when thermal shock is involved. Regardless of the potential failure mode, the ability of the designer to avoid these costly and potentially deadly problems requires a detailed understanding of the underlying thermal conditions (be they transient and/or steady-state) and the resulting thermoelastic stresses. While the advent of finite-element and other popular and widely available numerical techniques has simplified the process of determining thermal-stresses to some extent, the is still a strong need to understand the basis for their generation and the available relationships between transient and steady-state temperature-states and the ensuing stresses.

Contrary to what can be found in a vast majority (if not all) of engineering textbooks that cover the subject of thermal-stresses, there are actually two primary mechanisms for generating thermal-stresses in any structure that can include pipes, pressure vessels, and any other tubular components. The first and universally known mechanisms involves internal or external restraints that generate forces when coupled with the thermal expansion or contraction associated with a temperature change; with most materials exhibiting a positive coefficient of thermal expansion or CTE (water being a very notable exemption), they will expand for temperatures above reference (usually room temperature) and contract at lower temperatures. The bi-material strips found in many thermostats is a prime example where the differing thermal deformation of the two materials induces a bowing that switches a system on or off.

However, a similar effect can occur in a monolithic material if there is a temperature gradient since the thermal deformation will also vary with the gradient with the warmer sections deforming more than the cooler. In fact, thermal expansion differentials may develop as a result of the temperature gradients just mentioned, as well as thermoelastic or thermo-physical property variations (perhaps due to a gradient), phase differences, and/or material anisotropies. Under the scenario of internal gradients and restraint for tubular geometries, there have been numerous analytical models developed to quantify the thermal transient fields and thermoelastic stresses. However, these solutions have been primarily limited to the most severe and often unrealistic step (Unit) temperature changes over time occurring at a surface. While step boundary conditions have been useful for modeling very severe thermal shocks, many studies along with practical experience have demonstrated the potentially complicated temporal dependence of the surface temperature loading and the resulting stresses. Hence, such solutions may not adequately (or accurately) predict the time or location of a failure from a burst, fatigue, and/or fracture event.

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The second mechanism for generating thermal-stresses is absent in most, if not all textbooks on the subject. While certainly missing from the general engineering discussion (and only found in a handful of technical publications), the resulting stresses are certainly no less severe or threatening to a component's survival, so this obscurity can be dangerous from a design point of view. In fact, any gradient that extends along the length of a component has the potential to induce additional bending-like stresses under certain conditions. For instance, a gradient along the length of a pipe with a distinct hot-spot and with a material exhibiting a positive CTE, will cause local swelling much like an aneurism. Conversely, a cooler spot will cause the tube or vessel to locally contract as if it were squeezed. In either case, the resulting deformation will induce potentially severe bending-stresses that are proportional to the local curvature with concavity locally inducing compression and convexity tension. Since the curvature is in-turn proportional to the second derivative of the temperature gradient, this implies that constant or linear gradients will not induce these stresses. Unfortunately, linear assumptions are often used, but not very realistic in practice.

One common thread to the scenarios just discussed for the generation of thermal stresses is the underlying temperature gradients, be they internal or along some characteristic dimension. As such, it is important to understand the underlying thermal-states before the resulting stresses can be determined. However, given the complexity of two-dimensional conduction in a tube or vessel where both radial and axial gradients may develop (as well as the absence of useable relationships), the thermal analysis will initially be restricted to radial gradients only. Once these thermal conditions are fully understood, relationships for stresses due to radial and axial temperature gradients will be given. Stresses resulting from operational pressures can always be combined via superposition to determine the complete stress state.

2. Thermal Analysis

2.1. Thermal Conduction (General Considerations)

Under the stated assumption of radial temperature gradients through the wall of a tube or vessel, the analysis begins with the partial differential equation that defines the time-dependant axisymmetric thermal state in a hollow circular-cylinder subjected to mixed boundary conditions on the internal ($r = a$) and external ($r = b$) surfaces, respectively:

$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} = \frac{1}{\kappa}\frac{\partial T}{\partial t} \tag{1}$$

In Eq. (1), the variable T represents the temperature, r is the radial coordinate, κ is the thermal diffusivity (assumed to be independent of temperature), and t is time. In terms of solutions, the simplest situation is the later time solution when the structure reaches steady-state conditions and the terms dependent on time disappear. In this case, and with the assumption of no internal heat generation, Eq. (1) simplifies to the well-known Laplace equation:

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$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} = 0 \qquad (2)$$

with the resulting solution for the temperature distribution as a function of radius:

$$T(r) = \frac{\Delta T}{\ln\left(\frac{b}{a}\right)} \ln\left(\frac{b}{r}\right) + T_b \qquad (3)$$

where $\Delta T = T_a - T_b$ is the temperature difference between the inner and outer surfaces. As implied, the sign of the temperature difference will dictate the tensile or compressive nature of the inner ($r = a$) and outer ($r = b$) surfaces, respectively.

2.2. Transient Solutions (Unit Response)

The transient solution for Eq. (1) is far more difficult and usually can only be done for the simpler cases mentioned earlier where the surface undergoes a step change in temperature. However, and despite its unrealistic nature, a generalized solution for a unit response does actually have utility as will be demonstrated in the next section. For the boundary conditions representative of an internal surface subjected to a step in temperature with convection on the external surface allowed:

$$T(r,t) = T_0 = 0 \qquad t \le 0, \quad a \le r \le b \qquad (4a)$$

$$T_\infty = U = 1 \qquad t > 0, \quad r = a \qquad (4b)$$

$$k\frac{\partial T}{\partial r} + h(T - T_0) = 0 \qquad t > 0, \quad r = b \; t > 0, \quad r = b \qquad (4c)$$

a general purpose series solution can be derived for a hollow circular-cylinder. For such a cylinder subjected to the mixed boundary conditions described above, the solution takes on the following form involving an infinite series:

$$\Phi(r,t) = U\left[\frac{1 - Bi\ln(r/b)}{1 - Bi\ln(a/b)} + \pi\sum_{k=1}^{\infty} e^{-\kappa\beta_k^2 t}\,\frac{C_0(r,\beta_k)C_1^2(b,\beta_k)}{C_2(a,b,\beta_k)}\right] \qquad (5)$$

where Bi is the Biot number, $Bi = bh/k$. The parameter U can be set to any value beyond unity to scale for different step changes. Additional parameters include h as the convective coefficient for the external surface and the recurring functions $C_0(r,\beta)$, $C_1(b,\beta)$, and $C_2(a,b,\beta)$ that are defined in the following fashion:

$$C_0(r,\beta_k) = J_0(r\beta_k)Y_0(a\beta_k) - Y_0(r\beta_k)J_0(a\beta_k) \qquad (6a)$$

$$C_1(b,\beta_k) = k\beta_k J_1(b\beta_k) - hJ_0(b\beta_k) \qquad (6b)$$

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$$C_2(a,b,\beta_k) = \left(k^2\beta_k^2 + h^2\right)\left[J_0(a\beta_k)\right]^2 - \left[k\beta_k J_1(b\beta_k) - hJ_0(b\beta_k)\right]^2 \tag{6c}$$

In addition, J_0, Y_0, J_1, and Y_1 represent Bessel functions of various orders while β represents the real and simple roots of the characteristic equation:

$$J_0(a\beta)\left[\beta Y_1(b\beta) - \frac{h}{k}Y_0(b\beta)\right] - Y_0(a\beta)\left[\beta J_1(b,\beta) - \frac{h}{k}J_0(b,\beta)\right] = 0 \tag{7}$$

An analysis of the first term in Eq. (5) indicates that it represents the stationary distribution which would be established if the prescribed surface temperature would be fixed at time t (i.e., steady-state), whereas the second term accounts for the lag in the temperature distribution behind the stationary. As would be expected, the second term eventually vanishes and the solution approaches the expected logarithmic steady-state distribution at large values of time given by Eq. (3). While Eq. (5) is a versatile relationship that can be used to conservatively calculate the transient response of a cylinder to a very severe thermal event, most thermal-shock events are not instantaneous and will exhibit some time dependence of the surface excitation.

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Biographical Sketch

Albert E. Segall Received a Ph.D. in Engineering Science and Mechanics from the Pennsylvania State University in 1992. After completing his degree, Dr. Segall remained at the University and served as the Associate Director of the Center for Advanced Materials and a Senior Research Associate at the Applied Research Laboratory until 1999. In 1999, he joined the Washington State University Vancouver faculty as an Associate Professor of Mechanical and Manufacturing Engineering where he eventually became the Director of Engineering Programs. In the 2002, Dr. Segall returned to the Pennsylvania State University and the Engineering Science and Mechanics Department where he also served as the Co-Chair of the Intercollege Program in Materials Science and Engineering, as well as an Associate Editor of the Society for Tribologists and Lubrication Engineers (STLE) Tribology Transactions. His research interests have mainly focused on the thermo-structural behaviors and reliability of materials. This research includes the development of probabilistic fracture and brittle-design methodologies and their application to the understanding of thermal shock-behaviors of ceramics, the underlying thermal transients via direct and inverse approaches, and laser machining. Dr. Segall is also interested in the study of wear, friction, coatings, and the development of realistic tribo-test methods to assess wear-couples under industrially relevant conditions. As an avid science fiction fan, Dr. Segall is also working on innovative ways to integrate this genre as seen in movies and books with engineering education.

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