SUBSPACE IDENTIFICATION METHODS

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Contents

Summary

This chapter gives a short introduction to and survey of subspace identification algorithms. Deterministic, stochastic and combined deterministic-stochastic subspace identification algorithms are treated. These methods estimate state sequences directly from the given data, either explicitly or implicitly, through an orthogonal or oblique projection of the row spaces of certain block Hankel matrices of data into the row

spaces of other block Hankel matrices, followed by a singular value decomposition (SVD) to determine the order, the observability matrix and /or the state sequence. The extraction of the state space model is then achieved through the solution of a least squares problem. Each of these steps can be elegantly implemented using well-known numerical linear algebra algorithms such as the singular value decomposition and the QR decomposition.

1. Introduction

This Section contains a description of the central ideas of this chapter. First, in Section 1.1, we describe state space models, which is the type of models that is delivered by subspace identification algorithms. In Section 1.2 we explain how subspace identification algorithms work.

1.1. State Space Models

Models in this chapter are lumped, discrete time, linear, time-invariant, state space models. From the number of epithets used, this might seem like a highly restricted class of models (especially the fact that they are linear), but, surprisingly enough, many industrial processes can be described very accurately by this type of models, especially locally in the neighborhood of a working point. Moreover, there is a large number of control system design tools available to build controllers for such systems and models.

Mathematically, these models are described by the following set of difference equations:

$$\begin{cases} x_{k+1} &= Ax_k + Bu_k + w_k, \\ y_k &= Cx_k + Du_k + v_k, \end{cases} \tag{1}$$

with

$$\mathbf{E}[\begin{pmatrix} w_p \\ v_p \end{pmatrix}\begin{pmatrix} w_q^T & v_q^T \end{pmatrix}] = \begin{pmatrix} Q & S \\ S^T & R \end{pmatrix}\delta_{pq} \geq 0. \tag{2}$$

E denotes the expected value operator and δ_{pq} the Kronecker delta.

In this model, we have

- **vectors:** The vectors $u_k \in \mathbf{R}^m$ and $y_k \in \mathbf{R}^l$ are the observations at time instant *k* of respectively the *m* inputs and *l* outputs of the process. The vector $x_k \in \mathbf{R}^n$ is the state vector of the process at discrete time instant *k* and contains the numerical values of n states. $v_k \in \mathbf{R}^l$ and $w_k \in \mathbf{R}^n$ are unobserved vector signals, usually called the measurement, respectively process noise. It is assumed that they are zero mean, stationary, white noise vector sequences. (The Kronecker delta in (2) means $\delta_{pq} = 0$ if $p \neq q$, and

$\delta_{pq} = 1$ if $p = q$.) The effect of the process w_k is different from that of v_k: w_k as an input will have a dynamic effect on the state x_k and output y_k, while v_k only affects the output y_k directly and therefore is called a measurement noise.

- **matrices:** $A \in \mathbf{R}^{n \times n}$ is called the (dynamical) system matrix. It describes the dynamics of the system (as characterized by its eigenvalues). $B \in \mathbf{R}^{n \times m}$ is the input matrix, which represents the linear transformation by which the deterministic inputs influence the next state. $C \in \mathbf{R}^{l \times n}$ is the output matrix , which describes how the internal state is transferred to the outside world in the observations y_k. The term with the matrix $D \in \mathbf{R}^{l \times m}$ is called the direct feedthrough term. The matrices $Q \in \mathbf{R}^{n \times n}$, $S \in \mathbf{R}^{n \times l}$ and $R \in \mathbf{R}^{l \times l}$ are the covariance matrices of the noise sequences w_k and v_k. The block matrix in (2) is assumed to be positive definite, as is indicated by the inequality sign. The matrix pair $\{A, C\}$ is assumed to be observable, which implies that all *modes* in the system can be observed in the output y_k and can thus be identified. The matrix pair $\{A, [B\ Q^{1/2}]\}$ is assumed to be controllable, which in its turn implies that all *modes* of the system can be excited by either the deterministic input u_k and/or the stochastic input w_k.

A graphical representation of the system can be found in Figure 1.

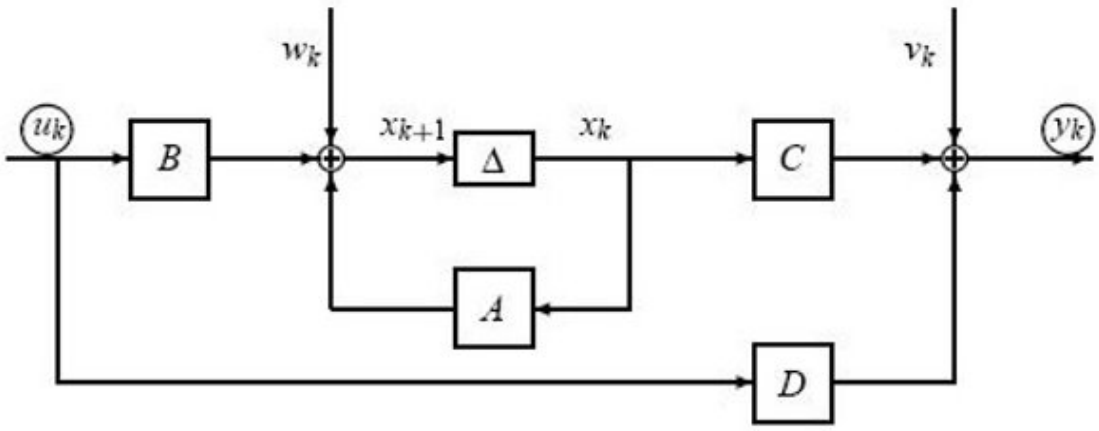

Figure 1: The (circled) vector signals u_k and y_k are available (observed) while v_k, w_k are unknown disturbances. The symbol Δ represents a delay. Note the inherent feedback via the matrix *A* (which represents the dynamics). Sensor or actual dynamics are completely contained in *A* too. It is assumed that u_k is available without measurement noise.

We are now ready to state the main mathematical problem of this chapter.

Given s consecutive input and output observations $u_0,\ldots,u_{s-1}$, and $y_0,\ldots,y_{s-1}$. Find an appropriate order n and the system matrices A,B,C,D,Q,R,S.

1.2. The Basic Idea behind Subspace Identification Algorithms

The goal of this Section is to provide a verbal description of the main principles on which subspace identification algorithms are based. The mathematical derivations will be elaborated on in the next sections.

Subspace identification algorithms are based on concepts from system theory, (numerical) linear algebra and statistics. The main concepts in subspace identification algorithms are

1. The *state sequence of the dynamical system* is determined first, directly from input/output observations, without knowing the model. That this is possible for the model class (1) is one of the main contributions of subspace algorithms, as compared to "classical" approaches that are based on an input-output framework. The difference is illustrated in Figure 2. So an important achievement of the research in subspace identification was to demonstrate how the Kalman filter states can be obtained directly from input-output data using linear algebra tools (QR and singular value decomposition) without knowing the mathematical model. An important consequence is that, once these states are known, the identification problem becomes a linear least squares problem in the unknown system matrices, and the process and measurement noise covariance matrices follow from the least squares residuals, as is easy to see from Eq. (1):

$$\underbrace{\begin{pmatrix} x_{i+1} & x_{i+2} & \cdots & x_{i+j} \\ y_i & y_{i+1} & \cdots & y_{i+j-1} \end{pmatrix}}_{\text{known}} = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \underbrace{\begin{pmatrix} x_i & x_{i+1} & \cdots & x_{i+j-1} \\ u_i & u_{i+1} & \cdots & u_{i+j-1} \end{pmatrix}}_{\text{known}} + \begin{pmatrix} w_i & w_{i+1} & \cdots & w_{i+j-1} \\ v_i & v_{i+1} & \cdots & v_{i+j-1} \end{pmatrix}. \quad (3)$$

The meaning of parameters i and j will become clear henceforth.

Even though the state sequence can be determined explicitly, in most variants and implementations, this is not done explicitly but rather implicitly. Said in other words, the set of linear equations above can be solved 'implicitly' as will become clear below, without an explicit calculation of the state sequence itself. Of course, when needed, the state sequence can be computed explicitly.

The two main steps that are taken in subspace algorithms are the following.

(a) Determine the model order n and a state sequence $\hat{x}_i, \hat{x}_{i+1},\ldots,\hat{x}_{i+j}$ (estimates are denoted by $\hat{\cdot}$). They are typically found by first projecting row spaces of

data block Hankel matrices and then applying singular value decomposition (see Sections 4, 5, 6).

(b) Solve a least squares problem to obtain the state space matrices:

$$\begin{pmatrix} \hat{A} & \hat{B} \\ \hat{C} & \hat{D} \end{pmatrix} = \min_{A,B,C,D} \left\| \begin{pmatrix} \hat{x}_{i+1} & \hat{x}_{i+2} & \cdots & \hat{x}_{i+j} \\ y_i & y_{i+1} & \cdots & y_{i+j-1} \end{pmatrix} - \begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} \hat{x}_i & \hat{x}_{i+1} & \cdots & \hat{x}_{i+j-1} \\ u_i & u_{i+1} & \cdots & u_{i+j-1} \end{pmatrix} \right\|_F^2, \tag{4}$$

where $\|\cdot\|_F$ denotes the Frobenius-norm of a matrix. The estimates of the noise covariance matrices follow from

$$\begin{pmatrix} \hat{Q} & \hat{S} \\ \hat{S}^T & \hat{R} \end{pmatrix} = \frac{1}{J} \begin{pmatrix} \rho_{w_i} & \rho_{w_{i+1}} & \cdots & \rho_{w_{i+j-1}} \\ \rho_{v_i} & \rho_{v_{i+1}} & \cdots & \rho_{v_{i+j-1}} \end{pmatrix} \begin{pmatrix} \rho_{w_i} & \rho_{w_{i+1}} & \cdots & \rho_{w_{i+j-1}} \\ \rho_{v_i} & \rho_{v_{i+1}} & \cdots & \rho_{v_{i+j-1}} \end{pmatrix}^T, \tag{5}$$

where $\rho_{w_k} = \hat{x}_{k+1} - \hat{A}\hat{x}_k - \hat{B}u_k$ and $\rho_{v_k} = y_k - \hat{C}\hat{x}_k - \hat{D}u_k \, (k = i, \ldots, i+j-1)$ are the least squares residuals.

2. Subspace system identification algorithms make full use of the well developed body of *concepts and algorithms from numerical linear algebra.* Numerical robustness is guaranteed because of

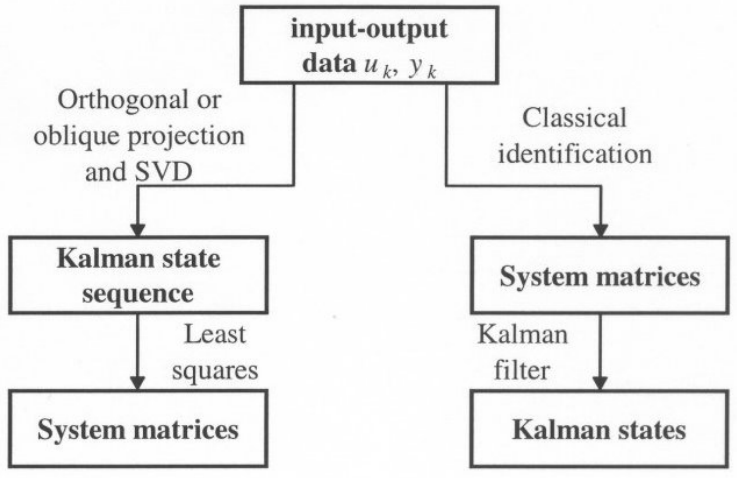

Figure 2: Subspace identification aims at constructing state space models from input-output data. The left hand side shows the subspace identification approach: first the (Kalman filter) states are estimated directly (either implicitly or explicitly) from input-output data, then the system matrices can be obtained. The right hand side is the classical approach: first obtain the system matrices, then estimate the states.

the well-understood algorithms, such as the QR-decomposition, the singular

value decomposition and its generalizations. Therefore, they are very well suited for large data sets ($s \to \infty$) and large scale systems (m, l, n large). Moreover, subspace algorithms are not iterative. Hence, there are no *convergence* problems. When carefully implemented, they are computationally very efficient, especially for large datasets (implementation details are however not contained in this survey).

3. The conceptual straightforwardness of subspace identification algorithms translates into *user-friendly software implementations*. To give only one example: since there is no explicit need for parameterizations in the geometric framework of subspace identification, the user is not confronted with highly technical and theoretical issues such as canonical parameterizations. The number of user choices is greatly reduced when using subspace algorithms because we use full state space models and the only parameter to be specified by the user, is the order of the system, which can be determined by inspection of certain singular values.

2. Notation

In this section, we set some notation. In Section 2.1, we introduce the notation for the data block Hankel matrices and in Section 2.2 for the system related matrices.

2.1. Block Hankel Matrices and State Sequences

Block Hankel matrices with output and/or input data play an important role in subspace identification algorithms. These matrices can be easily constructed from the given input-output data. Input block Hankel matrices are defined as

$$U_{0|2i-1} \overset{\text{def}}{=} \left(\begin{array}{ccccc} u_0 & u_1 & u_2 & \cdots & u_{j-1} \\ u_1 & u_2 & u_3 & \cdots & u_j \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ u_{i-1} & u_i & u_{i+1} & \cdots & u_{i+j-2} \\ \hline u_i & u_{i+1} & u_{i+2} & \cdots & u_{i+j-1} \\ u_{i+1} & u_{i+2} & u_{i+3} & \cdots & u_{i+j} \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ u_{2i-1} & u_{2i} & u_{2i+1} & \cdots & u_{2i+j-2} \end{array}\right) = \left(\begin{array}{c} U_{0|i-1} \\ \hline U_{i|2i-1} \end{array}\right) = \left(\begin{array}{c} U_p \\ \hline U_f \end{array}\right) \tag{6}$$

$$= \left(\begin{array}{ccccc} u_0 & u_1 & u_2 & \cdots & u_{j-1} \\ u_1 & u_2 & u_3 & \cdots & u_j \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ u_{i-1} & u_i & u_{i+1} & \cdots & u_{i+j-2} \\ u_i & u_{i+1} & u_{i+2} & \cdots & u_{i+j-1} \\ \hline u_{i+1} & u_{i+2} & u_{i+3} & \cdots & u_{i+j} \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ u_{2i-1} & u_{2i} & u_{2i+1} & \cdots & u_{2i+j-2} \end{array}\right) = \left(\frac{U_{0|i}}{U_{i+1|2i-1}}\right) = \left(\frac{U_p^+}{U_f^-}\right) \quad (7)$$

where:

- The number of block rows (i) is a user-defined index which is large enough, i.e. it should at least be larger than the maximum order of the system one wants to identify. Note that, since each block row contains *m* (number of inputs) rows, the matrix $U_{0|2i-1}$ consists of $2mi$ rows.

- The number of columns (j) is typically equal to $s-2i+1$, which implies that all *s* available data samples are used. In any case, *j* should be larger than $2i-1$. Throughout this chapter, for statistical reasons, we will often assume that $j, s \to \infty$. For deterministic (noiseless) models, i.e. where $v_k \equiv 0$ and $w_k \equiv 0$, this will however not be needed.

- The subscripts of $U_{0|2i-1}, U_{0|i-1}, U_{0|i}, U_{i|2i-1}, etc\ldots$ denote the subscript of the first and last element of the first column in the block Hankel matrix. The subscript "*p*" stands for "past" and the subscript "*f*" for "future". The matrices U_p (the past inputs) and U_f (the future inputs) are defined by splitting $U_{0|2i-1}$ in two equal parts of i block rows. The matrices U_p^+ and U_f^- on the other hand are defined by shifting the border between past and future one block row down. The superscript "+" stands for "add one block row" while the superscript "–" stands for "delete one block row".

  They are defined as $U_p^+ = U_{0|i}$ and $U_f^- = U_{i+1|2i-1}$.

The output block Hankel matrices $Y_{0|2i-1}, Y_p, Y_f, Y_p^+, Y_f^-$ are defined in a similar way.

State sequences play an important role in the derivation and interpretation of subspace identification algorithms. The state sequence X_i is defined as:

$$X_i \stackrel{\text{def}}{=} \begin{pmatrix} x_i & x_{i+1} & \cdots & x_{i+j-2} & x_{i+j-1} \end{pmatrix} \in \mathbf{R}^{n\times j}, \tag{8}$$

where the subscript i denotes the subscript of the first element of the state sequence.

2.2. Model Matrices

Subspace identification algorithms make extensive use of the observability and of its structure. The extended $(i > n)$ observability matrix Γ_i (where the subscript i denotes the number of block rows) is defined as:

$$\Gamma_i \stackrel{\text{def}}{=} \begin{pmatrix} C \\ CA \\ CA^2 \\ \cdots \\ CA^{i-1} \end{pmatrix} \in \mathbf{R}^{li\times n}. \tag{9}$$

We assume the pair $\{A,C\}$ to be observable, which implies that the rank of Γ_i is equal to n.

3. Geometric Tools

In Section 3.1 through 3.2 we introduce the main geometric tools used to reveal some system characteristics. They are described from a linear algebra point of view, independently of the subspace identification framework we will be using in the next sections.

In the following sections we assume that the matrices $A \in \mathbf{R}^{p\times j}, B \in \mathbf{R}^{q\times j}$ and $C \in \mathbf{R}^{r\times j}$ are given (they are dummy matrices in this section). We also assume that $j \geq \max(p,q,r)$, which will always be the case in the identification algorithms.

3.1. Orthogonal Projections

The orthogonal projection of the row space of A into the row space of B is denoted by A/B and its matrix representation is

$$A/B \stackrel{\text{def}}{=} AB^T(BB^T)^\dagger B, \tag{10}$$

where $(\bullet)^\dagger$ denotes the Moore-Penrose Pseudo-inverse of the matrix $(\bullet)$. $A/B^\perp$ is the projection of the row space of A into $B^\perp$, the orthogonal complement of the row space of B, for which we have $A/B^\perp = A - A/B = A(I_j - B(BB^T)^\dagger B)$. The projections Π_B and $\Pi_{B^\perp}$ decompose a matrix A into two matrices, the row spaces of which are

orthogonal:

$$A = A\Pi_B + A\Pi_{B^\perp}. \tag{11}$$

The matrix representations of these projections can be easily computed via the LQ decomposition of $\begin{pmatrix} B \\ A \end{pmatrix}$, which is the numerical matrix version of the Gram-Schmidt orthogonalization procedure.

Let A and B be matrices of full row rank and let the LQ decomposition of $\begin{pmatrix} B \\ A \end{pmatrix}$ be denoted by

$$\begin{pmatrix} B \\ A \end{pmatrix} = LQ^T = \begin{pmatrix} L_{11} & 0 \\ L_{21} & L_{22} \end{pmatrix} \begin{pmatrix} Q_1^T \\ Q_2^T \end{pmatrix}, \tag{12}$$

where $L \in \mathbf{R}^{(p+q)\times(p+q)}$ is lower triangular, with $L_{11} \in \mathbf{R}^{q\times q}, L_{21} \in \mathbf{R}^{p\times q}, L_{22} \in \mathbf{R}^{p\times p}$ and $Q \in \mathbf{R}^{j\times(p+q)}$ is orthogonal, i.e. $Q^T Q = \begin{pmatrix} Q_1^T \\ Q_2^T \end{pmatrix} \begin{pmatrix} Q_1 & Q_2 \end{pmatrix} = \begin{pmatrix} I_q & 0 \\ 0 & I_q \end{pmatrix}$. Then, the matrix representations of the orthogonal projections can be written as

$$A/B = L_{21}Q_1^T, \tag{13}$$

$$A/B^\perp = L_{22}Q_2^T. \tag{14}$$

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