GROUNDWATER FLOW THROUGH FRACTURED ROCKS

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Summary

Fractured formations are fractured rocks cut by an interconnected system of fractures, resulting in blocks surrounded by a fracture network. One may distinguish between fractured formations with impervious blocks, e.g., granite, and fractured formations with pervious blocks (matrix) so called of *double porosity* (Barenblatt et al., 1960), e.g., limestone and sandstone. Due to the small porosity of the fracture network aquifers with impervious blocks cannot store large amounts of water and therefore such aquifers are not candidate for water supply. However, due to the relatively large permeability of the fractures, pollutants e.g., resulting from accidental spills, and radioactive wastes, may be conveyed relatively fast through the fracture network and pollute adjacent aquifers, Therefore the study of fractured formations with impervious blocks is of interest in connection to the possible pollution of aquifers. A large class of fractured formations of double porosity, commonly encountered, is characterized by negligible block permeability compared with that of the fracture network and negligible fracture porosity compared with that of the blocks. As a result, the permeability of the entire formation is represented by that of the fracture network and the porosity is represented by that of the porous blocks. Fractured rocks of double porosity play in important role in petroleum engineering and most progress in the study of such formation is due to the research carried out in this field. Relatively large amounts of oil accumulated in the porous blocks may be displaced by imbibition by the water flowing in the surrounding fractures. The theoretical basis and the equations of flow through porous fractured formations of double porosity were formulated through the continuum approach by Barenblatt et al. (1960). Warren and Root's (1963) presented a solution to Barenblatt equations for radial flow through a homogeneous aquifer and Braester and Zeitoun (1993) presented a stochastic model for flow through inhomogeneous fractured aquifer

of double porosity, based on Barenblatt et al. (1960) approach. These mathematical models and solutions to the model equations are presented and discussed.

1. Introduction

Fractured formations are fractured rocks cut by an interconnected system of fractures, resulting in blocks surrounded by a fracture network.

One may distinguish between fractured formations with impervious blocks, e.g., granite, and fractured formations with pervious blocks (matrix) so called of *double porosity* (Barenblatt et al., 1960), e.g., limestone and sandstone.

Due to the small porosity of the fracture network aquifers with impervious blocks cannot store large amounts of water and therefore such aquifers are not candidate for water supply. However, due to the relatively large permeability of the fractures, pollutants e.g., resulting from accidental spills, and radioactive wastes, may be conveyed relatively fast through the fracture network and pollute adjacent aquifers. Therefore the study of fractured formations with impervious blocks is of interest in connection to the possible pollution of aquifers.

A large class of fractured formations of double porosity, commonly encountered, is characterized by negligible block permeability compared with that of the fracture network and negligible fracture porosity compared with that of the blocks. As a result, the permeability of the entire formation is represented by that of the fracture network and the porosity is represented by that of the porous blocks.

To get an insight on the order of magnitude of the permeability and of the porosity of a fractured formation with impervious blocks, we consider a formation consisting of cubes of length L separated by fractures with parallel walls of width b (the parallel plate model). By analogy with the Hele-Shaw law, the permeability of an individual fracture equals $k_1 = b^2/12$, and the average permeability over a section perpendicular to one of the fracture, including both fractures and solid blocks, is $k = b^2/12(L+b)$. The hydraulic conductivity is $K = k\rho g/\mu$ where ρ is density, g is acceleration of gravity and μ is dynamic viscosity. The porosity of the fracture system is $\phi = [(L+b)^3 - L^3]/(L+b)^3$. As an example we consider a fracture width $b = 10^{-4}\text{m}$ and a block of length $L = 1\text{m}$. For water density equal $\rho = 1000\text{kg/m}^3$ viscosity $\mu = 10^{-3}\text{Pa.s}$ and acceleration of gravity $g = 9.81\text{m/s}^2$, we obtain a porosity $\phi = 3\times10^{-4}$ and a hydraulic conductivity $K = 8.17\times10^{-3}\text{m/s}$ or 706m/d. These figures differ by order of magnitude by that of an ordinary granular formation, e.g., sand for which porosity is 0.15 to 0.2 and hydraulic conductivity 20 to 30m/d. The hydraulic conductivity of a block (matrix) of a fractured formation of double porosity is of order of magnitude of 10^3 m/d, while the porosity may be of the same order of magnitude as the one of a ordinary granular formation.

Relatively large amounts of water may be accumulated in the medium of the porous blocks. However, the relatively low permeability of the porous blocks prevents extraction of industrial amounts of water from these types of aquifers. As in case of fractured formations with impervious blocks, porous fractured aquifers are not candidates for water supply, and the study of such formations is of interest in connection to aquifer contamination.

Fractured rocks of double porosity play in important role in petroleum engineering and most progress in the study of such formation is due to the research carried out in this field. Relatively large amounts of oil accumulated in the porous blocks may be displaced by imbibition by the water flowing in the surrounding fractures. Expelled from the blocks, the oil flows to the production wells through the fractures of large permeability. The process of oil production from fractured formations of double porosity is a two-phase flow process, and it is beyond the purpose of the hydrological aspects treated here.

Discrete modeling of fractured formations, consistent with their occurrence in nature, is the only one capable of simulating all flow phenomena taking place in reality. Discrete modeling requires the knowledge of the fracture network topology and of the individual fracture properties, such as, fracture aperture and filling material or fracture permeability and porosity. For fractured formations of double porosity, the porosity and the permeability of the blocks should also be known. Such a scale of representation of a fractured formation, at the level of the fractures, similar to the description of a conventional granular media at the scale of the pore, is obviously impossible.

Due to their complicated structure, the details of fractured rock formations will always remain inaccessible, and methods others than the discrete approach were used. A method commonly used in physics and which circumvents the need for a detailed description of the discontinuous is the *continuum approach*. According to the continuum approach the discrete medium is transformed to a continuum, with properties averaged over finite elementary volumes, so called by the physicists *Physical Points*. Attributing the average properties to the centroids of the Physical Points one obtains a fictitious continuum with properties defined at each mathematical point. Then, flow may be described through partial differential equations. The practical determination of the size of the Physical Point of real fractured formations is impossible, and its existence is postulated rather than determined.

In both discrete and the continuum approaches, we face from the outset the difficulty of practical determination of the rock formation parameters included in the mathematical models. The size of the Physical Point, of a fractured formation, probably of order of magnitude of tens of meters, over which spatially average parameters such as porosity and permeability are to be determined, precludes laboratory techniques developed for conventional granular media, and due to the degree of formation inhomogeneity, determination by field experiments would require an unrealistic number of field experiments.

The theoretical basis and the equations of flow through porous fractured formations of double porosity were formulated by Barenblatt et al. (1960). Analytical solutions to these equations have been obtained under the assumptions of constant formation properties such as constant fracture and block permeabilities and porosities, and constant coefficient of fluid transfer between fractures and blocks over the entire flow domain. Such assumptions, far from reality, make questionable the use of these solutions as type curves for the practical determination of rock properties and for prediction of aquifer performances. Numerical solutions which avoid such assumptions require a description of the properties of the formation at the scale of the cell of the discrete mesh, which for a real fractured formation is not possible.

Stochastic methods require only a limited amount of information, e.g., obtained from core samples. Some attempts have been made in the past, and efforts are continuing, to develop methods for fracture network generation through this approach (e.g., Long et al., 1982).

2. Barenblatt's et al. Mathematical Model

Models of flow through double porosity formations were proposed by Barenblatt et al. (1960), Bokserman et al. (1964), Odeh (1965), Kazemi et al. (1969), Najurieta (1976), De Swaan (1976), Da Prat et al. (1981), and others. However, Barenblatt et al. (1960) model described in the following is the more accepted one, and solutions to the model equations were provided by analytical and numerical methods. Analytical solutions to Barenblatt et al. (1960) equations were presented by Barenblatt et al. (1960), Barenblatt (1963), Warren and Root (1963), Barenblatt (1990), and others.

Barenblatt et al. (1960) consider the medium of the blocks and that of the fractures as overlapping continua over the entire flow domain. Darcy's and conservation of mass equations are written separately for the fluid in each medium. The transfer of fluid between the two media, fractures and blocks, is represented by a steady-state source/sink function in the equation of conservation of mass. It is considered to be a function of the pressure differences between the fluid in fractures and blocks, through a coefficient proportional to block permeability and to the block specific surface.

Denoting the block and fracture media by subscripts 1 and 2, respectively, for horizontal flow the equations are

Darcy's law

$$\mathbf{u}_1 = -\frac{1}{\mu}\nabla\cdot(k_1\nabla p_1) = 0 \qquad (1)$$

and

$$\mathbf{u}_2 = -\frac{1}{\mu}\nabla\cdot(k_2\nabla p_2) = 0\,, \qquad (2)$$

where $\mathbf{u}$ is Darcy's flux, k is permeability, μ is dynamic viscosity, and p is pressure.

The equations of conservation of mass are

$$\phi_1 c_1 \frac{\partial p_1}{\partial t} + u_1 + \frac{ak_1}{\mu}(p_1 - p_2) = 0 \;, \qquad (3)$$

$$\phi_2 c_2 \frac{\partial p_2}{\partial t} + \mathbf{u}_2 - \frac{ak_1}{\mu}(p_1 - p_2) = 0 \,, \qquad (4)$$

where ϕ porosity, c is compressibility and a is a parameter, proportional to the specific surface of the block, controlling the transfer of liquid between fractures and blocks. Substitution of Darcy's law in the equations of conservation of mass yields

$$\phi_1 c_1 \frac{\partial p_1}{\partial t} - \frac{1}{\mu}\nabla \cdot (k_1 \nabla p_1) + \frac{ak_1}{\mu}(p_1 - p_2) = 0 \quad , \qquad (5)$$

$$\phi_2 c_2 \frac{\partial p_2}{\partial t} - \frac{1}{\mu}\nabla \cdot (k_2 \nabla p_2) - \frac{ak_1}{\mu}(p_1 - p_2) = 0 \quad . \qquad (6)$$

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Biographical Sketch

Carol Braester is Professor with Technion-Israel Institute of Technology. He holds a degree in Engineering from the Polytechnic Institute of Romania, and MSc. and DSc. degrees form Technion-Israel Institute of Technology. He taught at the Norwegian Institute of Technology, at the Swedish Royal Institute of Technology, at the Western Australian Institute of Technology, and has been scientific adviser to International Research Organizations. His research activity covers a large spectrum of problems such as groundwater artificial replenishment, groundwater flow with heat transfer, pollution of aquifers by radioactive waste repositories, aquifer contaminated restoration, storage of compressed air in aquifers, storage of oil in unlined caverns in aquifers, and drilling strategies of fractured rock formations. His research achievements are summarized in more than 90 publications. Braester got international recognition in the research of fractured rock formations. He showed that the *block size*, a significant parameter of fractured formations cannot be determined using the transition curve of a pumping test as previously believed, and presented analytical solutions to generalized Barenblatt's equations to two-phase immiscible flow, and to flow through inhomogeneous fractured formations.