THE SCALING PROPERTIES OF GEOLOGICAL MEDIA WITH RESPECT TO GROUNDWATER FLOW AND TRANSPORT

Charles W. Rovey II
Southwest Missouri State University, Springfield, MO, USA

Keywords: hydraulic conductivity, dispersivity, scale effects.

Contents

Summary

The scale dependency of dispersivity (α) is well established, but the scale dependency of hydraulic conductivity is less known and more controversial. The best argument in favor of a true scale dependence in hydraulic conductivity (K) is the set of *ad hoc* assumptions, which must simultaneously be true, if the scale increase is caused solely by systematic bias: (1) The degree of bias must increase in direct proportion to decreasing sample size; (2)

This same bias must occur whether testing with different methods of different support or with a single method with changing support; (3) The degree of bias must be consistent for similar media, but differ substantially among media with different types of pore systems; (4) The bias must decrease in proportion to the degree of homogeneity and disappear in the most homogeneous media; and (5) The correspondence between field results and digital simulations must be caused by some heretofore unknown flaw in the models which mimics the (postulated) biases of field tests.

Implications of the scale/dimension dependence of K are both subtle and obvious. Obviously, investigators should conduct test measurements at the same scale or dimension as the desired application. Small-scale lab and slug tests will not generally predict large-scale water flux. Alternatively, several authors have shown, that in at least some media, the K applicable to transient solute migration is closer to the smaller-scale, two-dimensional (2-D) value measured by slug tests.

Due to dispersion and diffusion the solute cannot remain isolated within the highest-K heterogeneity which conveys a disproportionate amount of water flux in a regional-flow system or during a pumping test. Thus, solute plumes must cross heterogeneity more than the pure water flux, and in effect, the K applicable to solute migration has a lower dimensionality. Therefore, the K applicable to transient solute flux is weighted more toward the low-K heterogeneity and is closer to the 2-D value measured by slug tests.

Many workers have recognized that small-scale values of K do not accurately predict the large-scale behavior of well fields and oil reservoirs. Therefore, much geostatistical work has been devoted to the problem of "upscaling," an attempt to extrapolate small-scale measurements (e.g. permeameter) of K to more representative values over the entire medium. However, these models are invariably based on the assumption that such lab values are true "point values" of K.

Therefore, the large-scale K would be bound between the harmonic and arithmetic mean of the lab measurements and could be determined by finding the proper weighting function for the "point values." However, such "point values" are hypothetical constructs based on the assumption of homogeneous conditions about a representative elementary volume (REV), such that flow and boundary conditions of the test have no effect on the measured value.

As shown by the data in Table 2, however, such "point values" are generally dubious. Because heterogeneity is usually present, even within small test volumes of a permeameter, the resulting values tend toward the harmonic mean of the local K distribution. Therefore, the K_{eff} at a larger scale can be substantially greater than even the arithmetic mean of these lab values.

Finally, consideration of the scale dependency in K has led to the realization that convergent flow in heterogeneous media locally increases drawdown in the immediate vicinity of the well, relative to that predicted by the measured K. Therefore, standard methods for measuring wellskins are suspect.

The standard procedure typically involves a comparison of the measured drawdown in the pumping well with that predicted from the measured K (and hence transmissivity). If the measured drawdown is greater than predicted, a positive wellskin and corresponding "formation damage" is inferred.

However, these results show that at least part of any additional drawdown is an effect of the pseudoskin caused by the convergent flow near the well. Therefore, the actual skin and corresponding well damage are routinely overestimated, because the drawdown comparison is inappropriate.

1. Introduction

Two of the most fundamental parameters in groundwater hydrology, hydraulic conductivity (K) and dispersivity (α), are scale dependent. The term "scale dependent" means that the average measured value at a small scale (or support) does not equal the average value in the same medium measured over some larger volume or support. Typically K and α increase with support to a certain threshold, beyond which respective values are approximately constant (Figure 1).

This fact is important for virtually all hydrogeologic problems and investigations. Small-scale tests and measurements are easier and more economical to complete, but the resulting values may be irrelevant to the desired scale of application.

Since testing is generally done for predictive purposes, several practical questions arise: (1) How similar must the test scale be to the problem scale? (2) Is there a valid threshold of support above which the medium functions as an "equivalent homogeneous medium" and the parameters are constant with scale? and (3) Can small-scale measurements be manipulated with statistical parameters to predict the effective value at a larger scale?

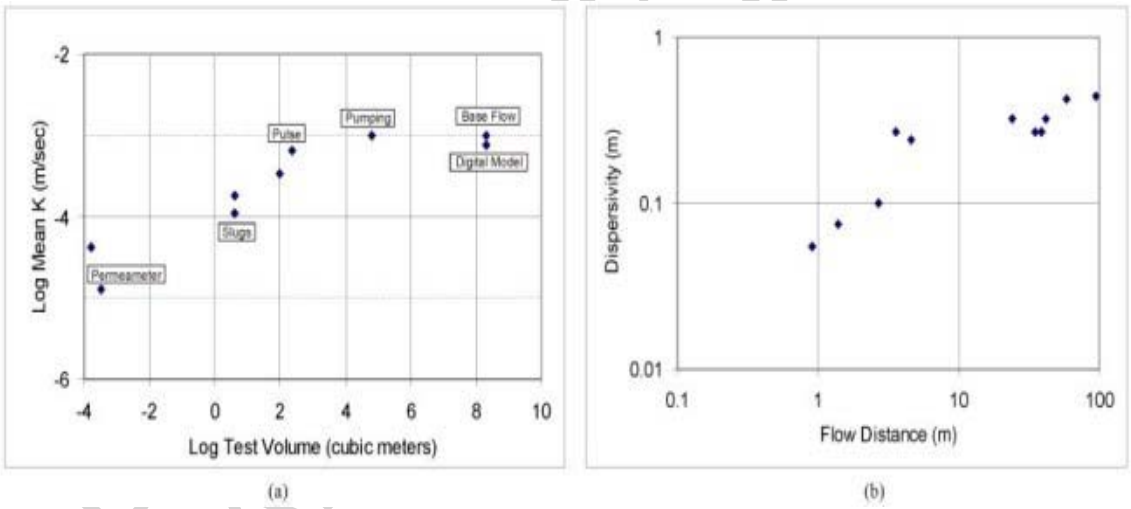

Figure 1. Scale dependency of hydraulic conductivity and dispersivity. (a) Arithmetic mean hydraulic conductivity versus test volume. Material is glacial outwash at the Camp Dodge site (see Bibliography). For radial tests (slugs, pulse, and pumping) the test volume is calculated using aquifer thickness and radius of influence as defined and measured by Rovey (1988). Note the decrease in mean K below a critical volume corresponding to a radius of influence of approximately 12 m, coinciding closely with the range (length of statistical correlation) in the spatial distribution of K. See Figure 2 for number of measurements and variance of each respective test method. (b) Dispersivity versus distance at the Borden site (see Bibliography). Source: Freyberg (1986).

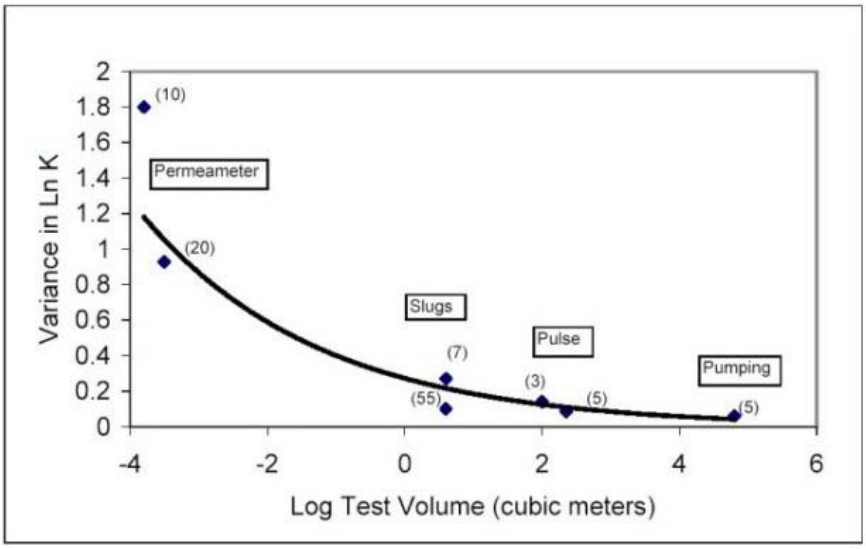

Figure 2. Decrease in variance of lnK with increasing measurement support: outwash sand, Camp Dodge site. Appended numerals are the number of measurements for each respective group.

2. Dispersivity

2.1. Classical Approach

The scale dependency of α was noticed soon after hydrologists began applying the mass-transport equation to model solute transport. Assuming a conservative tracer:

$$\underset{(1)}{\frac{\partial(D_{ij}\,\partial c)}{\partial x_i}} - \underset{(2)}{\frac{\partial(cv_i)}{\partial x_i}} = \underset{(3)}{\frac{\partial c}{\partial t}} \qquad (1)$$

where D_{ij} is the dispersion coefficient (l^2/t), c is the solute concentration, v_i is the velocity, and:

$$D_{ij} = \alpha_{ij} v_i \qquad (2)$$

where α_{ij} (l) is the dispersivity tensor. Here the tensor form of the dispersion coefficient is treated as a vector, concentrating on the longitudinal coefficients D_l and $\alpha_{l,}$, those applicable to spreading in the mean direction of groundwater flow. The second term in Eq. 1 describes migration of a solute's center of mass in the direction of the mean macroscopic groundwater velocity. Since the actual velocity varies in magnitude and direction at a small immeasurable scale, the first term in Eq. 1 is employed to describe dispersion (or spreading) of the solute away from the plume's center of mass because of random small-scale variation in flow velocity. Because the spreading or dispersion is proportional to lateral variation in velocity, which in turn is caused by heterogeneity in K, α represents a macroscopic measure of the variability in the K field: that is, its heterogeneity. In simple terms, the greater the heterogeneity, the larger the α.

The portion within parentheses in the first term of Eq. 1 follows from Fick's Law, which assumes that the dispersive flux is directly proportional to the concentration gradient. Thus *D*, and indirectly α, can be expressed as proportionality constants. Although it is not directly obvious from Eq. 1, *D* can then be expressed as a ratio between the spatial variance in solute concentration within a plume and the mean travel distance:

$$D_1 = \sigma_1^2 v_1 / 2x_1 \Rightarrow \alpha_1 = \sigma_1^2 / 2x_1, \qquad (3)$$

where σ_l^2 is the variance in displacement about the mean travel distance, and x is the distance from the source. Eq. 3 implies that for a constant D_l (and α_l) the spatial variance in concentration will vary linearly with flow distance. Systems adequately described by Eqs. 1 to 3 using a constant *D* (or α) are therefore termed "Fickian."

By the 1960s it was widely known that α measured at the lab scale (0.1 to 1.0 m) in column experiments cannot be applied to field-scale problems. Column experiments typically yield values of α < 0.01 m (Figure 3). Field-measured αs, however, are typically > 0.05m and increase with flow distance.

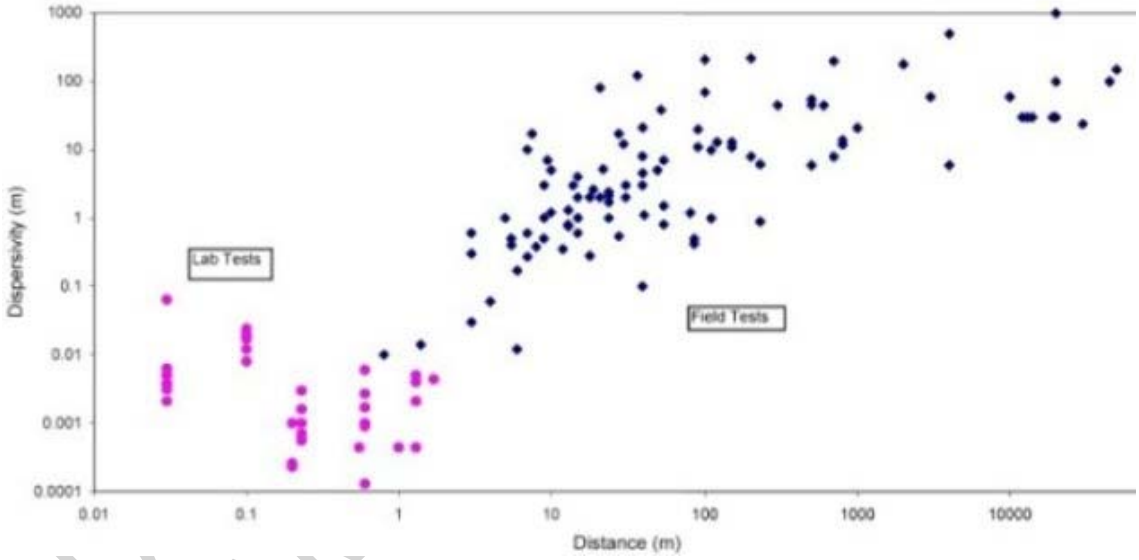

Figure 3. Measured dispersivity at various sites. Field values are from Figure 3 in Gelhar et al., 1992; lab values are from Neuman (1990) and Schulze-Makuch (1996).

The obvious qualitative explanation for the scale dependency shown in Figures 1b and 3 is that field tracers encounter a progressively greater range in heterogeneity, and hence, field αs must increase with flow distance. Nevertheless, prior to the 1980s the general notion of scale-dependent α was still contentious, because very few tracer tests had been reported for sequential distances at the same site.

Alternative explanations could account for some of the scale dependency shown in Figure 3. Some field determinations are biased toward large values by inappropriate sampling techniques. If the vertical scale of sampling (e.g. in a fully penetrating well) is greater than the plume thickness, the sampling process will mix the solute with

background water, thereby artificially increasing mixing and yielding an inappropriately large α. Additionally, employing analytical solutions based on the assumption of steady flow can cause substantial overestimation of α, if the solutions are applied to field problems with large temporal variations in hydraulic gradient. Finally, scale dependency could also be partly attributable to the use of inappropriate analytical expressions (solutions to Eq. 1) to solve for α from field-measured concentrations. Specifically, if one applies an expression of lower dimensionality than the actual field problem (i.e. using a simpler 2-D solution to Eq. 1, instead of a more complex 3-D form), the solution will progressively overestimate α along the flow path. Some spreading of solute occurs in the neglected dimension, so the calculated value of α ascribes the neglected spreading to the included dimension(s), thus overestimating α.

Another complication is apparent upon close inspection of Figure 3. Although the extrapolated trend of field-α decreases to values within the range of laboratory (column) tests, the two sets of values have distinctly different trends. Field values tend to increase with distance, while lab values apparently decrease. Therefore, additional factors beyond heterogeneity may account for the differences.
In summary, (1) Field-measured αs are greater than laboratory values; (2) Field αs generally increase with flow distance; (3) Some of this increase is likely due to bias, specifically test and analytical shortcomings; and (4) A complete explanation for the difference in magnitude between lab and field values may require additional factors beyond heterogeneity. Over the past 25 years these results and uncertainties have provided the impetus for a number of carefully controlled field studies to characterize the dispersion process empirically, and an even greater number of theoretical studies treating mass transport as a stochastic process.

2.2. The Stochastic Approach

2.2.1. Geostatistical Models

Most stochastic models are ultimately based on geostatistical models of heterogeneity. Geostatistical models reflect the fact that parameter values such as K are spatially correlated, meaning that values at nearby locations are generally more similar than those separated by larger distances. Spatial correlation can be quantified and presented in a number of related ways. The most common is the autovariance (or semivariance, Figure 4a):

$$\Gamma(\mathrm{h}) = 1/2\mathrm{E}[\mathrm{K}(\mathrm{z}+\mathrm{h}) - \mathrm{K}(\mathrm{z})]^2 \qquad (4)$$

where $\Gamma(h)$ is the variance between values at a specified separation or lag; z is a given spatial location; h is the distance or lag to z; $K(z)$ denotes hydraulic conductivity measured at z; and $E[\ ]$ denotes the expected or average value over all paired samples at a given lag. Thus, $\Gamma(h)$ is half the average squared difference between paired values of K separated by a distance h. As the lag (h) increases, paired values of K become increasingly dissimilar (uncorrelated), and the magnitude of $\Gamma(h)$ increases. When the maximum distance of spatial correlation is reached $\Gamma(h)$ becomes constant. The constant value of $\Gamma(h)$ is the sill, equal to the population sample variance, and the lag distance (h)

to the sill is the range. Thus, the variogram quantifies the magnitude of heterogeneity at different scales.

The covariance function (autocovariance) is an alternative yet equivalent way of expressing the same spatial correlation (Figure 4b):

$$C(h) = E[K(z+h)*K(z)] - E[K(z)]^2 = C(0) - \Gamma(h), \qquad (5)$$

where $C(h)$ is the covariance between points at a given lag, and $C(0)$ equals the variogram sill value. The correlogram (Figure 4c) is normalized from the correlation function:

$$\rho(h) = C(h)/\sigma_{-h}\sigma_{+h} = C(h)/C(0) \qquad (6)$$

where $\rho(h)$ is the correlation function (covariance normalized by respective standard deviations at plus or minus one lag distance). The lag distance at which $\rho(h)$ decreases to e^{-1} (0.37) is the correlation scale (λ), the *average* distance over which K is statistically correlated.

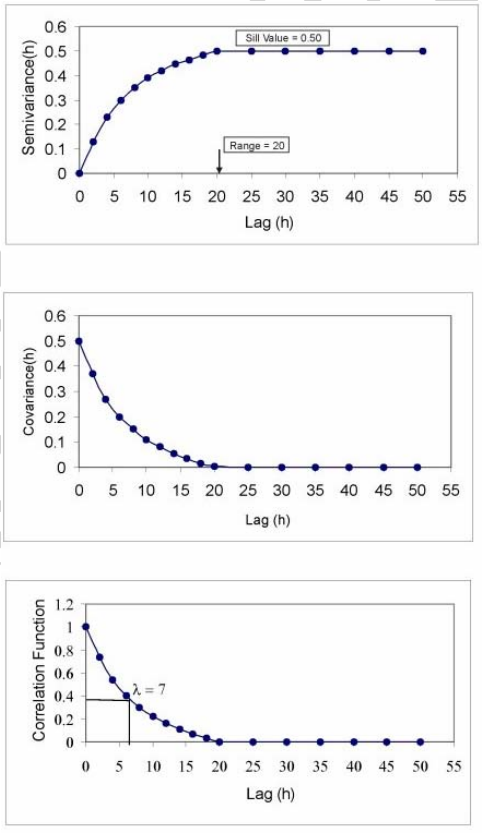

Figure 4. Geostatistical parameters. (a) Semivariance. The range value is the maximum distance over which the measurement parameter is spatially correlated. (b) Covariance. (c) Correlation function. The correlation scale (λ) is the average distance over which the measurement parameter is spatially correlated.

-
-
-

Bibliography

Adams E.E. and Gelhar L.W. (1992). Field study of dispersion in a heterogeneous aquifer 2. Spatial moments analysis. *Water Resources Research* **28**(12), 3293–3307. [Field data from the Columbus Air Force Base, Table 1.]

Berg P.L., Hinsby K., Christensen T.H., and Gravesen P. (1992). Spatial variability of hydraulic conductivity of an unconfined sandy aquifer determined by a min slug test. *Journal of Hydrology* **136**, 107–122. [Field data from the Vejen Site, Table 1.]

Boggs J.M., Young S.C., and Beard L.M. (1992). Field dispersion in a heterogeneous aquifer 1. Overview and site description. *Water Resources Research* **28**(12), 3281–3291. [Field data from the Columbus Air Force Base, Table 1.]

Bradbury K.R. and Muldoon M.A. (1990). Hydraulic conductivity determinations in unlithified glacial and fluvial materials. *Ground Water and Vadose Zone Monitoring* (ed. D.M. Nielson and A.I. Johnson), pp. 138–151. American Society for Testing and Materials, ASTM STP 1142. [One of the original papers demonstrating a scale dependency in K.]

Clausser C. (1992). Permeability of crystalline rocks. *EOS* **73**(21), 233–240. [A summary of scale-dependent K in crystalline rocks.]

Desbarats A.J. (1987). Numerical estimation of effective permeability in sand-shale formations. *Water Resources Research* **23**(2), 273–286. [Numerical verification that the K_{eff} of 3-D flow systems is generally between the geometric and arithmetic means of the local K values.]

Desbarats A.J. (1992). Spatial averaging of transmissivity in heterogeneous fields with flow toward a well. *Water Resources Research* **28**(3), 757–767. [Demonstrates the existence of a pseudoskin around a wellbore caused by radial convergent flow across heterogeneity.]

Deutsch C. (1989). Calculating effective absolute permeability in sandstone/shale sequences. *SPE Formation Evaluation*, September, 343–348. [Additional demonstration that the 3-D K is between the geometric and arithmetic means of the local K values.]

Döll P. and Schneider W. (1995). Lab and field measurements of the hydraulic conductivity of clayey silts. *Ground Water* **33**(6), 884–891. [Data for Figure 5.]

Earlougher R.C. Jr. (1977). *Advances in Well Test Analysis.* Society of Petroleum Engineers Monograph 5. Dallas, Texas. 201 pp. [Standard reference for skin and borehole storage effects; data for Figure 7a.]

Freyberg D.L. (1986). A natural gradient experiment on solute transport in a sand aquifer 2. Spatial moments and the advection and dispersion of nonreactive tracers. *Water Resources Research* **22**(13), 2031–2046. [Field data for the Borden site, Table 1.]

Garabedian S.P., LeBlanc D.R., Gelhar L.W., and Celia M.A. (1991). Large-scale natural gradient tracer test in sand and gravel, Cape Cod, Massachusetts 2. Analysis of spatial moments for a nonreactive tracer. *Water Resources Research* **27**(5), 911–924. [Field data for the Cape Cod site, Table 1.]

Gelhar L.W. and Axness C.L. (1983). Three-dimensional stochastic analysis of macrodispersion in aquifers. *Water Resources Research* 1**9**(1), 161–180. [Derivation of Eq. 9.]

Gelhar L.W., Gutjahr A.L., and Naff R.L. (1979). Stochastic analysis of macrodispersion in a stratified aquifer. *Water Resources Research* **15**(6), 1387–1397. [Derivation of Eq. 8.]

Gelhar L.W., Welty C., and Rehfeldt K.R. (1992). A critical review of data on field-scale dispersion in aquifers. *Water Resources Research* **28**(7), 1955–1974. [Data for Figure 3.]

Hess K.M., Wolf S.H., and Celia M.A. (1992). Large-scale natural gradient tracer test in sand and gravel, Cape Cod, Massachusetts 3. Hydraulic conductivity variability and calculated macrodispersivities. *Water Resources Research* **28**(8), 2011–2027. [Data for Cape Cod site, Table 1.]

Hinsby K., Bjerg P.L., Andersen L.J., Skov B., and Clausen E.V. (1992). A mini slug test method for determination of a local hydraulic conductivity of an unconfined sandy aquifer. *Journal of Hydrology* **136**, 87–106. [Data for Vejen site, Table 1.]

Hvilshøj S., Jensen K.H., Barlebo H.C., and Madsen B. (1999). Analysis of pumping tests of partially penetrating wells in an unconfined aquifer using inverse numerical optimization. *Hydrogeology Journal* **7**, 365–379. [Data for Vejen site, Table 1.]

Isaaks E.H. and Srivastava R.M. (1989). *An Introduction to Applied Geostatistics.* New York: Oxford University Press, 565 pp. [A general discussion of geostatistics and development of Eqs. 4–6.]

Jensen K.H., Bitsch K., and Bjerg P.L. (1993). Large-scale dispersion experiments in a sandy aquifer in Denmark: observed tracer movements and numerical analyses. *Water Resources Research* **29**(3), 673–696. [Data for Vejen site, Table 1.]

Killey R.W.D. and Moltyaner G.L. (1988). Twin Lake tracer tests: setting, methodology, and hydraulic conductivity distribution. *Water Resources Research* **24**(10), 1585–1612. [Data for Twin Lake site, Table 1.]

LeBlanc D.R., Garabedian S.P., Hess K.M., Gelhar L.W., Quadri R.D., Stollenwerk K.G., and Wood W.W. (1991). Large-scale natural gradient tracer test in sand and gravel, Cape Cod, Massachusetts 1. Experimental design and observed tracer movement. *Water Resources Research* **27**(5), 895–910. [Data for Cape Cod site, Table 1.]

Mackay D.M., Freyberg D.L., Roberts P.V., and Cherry J.A. (1986). A natural gradient experiment on solute transport in a sand aquifer 1. Approach and overview of plume movement. *Water Resources Research* **22**(13), 2031–2046. [Data for Borden site, Table 1.]

Mercado A. (1967). The spreading pattern of injected water in a permeability stratified aquifer. *Symposium of Haifa, Artificial Recharge and Management of Aquifers,* pp. 23–36. IHSA publication 72. Gentbrugge, Belgium: International Association of Scientific Hydrology. [The original derivation of Eq. 7.]

Moltyaner G.L. (1986). Stochastic versus deterministic: a case study. *Hydrogeologie* **2**, 183–196. [Data for Twin Lake site, Table 1.]

Moltyaner G.L. and Killey R.W.D. (1988). Twin Lake tracer tests: longitudinal dispersion. *Water Resources Research* **24**(10), 1613–627. [Data for Twin Lake site, Table 1.]

Niemann W.L. and Rovey C.W. (2000). Comparison of hydraulic conductivity values obtained from aquifer pumping tests and conservative tracer tests. *Ground Water Monitoring and Remediation* **20**(3), 122–128. [Description of the Camp Dodge site; demonstrates the importance of dimensionality in measuring K.]

Neuman S.P. (1990). Universal scaling of hydraulic conductivities and dispersivities in geologic media. *Water Resources Research* **26**(8), 1749–1758. [Data for Figure 3.]

Neuman S.P. (1994). Generalized scaling of permeabilities: validation and effect of support scale. *Geophysical Research Letters* **21**(5), 349–352. [Presentation of Eq. 11.]

Parker J.C. and Albrecht K.A. (1987). Sample volume effects on solute transport predictions. *Water Resources Research* **23**(12), 2293–2301. [Data for Figure 5.]

Rayne T.W. (1993). *Variability of Hydraulic Conductivity in Sandy Till: The Effects of Scale and Method.* 134 pp. Ph.D. dissertation, University of Wisconsin–Madison. [Demonstrates scale dependency of K in synthetic media, wherein skin effects are impossible.]

Rehfeldt K.R., Boggs J.M., and Gelhar L.W. (1992). Field study of dispersion in a heterogeneous aquifer 3. Geostatistical analysis of hydraulic conductivity. *Water Resources Research* **28**(12), 3309–3324. [Data for Columbus Air Force Base, Table 1.]

Rovey C.W. (1998). Digital simulation of the scale effect in hydraulic conductivity. *Hydrogeology Journal* **6**, 216–225. [Data for Figure 6.]

Rovey C.W. and Cherkauer D.S. (1995). Scale dependency of hydraulic conductivity measurements. *Ground Water* **33**(5), 769–780. [One of the first papers showing empirical evidence for the scale dependency of K.]

Rovey C.W. and Niemann W.L. (2001). Wellskins and slug tests: where's the bias? *Journal of Hydrology* 243, 120–132. [Shows that the smaller value of slug-test K, compared to pumping tests, is not an artifact of wellskins.]

Schulze-Makuch D. (1996). *Facies Dependent Scale Behavior of Hydraulic Conductivity and Longitudinal Dispersivity in the Carbonate Aquifer of Southeastern Wisconsin*. Ph.D. dissertation, University of Wisconsin–Milwaukee. [Data for Figure 3.]

Schulze-Makuch D., Carlson D.A., Cherkauer D.S., and Malik P. (1999). Scale dependency of hydraulic conductivity in heterogeneous media. *Ground Water* **37**(6), 904–919. [The most comprehensive report to date on the scale dependence of K; presentation of Eq. 10.]

Schulze-Makuch D. and Cherkauer D.S. (1998). Variations in hydraulic conductivity with scale of measurement during aquifer testing in heterogeneous, porous carbonate rocks. *Hydrogeology Journal* **6**, 204–215. [Tabulation of pumping-test results with an increase in K during the test.]

Sudicky E.A. (1986). A natural gradient experiment on solute transport in a sand aquifer: spatial variability of hydraulic conductivity and its role in the dispersion process. *Water Resources Research* **22**(13), 2069–2082. [Data for Borden site, Table 1.]

Tidwell V.C. and Wilson J.J. (1997). Laboratory method for investigating permeability upscaling. *Water Resources Research* **33**(7), 1607–1616. [Data for Table 2.]

Tidwell V.C. and Wilson J.L. (1999). Permeability upscaling measured on a block of the Berea Sandstone: results and interpretation. *Mathematical Geology* **31**(7), p. 759–769. [Data for Table 2.]

Biographical Sketch

Charles W. Rovey II received a B.A. from Augustana College (Rock Island, Illinois, USA) and M.S. and PhD. degrees in geoscience from the University of Wisconsin–Milwaukee. He is currently an associate professor in the Department of Geography, Geology, and Planning at Southwest Missouri State University. Research interests include regional groundwater flow through carbonate sequences, scale dependency of hydraulic parameters, and stratigraphy and paleomagnetism of the pre-Illinoian glacial sequence in the midcontinental USA.