INTRODUCTION TO TRANSIENT ANALYSIS OF POWER SYSTEMS

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Summary

This chapter provides an overview of the transient phenomena in electric–power supply–systems, as well as of the methodology being employed in their analysis. Power system elements are represented by diverse models which depend on the type of transient to be analyzed. Depending on the frequencies involved in the transient phenomena being studied, these elements may require a distributed-parameter representation, or a lumped one. Traveling wave phenomena are a central topic in the analysis of electromagnetic transient.

A method to analyze traveling waves developed by Bewley is described here in detail as it provides deep understanding of the phenomena and their effects. Several techniques have been developed for transient analysis. Three of them are detailed in this chapter, namely: the analytic Laplace transform, the numerical method for time domain analysis developed by Dommel, and the Laplace numerical analysis method known as the Numerical Laplace Transform. Several examples are included with the purpose of showing the applicability of the three techniques here described.

The examples aim as well at illustrating how these three methods complement each others. All these examples are based on single-phase models; they are aimed at introducing electromagnetic transients in power systems.

1. Transients in Power Systems

A transient phenomenon in any type of system can be caused by a change of the operating conditions or of the system configuration. Power system transients can be caused by faults, switching operations, lightning strokes or load variations. The importance of their study is mainly due to the effects the disturbances can have on the system performance or the failures they can cause to power equipment.

Stresses that can damage power equipment are of two types: overcurrents and overvoltages. Overcurrents may damage some power components due to excessive heat dissipation; overvoltages may cause insulation breakdowns (failure through solid or liquid insulation) or flashovers (insulation failure through air). Protection against overcurrents is performed by specialized equipment whose operation is aimed at disconnecting the faulted position from the rest of the system by separating the minimum number of power components from the unfaulted sections.

Protection against overvoltages can be achieved by selecting an adequate insulation level of power equipment or by installing devices aimed at mitigating voltage stresses. In order to select an adequate protection against both types of stresses, it is fundamental to know their origin, to estimate the most adverse conditions and to calculate the transients they can produce.

Several criteria can be used to classify power system transients:

- According to their origin, disturbances can be external (lightning strokes) or internal (faults, switching operations, load variations).
- According to the nature of the physical phenomena, power system transients can be electromagnetic, when it is necessary to analyze the interaction between the (electric) energy stored in capacitors and the (magnetic) energy stored in inductors, or electromechanical, when the analysis involves the interaction between the electric energy stored in circuit elements and the mechanical energy stored in rotating machines.

An accurate calculation of transients in power systems is a difficult task because of the equipment involved and of the interaction between components. The solution of most transients is not easy by hand calculation, even for small size systems.

For some cases, one can drastically reduce the size of the equivalent circuit and obtain an equation whose solution can be found in textbooks. For a majority of transients, an accurate, or even an approximate, solution can be only obtained by using a computer.

Transients in power systems were initially analyzed with network analyzers. Since the release of the first digital computers, a significant effort has been dedicated to the development of numerical techniques and simulation tools aimed at solving transients in power systems.

Hardware and software developments through years have also motivated the development of more powerful techniques and simulation tools. Computer transients programs have significantly reduced time and cost of simulations, and most transients studies are presently based on the application of a digital computer. The main goal of this chapter is to introduce some of the techniques used to analyze electromagnetic transients in power systems, with emphasis on numerical techniques and the application of a digital computer.

Several techniques have been developed to date for computation of electromagnetic transients in power systems. They can be classified into two groups: time domain and frequency domain. Some hybrid approaches (i.e., a combination of both techniques) have been also proposed.

Among the time-domain solution methods, the most popular one is the algorithm developed by Dommel (1969, 1996), which is a combination of the trapezoidal rule and the method of characteristics, also known as Bergeron's method. This algorithm was the origin of the ElectroMagnetic Transients Program (EMTP) (Dommel, 1969; Scott Meyer & Dommel, 1974). This acronym is nowadays used to designate a family of simulation tools based on the Dommel’s scheme.

A general procedure for transients analysis could consist of the following steps:

1. Collect the information about the origin of the transient phenomenon and data of power equipment that could get involved or affected by the transient.
2. Select the zone of the system that should be included in the model to be analyzed.

3. Choose the best representation for each component included in the study zone.
4. Perform the calculation using one of the techniques developed for transients analysis.
5. Analyze the results.

The tasks that will be carried out in the last step will depend on the main goals of the study. For example, if the goal is to analyze the equipment failure rate caused by overvoltages, the results could force some design improvements in the system. These changes could affect the insulation level of some components or the installation of overvoltage mitigation devices.

The selection of the most adequate representation of a power component in transients studies is not an easy task due to the frequency ranges of the transients that can appear in power systems and to the different behavior that a component can have for each frequency range. Section 2 is dedicated to analyze this aspect; it presents a short introduction to modeling for transients analysis and to the basic elements that will be included in transient models.

Switch operations are one of the most common causes of electromagnetic transients in power systems. Both, the closing and the opening of a switch introduce a change in the system structure that can cause overcurrents and overvoltages. The analysis of switching transients in linear systems can be made by applying the superposition principle. Section 3 introduces some fundamental concepts for analysis of switching transients in linear systems.

The performance of power components during a transient phenomenon depends on their physical dimensions and on the main characteristics of the transient phenomenon. Voltages and currents propagate along conductors with finite velocity; so, by default, certain models for electromagnetic transients analysis should consider that electrical parameters are distributed.

Only when physical dimensions of those parts of a component affected by a transient are small, compared with the wavelength of the main frequencies, a representation based on lumped parameters could be used. The main concepts of wave propagation are introduced in Section 4.

2. Power System Components

2.1. Introduction

The goal of a power system is to satisfy the energy demand of a variety of users by generating, transmitting and distributing the electric energy. These functions are performed by components whose design and behavior are very complex. An accurate analysis of most transient phenomena in power systems is a difficult task due to the complexity of power components and to the interactions that can occur among these. For an introduction to this area, such as the one in this chapter, it is convenient to proceed simplifying models of those components involved in the phenomena to be analyzed. Initially only single-phase models are used for representing power components in transient phenomena. On the other hand, since only transients of electromagnetic nature are analyzed in this chapter, the representation of mechanical parts is omitted.

This section is dedicated to illustrate the importance that the mathematical model selected for representing power components can have on simulation results, to discuss some aspects that have to be accounted for when preparing models for transients calculations, and to classify the basic circuit elements that will be used to construct simplified models for electromagnetic transients analysis.

2.2. Modeling for Transient Analysis

Accurate simulation of electromagnetic transients should be based on an adequate representation of power components. The frequency range of transient phenomena in power systems can cover a broad spectrum.

An accurate simulation of any electromagnetic transient phenomenon could be based on power component models valid for a frequency range that varies from DC to several MHz. However, a representation valid throughout this range of frequencies is not practically possible for most components. Consider, for instance, the behavior of a transformer during electromagnetic transients.

A transformer is a device whose behavior is dominated by magnetic coupling between windings and core saturation when transients are of low or mid frequency; that is, for frequencies well below the first winding resonance. However, when the transients are caused by high frequency disturbances, such as lightning strokes, then the behavior of the transformer is dominated by stray capacitances as well as capacitances among windings.

Modeling of power components taking into account the frequency dependence of parameters can be practically made by developing mathematical models that are accurate enough for a specific range of frequencies. Each range of frequencies usually corresponds to some particular transient phenomena (CIGRE WG33.02, 1990; Gole, Martinez-Velasco, & Keri, 1999).

The following aspects are to be considered in digital simulations of electromagnetic transients:

- *Parameters*: Very often only approximated or estimated values are used for some parameters whose influence on the representation of a component can be important or very important. In general, this happens with frequency-dependent parameters in simulations of high frequency transients; e.g., above 100 kHz. It is also important to take into account that some parameters may change due to climatic conditions or be dependent on maintenance.

- *Type of study*: In some studies, the maximum peak voltage is the only information of concern. This maximum usually occurs during the first oscillation after the transient phenomenon starts. In many transients, the peak value will not be very affected by inductance or capacitance values.

- *The study zone*: The more components in the system under study, the higher the probability of insufficient or wrong modeling. In addition, a very detailed representation of a system will require very long simulation time. Therefore, some experience will often be needed to decide the degree of modeling detail for the study zone, as well as for the model selection with the most important components.

Guidelines for representing power components at digital simulations in time domain have been the main subject of several publications (CIGRE WG33.02, 1990; Gole, Martinez-Velasco, & Keri, 1999; IEC 60071-4, 2004).

2.3. Basic Circuit Elements

Power component models for electromagnetic transient analysis will be constructed by using basic circuit elements that can be classified into three categories: sources, passive elements and switches.

1. *Sources*: They are used to represent power generators and external disturbances that can be the origin of some transients (e.g., lightning strokes). Two types of sources can be distinguished: a voltage source (Thevenin representation) and a current source (Norton representation). The equivalent scheme of a voltage source includes a series impedance, while the equivalent scheme of a current source incorporates a parallel admittance. An ideal behavior for each type of source can be assumed by decreasing to zero the series impedance of a voltage source and the parallel admittance of a current source.

2. *Passive elements*: Depending on the transient phenomenon, the behavior of some components can be either linear or non linear. The transformer is an example for which either a linear or a saturable model can be required. The representation of linear components will be based on lumped-parameter elements (resistance, inductance, capacitance) and distributed-parameter elements (single-phase loss-less line). Figure 1 shows their symbols and mathematical models. This list can be expanded by adding other basic elements such as the magnetic coupling or the ideal transformer. If the behavior of a component is non linear, then its representation can include non linear resistances or saturable inductances.

3. *Switches*: They modify the topology of a network by connecting or disconnecting components, although they will be also used to represent faults or short-circuits. The behavior of an ideal switch can be summarized as follows: its impedance is infinite when it is opened and zero when it is closed. It can close at any instant regardless of the voltage value at the source side; however, it will open only when the current goes through zero. The possibility of opening an ideal switch with non-zero currents can be needed to analyze some phenomena; e.g., the *current chopping* phenomenon. Several types of switches can be modeled by using different criteria to determine when they should open or close.

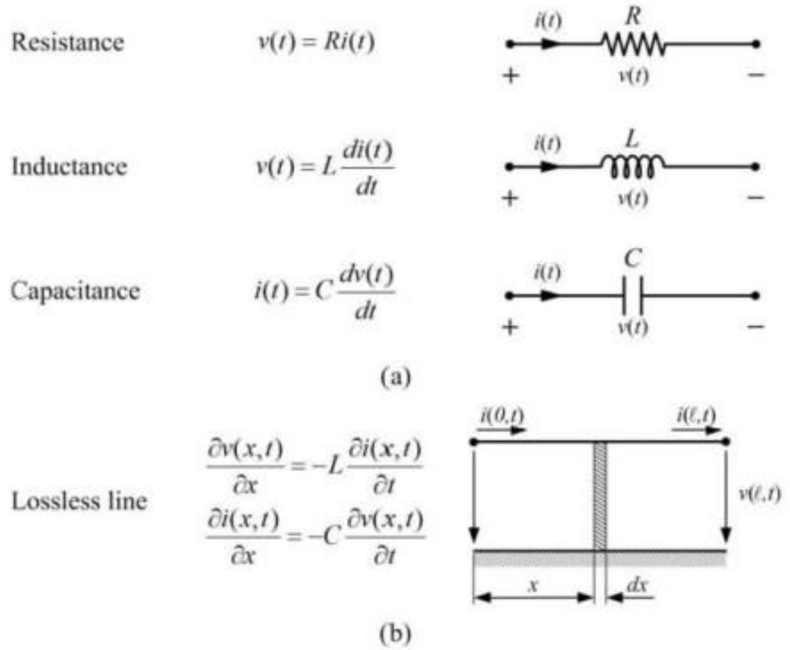

Figure 1. Basic linear circuit elements. (a) Lumped-parameter circuit elements. (b) Distributed-parameter circuit element.

3. Analysis of Switching Transients in Linear Systems

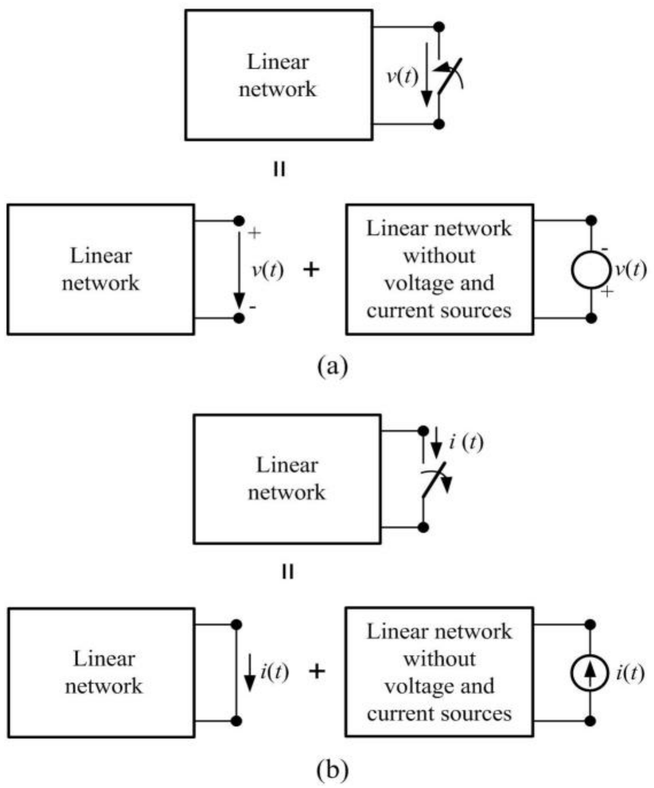

Figure 2. Application of the superposition principle: (a) Closing of a switch. (b) Opening of a switch.

Transients caused by switching operations in linear systems can be analyzed by using the superposition principle. The schemes that result from the application of this principle for analysis of the closing and the opening of a switch are respectively shown in Figure 2. The details of each case are described in the following paragraphs.

1. Transients caused by the closing of a switch can be analyzed by adding the steady state solution, which exists prior to the closing operation, and the transient response of the system that results from short-circuiting voltage sources and open-circuiting current sources to a voltage injected across the switch contacts. Since the voltage across the switch terminals after the operation will be zero, the injected voltage must be the negative of the one that would have existed between switch terminals if it remained open. Voltages and currents are then calculated by adding the values corresponding to the transient response to the steady state solution. In some studies the variable of concern is the current through the switch. To obtain the short-circuit current value, the analysis of the transient response will suffice, since the value of this current during steady state, i.e. prior to the closing operation, is zero, see Figure 2a.
2. Transients caused by the opening of a switch can be analyzed by adding the steady state solution, which exists prior to the opening operation, and the transient response of the system that results from short-circuiting voltage sources and open-circuiting current sources to a current injected through the switch contacts. Since the current through the switch terminals after the operation will be zero, the injected current must be the negative of the one that would have existed between switch terminals if it remained closed. When the contacts of a switch start opening, a transient voltage is developed across these. In many transients studies this voltage, known as transient recovery voltage (TRV), is the variable of concern. To obtain the TRV waveform, the analysis of the transient response will suffice, since this voltage is zero during steady state, i.e. prior to the opening operation, see Figure 2b.

4. Wave Propagation in Power Systems

4.1. Introduction

The analysis of electromagnetic transients in power components has to consider that electrical parameters are distributed. During a transient phenomenon, only the conductors whose lengths are short, when compared to the significant wavelengths in the phenomenon, can be represented by lumped-parameter models. This means that the analysis of electromagnetic transients at very high frequency (e.g., above 1 MHz) may have to be based on distributed-parameter models, even if conductors are a few meters short.

Voltages and currents on a component represented by a distributed-parameter model do not have the same value over the entire length of the conductor, and their travel occurs at finite velocities whose values depend on the physical characteristics of the component. The analysis of transient phenomena in distributed-parameter models is based on the concept of traveling waves.

The following subsections introduce the concepts associated to wave propagation in power components. The analysis is based on the model of a single-phase lossless distributed-parameter line which, for the rest of this chapter, will be also named as *ideal line*.

The most popular methods for analysis of electromagnetic transients in ideal lines are those developed by Bergeron (1949) and Bewley (1963). The Bergeron's method will serve as a basis for the numerical technique presented in this chapter. The method proposed by Bewley, also known as the *lattice diagram*, will as well be detailed and applied in this section.

4.2. Solution of the Single-phase Lossless Line Equations

Figure 3 shows the scheme of a single-phase line and the equivalent circuit of a line element. The equations of this circuit can be written as follows:

$$\begin{aligned} \frac{\partial v(x,t)}{\partial x} &= -L\frac{\partial i(x,t)}{\partial t} \\ \frac{\partial i(x,t)}{\partial x} &= -C\frac{\partial v(x,t)}{\partial t} \end{aligned} \qquad (1)$$

where L, C are respectively the inductance and capacitance per unit length, and x is the distance with respect to the sending end of the line.

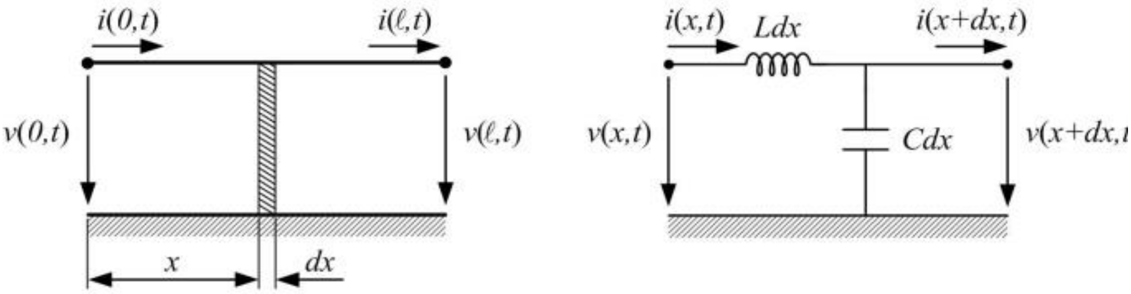

Figure 3. Scheme of a single-phase lossless line.

After differentiating with respect to the variable x, these equations become:

$$\begin{aligned} \frac{\partial^2 v(x,t)}{\partial x^2} &= LC\frac{\partial^2 v(x,t)}{\partial t^2} \\ \frac{\partial^2 i(x,t)}{\partial x^2} &= LC\frac{\partial^2 i(x,t)}{\partial t^2} \end{aligned} \qquad (2)$$

The general solution of the voltage equation has the following form:

$$v(x,t) = f_1(x-\upsilon t) + f_2(x+\upsilon t) \qquad (3)$$

where

$$\upsilon = \frac{1}{\sqrt{LC}} \tag{4}$$

Both f_1 and f_2 are voltage functions, and υ is the so-called *propagation velocity*. Since the current equation in (2) has the same form as the voltage equation, its solution will also have a similar expression. It is, however, possible to obtain a general solution of the current equation based on that for the voltage. This solution could be expressed as follows:

$$i(x,t) = \frac{f_1(x-\upsilon t) - f_2(x+\upsilon t)}{Z_\mathrm{c}} \tag{5}$$

where

$$Z_c = \sqrt{\frac{L}{C}} \tag{6}$$

is the *surge impedance* of the line.

Note that $f_1(x-\upsilon t)$ remains constant if the value of the quantity $(x-\upsilon t)$ also is constant. Thus, the function $f_1(x-\upsilon t)$ represents a voltage traveling wave towards increasing x, while $f_2(x+\upsilon t)$ represents a voltage traveling wave towards decreasing x. Both waves are neither distorted nor damped while propagating along the line. The general solution of the voltage and the current at any point along an ideal line is, therefore, constructed by superposition of waves that travel in both directions. The expressions for f_1 and f_2 are determined at a specific case from the boundary and the initial conditions.

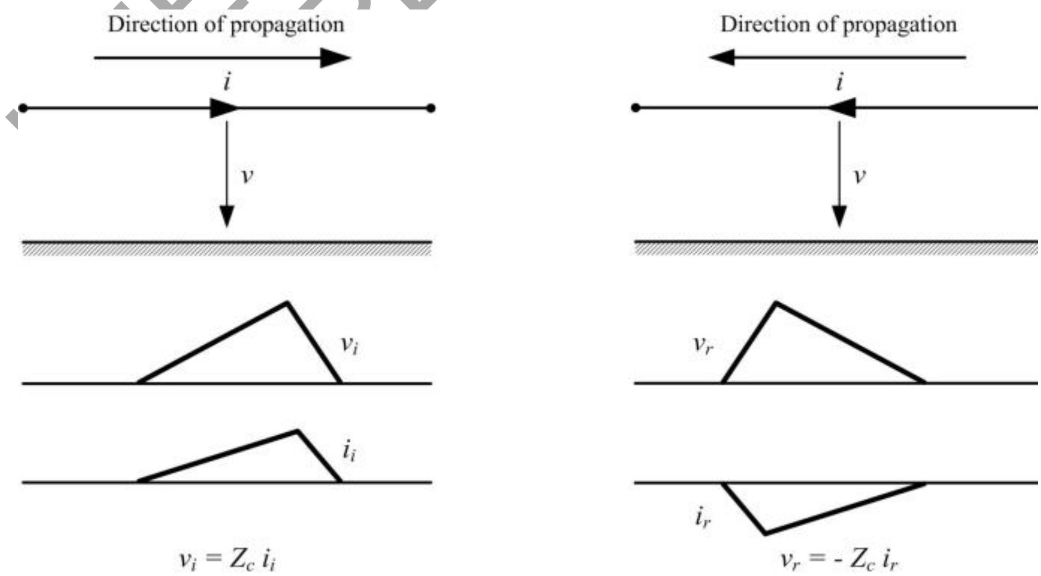

Figure 4. Relationships between voltage and current waves.

From (3) and (5) one can deduce that the ratio between voltage and current waves is always the surge impedance of the line. However, this ratio can be positive or negative depending on the direction of propagation, see Figure 4.

4.3. Wave Propagation and Reflection

The presence of traveling waves propagating in both directions along a line can be justified by the presence of discontinuity points; that is, points where waves meet a propagation media with different characteristic values (surge impedance and propagation velocity). The following cases will be useful to analyze the physical phenomena that occur when a traveling wave meets a discontinuity point.

Line termination: Figure 5 shows the particular case to be analyzed. Consider that a wave, which propagates along an ideal line, reaches an end where a resistance R_t has been installed.

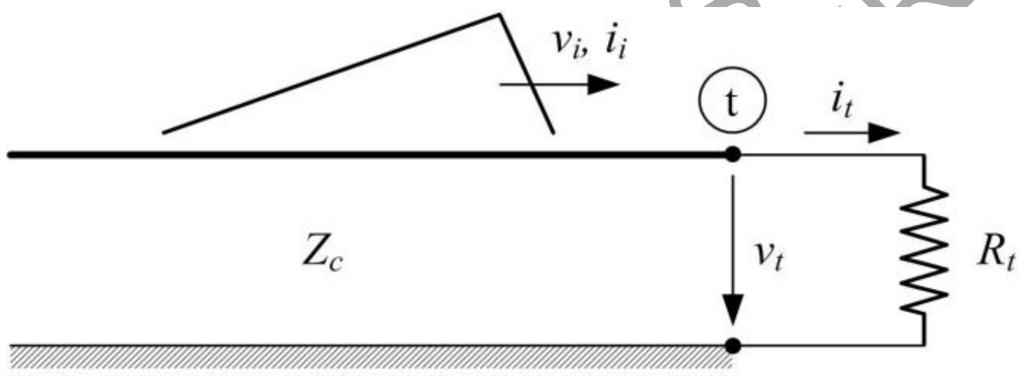

Figure 5. Line termination.

Voltage and current for the traveling wave, known as *incident wave*, are related as follows:

$$v_i = Z_c i_i \tag{7}$$

where Z_c is the surge impedance of the line along which the incident wave is traveling.

At the receiving end of the line, the relation between voltage and current is the following one (see Figure 5):

$$v_t = R_t i_t \tag{8}$$

The surge impedance and the terminal resistance are different, but a mismatch of both voltage and current exists at the discontinuity point; therefore, an adjustment is needed for both variables. A new wave, known as *reflected wave*, is produced when the incident wave reaches the discontinuity point.

Relationships between voltage and current waves at this terminal can be expressed as follows:

$$v_{\mathrm{t}} = v_{\mathrm{i}} + v_{\mathrm{r}} \qquad i_{\mathrm{t}} = i_{\mathrm{i}} + i_{\mathrm{r}} \tag{9}$$

where subscripts i,r,t are used to denote respectively incident, reflected and transmitted waves.

The reflected wave travels back along the line, far away from the receiving end of the line. Since this wave propagates in an opposite sense to that of the incident wave, voltage and current for this wave are related as follows:

$$v_{\mathrm{r}} = -Z_{\mathrm{c}} i_{\mathrm{r}} \tag{10}$$

Substitution of Eqs. (7), (8) and (9) into (10) yields:

$$v_{\mathrm{r}} = \Gamma v_{\mathrm{i}} \qquad i_{\mathrm{r}} = -\Gamma i_{\mathrm{i}} \tag{11}$$

where

$$\Gamma = \frac{R_{\mathrm{t}} - Z_{\mathrm{c}}}{R_{\mathrm{t}} + Z_{\mathrm{c}}} \tag{12}$$

is known as the *reflection coefficient* at the discontinuity point, in this particular case at the receiving end of the line.

Voltage and current waves at the discontinuity point are obtained from the superposition of incident and reflected waves

$$v_{\mathrm{t}} = (1+\Gamma) v_{\mathrm{i}} \qquad i_{\mathrm{t}} = (1-\Gamma) i_{\mathrm{i}} \tag{13}$$

The quantity $(1+\Gamma)$ is also known as the *refraction coefficient*.

Transition point: Figure 6 shows the diagram of a system that consists of two ideal lines with different surge impedances. Assume that the incident wave travels along Line 1 towards Line 2.

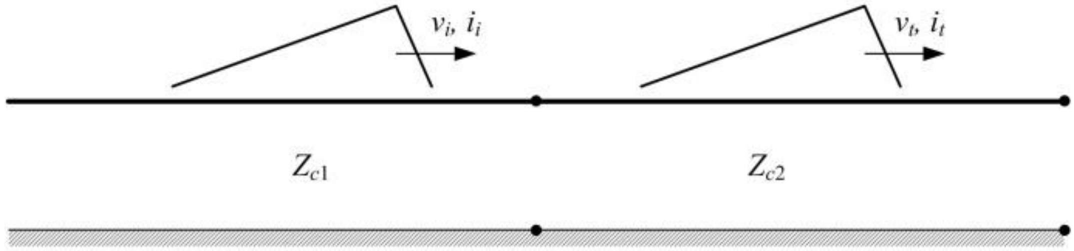

Figure 6. Transition point.

At the transition (or discontinuity) point this wave will be affected by a change in the characteristic parameters of the propagation mean. The traveling wave passing to Line 2 will depend on the parameters of both lines.

Following a similar reasoning as for the previous case, one can conclude that when the incident wave $\left(v_{\mathrm{i}};i_{\mathrm{i}}\right)$ reaches the transition point, two new waves are generated: a reflected wave $\left(v_{\mathrm{r}};i_{\mathrm{r}}\right)$, which will travel back along Line 1, and a refracted or transmitted wave $\left(v_{\mathrm{t}};i_{\mathrm{t}}\right)$, which will pass and travel along Line 2.

Relationships between voltage and current waves at the transition point can be expressed as follows:

$$v_{\mathrm{t}} = v_{\mathrm{i}} + v_{\mathrm{r}} \qquad i_{\mathrm{t}} = i_{\mathrm{i}} + i_{\mathrm{r}} \tag{14}$$

On the other hand

$$v_{\mathrm{i}} = Z_{\mathrm{c1}} i_{\mathrm{i}} \qquad v_{\mathrm{r}} = -Z_{\mathrm{c1}} i_{\mathrm{r}} \qquad v_{\mathrm{t}} = Z_{\mathrm{c2}} i_{\mathrm{t}} \tag{15}$$

Substitution of these expressions into the current equation of (14) yields:

$$v_{\mathrm{r}} = \Gamma v_{\mathrm{i}} \qquad v_{\mathrm{t}} = \left(1+\Gamma\right) v_{\mathrm{i}} \tag{16}$$

where

$$\Gamma = \frac{Z_{\mathrm{c2}} - Z_{\mathrm{c1}}}{Z_{\mathrm{c2}} + Z_{\mathrm{c1}}} \qquad 1+\Gamma = \frac{2}{Z_{\mathrm{c2}} + Z_{\mathrm{c1}}} \tag{17}$$

Γ is the reflection coefficient and $\left(1+\Gamma\right)$ is the refraction coefficient at the transition point.

As for the wave currents, the following results are obtained:

$$i_{\mathrm{r}} = -\Gamma i_{\mathrm{i}} \qquad i_{\mathrm{t}} = \left(1-\Gamma\right) i_{\mathrm{i}} \tag{18}$$

It is important to keep in mind that the incident wave could have been traveling along Line 2 towards Line 1. In such case, the reflection coefficient would have been:

$$\Gamma' = \frac{Z_{\mathrm{c1}} - Z_{\mathrm{c2}}}{Z_{\mathrm{c1}} + Z_{\mathrm{c2}}} \tag{19}$$

One can observe that the expressions of reflection coefficients follow a very simple rule. When an incident wave reaches a discontinuity point, the reflection coefficient is obtained from the equivalent surge impedance that the wave sees at the discontinuity point, Z_{eq}, and the surge impedance of the mean in which the wave is traveling, Z_{c}, that is:

$$\Gamma = \frac{Z_{\mathrm{eq}} - Z_{\mathrm{c1}}}{Z_{\mathrm{eq}} + Z_{\mathrm{c1}}} \tag{20}$$

The reflection coefficients of the following cases are of interest since they are encountered in many practical cases.

Open-ended line: The traveling waves that reach an open–ended terminal meet an infinite impedance. For this particular case Eq. (12) becomes:

$$\Gamma = 1 \tag{21}$$

Therefore, at the open-ended terminal

$$v_{\mathrm{r}} = v_{\mathrm{i}} \qquad i_{\mathrm{r}} = -i_{\mathrm{i}} \tag{22}$$

from where

$$v_{\mathrm{t}} = v_{\mathrm{i}} + v_{\mathrm{r}} = 2v_{\mathrm{i}} \qquad i_{\mathrm{t}} = i_{\mathrm{i}} + i_{\mathrm{r}} = 0 \tag{23}$$

According to these results, when a traveling wave reaches an opened terminal, the voltage wave is doubled, while the current wave is cancelled, as expected.

Short-circuited line: The incident waves that reach a short-circuited terminal meet a zero impedance. In this condition Eq. (12) becomes:

$$\Gamma = -1 \tag{24}$$

Therefore, at a short-circuited terminal

$$v_{\mathrm{r}} = -v_{\mathrm{i}} \qquad i_{\mathrm{r}} = i_{\mathrm{i}} \tag{25}$$

from where

$$v_{\mathrm{t}} = v_{\mathrm{i}} + v_{\mathrm{r}} = 0 \qquad i_{\mathrm{t}} = i_{\mathrm{i}} + i_{\mathrm{r}} = 2i_{\mathrm{i}} \tag{26}$$

According to these results, when a traveling wave reaches a short-circuited terminal, the voltage wave is cancelled, as expected, while the current wave is doubled.

Matched line: A line is matched at one terminal when the incident wave reaching this terminal meets an impedance equal to the line surge impedance.

The reflection coefficient at this particular case becomes:

$$\Gamma = 0 \tag{27}$$

Therefore, when a line is matched:

$$v_{\mathrm{r}} = 0 \qquad i_{\mathrm{r}} = 0 \tag{28}$$

from where

$$v_{\mathrm{t}} = v_{\mathrm{i}} \qquad i_{\mathrm{t}} = i_{\mathrm{i}} \tag{29}$$

When a traveling wave reaches a matched terminal there will be no reflected wave. This makes sense, since the incident wave does not see any change in the characteristic parameters of the propagation mean.

4.4. The Lattice Diagram

The solution of voltages and currents caused during a given transient phenomenon can be deduced from the boundary and initial conditions of the system under study. With the method presented in this subsection the voltages and currents at a given point are obtained as the superposition of the traveling waves passing by that point after successive reflections. This method, which assumes that lines are ideal, is illustrated as follows by a very simple case.

The conclusions derived from the cases analyzed above can be summarized as follows:

1. When a line is energized, voltage and current waves start to propagate along the line. The propagation takes place at a finite velocity without distortion and damping. The relation between the voltage and the current waves traveling in the same direction is given by the surge impedance of the line.
2. When a traveling wave encounters a discontinuity point, such as another line or a load, it will generate two waves: (i) the reflected one that will travel back and (ii) the transmitted one that will pass the discontinuity point.

Consider the system shown in Figure 7, a single-phase lossless line is energized from a voltage source. The internal series impedance of the source is a resistance R_0. The voltage equation at the sending end of the line can be written as follows:

$$e(t) = v(t) + R_0 i(t) \tag{30}$$

where $v(t)$ and $i(t)$ are respectively the voltage and the current waves that will appear at the sending end of the line and that will propagate along this line immediately after the closing of the switch.

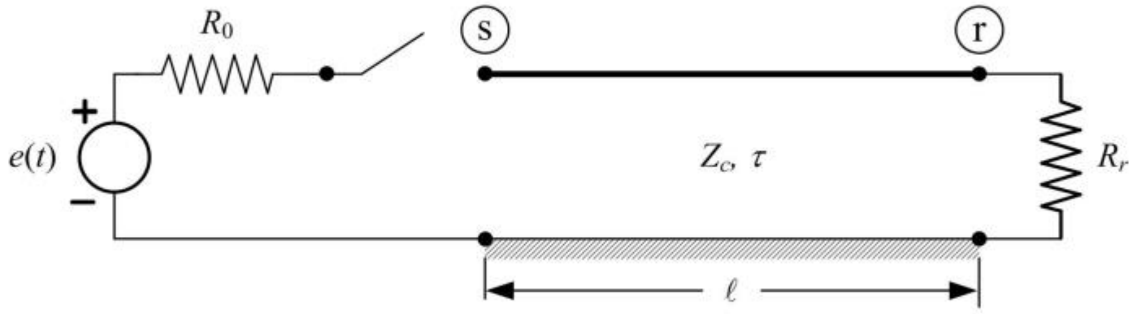

Figure 7. Energization of an ideal line.

Since the relationship between both waves is $v(t) = Z_c i(t)$, the voltage equation becomes:

$$e(t) = v(t) + R_0 \frac{v(t)}{Z_c} = \left(1 + \frac{R_0}{Z_c}\right) v(t)$$

from where

$$v(t) = \frac{Z_c}{R_0 + Z_c} e(t) \tag{31}$$

This expression is valid for as long as the waves reflected at the line far-end r have not yet arrived at the sending end s.

The waves produced by the closing of the switch will be reflected back at the receiving end; the reflected waves will encounter a discontinuity point at the sending end, where new reflected waves will be produced which will travel to the receiving end and so on. The transient phenomenon will continue with incident and reflected waves propagating between both line ends.

The values of the reflection coefficients at each end of the line are:

- Sending end

$$\Gamma_s = \frac{R_0 - Z_c}{R_0 + Z_c} \tag{32a}$$

- Receiving end

$$\Gamma_r = \frac{R_r - Z_c}{R_r + Z_c} \tag{32b}$$

The lattice diagram is a very useful tool to keep track of all traveling waves. The diagram shows the location of the discontinuity points, as well as the incident, the reflected and the transmitted waves at each point. The voltages and currents at one point are determined by the superposition of all traveling waves present at that point.

Assume that τ is the *travel time*; that is, the time that a wave needs for traveling between the two ends of the line (see Figure 7). Then:

$$\tau = \frac{\ell}{\upsilon} \tag{33}$$

where ℓ is the line length and υ is the propagation velocity.

Figure 8 depicts the lattice diagram that corresponds to this test system. The diagram shows the traveling waves at each line end. Since the behavior of the line is linear, the term $v(t)$ has been omitted in the diagram; that is, all traveling waves should be multiplied by the voltage wave that starts to travel after the switch is closed.

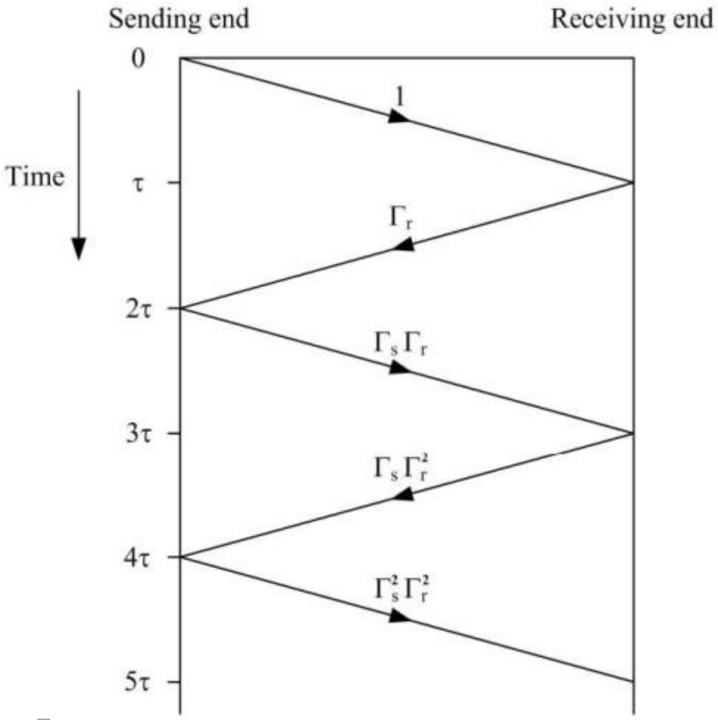

Figure 8. The lattice diagram.

The following voltage waves arrive to each end of the line:

- Sending end

$$\begin{aligned} & v(t)\varepsilon(t) \\ & v(t-2\tau)\varepsilon(t-2\tau)(\Gamma_r + \Gamma_s\Gamma_r) = v(t-2\tau)\varepsilon(t-2\tau)(1+\Gamma_s)\Gamma_r \\ & v(t-4\tau)\varepsilon(t-4\tau)(\Gamma_s\Gamma_r^2 + \Gamma_s^2\Gamma_r^2) = v(t-4\tau)\varepsilon(t-4\tau)(1+\Gamma_s)\Gamma_s\Gamma_r^2 \end{aligned} \tag{34a}$$

- Receiving end

$$
\begin{aligned}
& v(t-\tau)\varepsilon(t-\tau)(1+\Gamma_{\mathrm{r}}) \\
& v(t-3\tau)\varepsilon(t-3\tau)(\Gamma_{\mathrm{s}}\Gamma_{\mathrm{r}}+\Gamma_{\mathrm{s}}\Gamma_{\mathrm{r}}^2) = v(t-3\tau)\varepsilon(t-3\tau)(1+\Gamma_{\mathrm{r}})\Gamma_{\mathrm{s}}\Gamma_{\mathrm{r}} \\
& v(t-5\tau)\varepsilon(t-5\tau)(\Gamma_{\mathrm{s}}^2\Gamma_{\mathrm{r}}^2+\Gamma_{\mathrm{s}}^2\Gamma_{\mathrm{r}}^3) = v(t-5\tau)\varepsilon(t-5\tau)(1+\Gamma_{\mathrm{r}})\Gamma_{\mathrm{s}}^2\Gamma_{\mathrm{r}}^2
\end{aligned}
\tag{34b}
$$

where $\varepsilon(t-n\tau)$ is the unit step delayed an interval $n\tau$.

Note that all voltage waves at the sending end include a delay, except the first one.

5. Techniques for Electromagnetic Transients Analysis

5.1. Introduction

Several techniques can be used to analyze electromagnetic transients in power systems. Although most transients studies are presently based on digital simulation, those techniques applied before the development of the digital computer are still very useful, mainly for educational purposes.

Methods for transients analysis can be classified into two groups: analytical and numerical. Among the numerical methods there are two subgroups: time domain and frequency domain. The lattice diagram, introduced in the previous section, could be seen as a third category that is very useful when components with distributed parameters are included in the system model.

Before the development of the digital computer, many transient studies were performed with the help of network analyzers. A description of the characteristics of these tools or of those of the new real-time digital systems is out of the scope of this chapter.

The following subsections introduce the main principles of each technique, and include some examples that will illustrate their advantages and limitations.

5.2. Analytical Solution Techniques

5.2.1. The Laplace Transform

The performance of a power system during a transient can be described by a set of differential equations that often are derived from Electromagnetic and Circuit Analysis considerations. The simultaneous solution of these equations can be a difficult task. Several analytical techniques have been developed to provide an efficient and systematic procedure for solution of this problem. One of these techniques is the Laplace transform that converts integral and differential equations into algebraic relations.

The Laplace transform of a function of time, $f(t)$, is defined by the following expression (Van der Pol & Bremmer, 1955; Lathi B.P., 1992):

$$F(s) = \int_0^\infty f(t)e^{-st}dt \tag{35}$$

This integral transforms $f(t)$ into another function whose variable is s, which is a complex variable.

For a detailed description of the Laplace transform, the reader is referred to the specialized literature (Van der Pol & Bremmer, 1955; Lathi B.P., 1992).

The following subsection details the derivation of the Laplace-domain equivalents for basic circuit elements. A methodology for application of these equivalents in transient analysis is presented and applied to some simple test cases.

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Bibliography

Ametani A. (1973). The application of the Fast Fourier Transform to electrical transient phenomena, *Int. J. Elect. Eng. Educ.* 10, 277-287. [The use of the Fast Fourier Transform algorithm is introduced in this paper for the calculation of power system transients in combination with the Modified Fourier Transform technique].

Bergeron, L. (1949). *Du Coup de Belier en Hydraulique au Coup du Foudre en Électricité*, Paris, France: Dunod. [Classic text on Bergeron's method to analyze and predict both effects, water hammer in hydraulic pipes and voltage doubling at transmission lines].

Bewley L.V. (1963). *Traveling Waves on Transmission Systems*, New York, NY: Dover Publications Inc. [Text on traveling wave analysis of transmission lines. A graphic method developed by the author and known as the lattice diagram is described in detail].

CIGRE WG 33.02 (1990). Guidelines for Representation of Network Elements when Calculating Transients, CIGRE Brochure no. 39. [This text provides guidelines for selecting the proper models and representations of power systems elements at the various types of transient phenomena being analyzed; *i.e.*, slow, fast, very fast and ultrafast transients].

Day S.J., Battisson M.J., Mullineux N., Reed J.R. (1968). Developments in obtaining transient response using Fourier Transforms. Part III: Global response, *Int. J. Elect. Eng. Educ.* 6, 259-265. [This paper explores the tradeoffs when analyzing with frequency domain methods fast versus slow transients; or, short versus long term responses].

Day S.J., Mullineux N., Reed J.R. (1965). Developments in obtaining transient response using Fourier Transforms. Part I: Gibbs phenomena and Fourier integrals, *Int. J. Elect. Eng. Educ.* 3, 501-506. [Pioneering work on the numerical use of Fourier techniques in the analysis of electromagnetic power–system transients. It is shown here that frequency range truncation introduces Gibbs' errors and that these errors are decreased with the use of a Laczos window].

Day S.J., Mullineux N., Reed J.R. (1966). Developments in obtaining transient response using Fourier Transforms. Part II: Use of the modified Fourier Transform, *Int. J. Elect. Eng. Educ.* 4, 31-40. [The numerical treatment of transients by Fourier methods requires the sampling of all the involved signal spectra. The aliasing errors introduced by this sampling are controlled by the introduction of a damping factor, the resulting technique is referred to as the Modified Fourier Transform].

Dommel H.W. (1969). Digital computer solution of electromagnetic transients in single-and multiphase networks, *IEEE Trans. on Power Apparatus and Systems* 88, 388-399. [This is a classic paper on the basis of the Electromagnetic Transients Program (EMTP). Each basic circuit element (lumped) is represented in EMTP by a companion equivalent consisting of a conductance in parallel with an auxiliary current source. The companion circuits are obtained by the use of the trapezoidal rule of integration. As for distributed elements, each terminal of an ideal line (*i.e.*, the pure delay element) is represented by a companion circuit with the same form as the one for lumped elements which is obtained by means of Bergeron's method. A whole power system is thus represented by a matrix of conductances equipped with auxiliary current sources. The resulting system of nodal equations is solved and updated iteratively to predict the transient response for the represented power system].

Dommel H.W. (1996). *EMTP Theory Book*, Portland, OR, USA: Bonneville Power Administration. [A thorough description of the models and methods being employed in the Electromagnetic Transients Program].

Gole A.M., Martinez-Velasco J.A., Keri A.J.F. (Eds.) (1999). Modeling and Analysis of System Transients Using Digital Programs, *IEEE PES Special Publication*, TP-133-0. [This special publication presents an introduction to time-domain solution of electromagnetic transients in power systems using a digital computer. The publication covers two main topics: solution techniques and modeling of power components].

IEC TR 60071-4 (2004). Insulation co-ordination - Part 4: Computational guide to insulation co-ordination and modelling of electrical networks. [A technical report aimed at providing guidelines in terms of methods and models adapted to the use of numerical programs for conducting insulation co-ordination studies based of the approaches presented in IEC standard 60071].

Lathi B.P. (1992). *Linear Systems and Signals*, Berkeley: Cambridge Press. [This book provides a unified treatment of analysis techniques for continuous–time and discrete–time systems].

Mahseredjian J., Alvarado F. (1997). Creating an electromagnetic transients program in MATLAB: MatEMTP, *IEEE Trans. on Power Delivery* 12, 380-388. [This paper presents the creation of an EMTP-like transient analysis numerical simulator in the computational engine frame of MATLAB, an approach to software engineering that can afford a dramatic coding simplification for sophisticated algorithmic structures].

Mahseredjian J., Dubé L., Zou M., Dennetière S., Joos G. (2006). Simultaneous solution of control system equations in EMTP, *IEEE Trans. on Power Systems* 21, 117-124. [This paper presents a new Jacobian matrix-based formulation for eliminating numerical delays in the solution of control system equations].

Moreno P., Ramirez A. (2008). Implementation of the Numerical Laplace Transform: A review, *IEEE Trans. on Power Delivery* 23, 2599-2609. [This paper provides an updated review of the Numerical Laplace Transform (NLT) technique. Basic computer code in MatLab M–language is provided at the paper appendix to facilitate the use of the NLT by the paper readers].

Mullineux N., Reed J.R. (1973). Developments in obtaining transient response using Fourier transforms. Part IV: Survey of the theory, *Int. J. Elect. Eng. Educ.* 10, 256-267. [This paper provides a complete overview of the Modified Fourier Transform (MFT) technique for calculating power system transient responses. This technique served as basis for the Numerical Laplace Transform (NLT) technique].

Naredo J.L., Mahseredjian J., Kocar I., Gutiérrez-Robles J.A., Martinez-Velasco J.A. (2010). "Frequency domain aspects of electromagnetic transient analysis of power systems, in *IEEE/PES Tutorial on Electromagnetic Transients in Power Systems. Solution Techniques, Tools and Applications* (ed. J.A. Martínez-Velasco), IEEE Catalog Nr. 11TP255E, ISBN 978-1-4577-1501-3. [This text provides a tutorial on the use of frequency domain methods in the calculation of power system transients. It is aimed at providing full understanding of the basis of the Numerical Laplace Transform technique, as well as of its strengths and of its limitations].

Proakis J.G., Manolakis D.G. (2007). *Digital Signal Processing. Principles, Algorithms and Applications*, 4th Edition, Prentice-Hall. [An obligated text for the modern analysis of systems, signals and spectra by digital means].

Scott Meyer W., Dommel H.W. (1974). Numerical modeling of frequency dependent transmission-line parameters in an electromagnetic transients program, *IEEE Trans. on Power Apparatus Systems* **93**, 1401-1409. [Pioneering work is described in this paper on the accurate representation of frequency dependence effects on transient waves propagating along power lines. This paper introduces the long–convolution algorithm for the calculation of these frequency effects].

Van der Pol B., Bremmer H. (1955). *Operational Calculus Based on the Two-Sided Laplace Transform*, Cambridge, UK: Cambridge at the University Press. [This book provides a thorough review of the Laplace Transform theory. It is perhaps the most authoritative text in this subject].

Wedepohl L.M. (1983). Power system transients: errors incurred in the numerical inversion of the Laplace Transform", 26th *Midwest Symposium on Circuits and Systems*, 174-178. [This paper represents a major breakthrough in the accuracy that can be accomplished with the use of the Numerical Laplace Transform (NLT) technique].

Wilcox D.J. (1978). Numerical Laplace transformation and inversion, *Int. J. Elect. Eng.. Educ.* 15, 247-265. [In this paper the previously developed methodologies for the Modified Fourier Transform are conveniently translated into the theoretical framework of the Laplace Transform. The term Numerical Laplace Transform is coined here and, apparently, for the first time a criterion is given for the selection of a suitable Laplace damping coefficient].

Biographical Sketches

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