ROBUST CONTROL OF NONLINEAR SYSTEMS: A CONTROL LYAPUNOV FUNCTION APPROACH

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Contents

Summary

This chapter presents selected results in robust nonlinear control, that is, feedback control of nonlinear systems with bounded unknown disturbances. Our presentation follows a "control Lyapunov function" approach and solves "the disturbance attenuation problem" by using feedback control to achieve boundedness in the presence of such disturbances.

For a class of nonlinear systems the solution is constructive via a recursive procedure ("backstepping") employed to build a desired robust Control Lyapunov Function. When only an output signal is available for feedback, the problem is more complex, as briefly discussed in the last section.

1. Robust Control Lyapunov Function (RCLF)

The concept of a *Control Lyapunov Function* (CLF) is a tool for solving stabilization tasks. One way to stabilize a nonlinear system is to *select a Lyapunov function* $V(x)$ and then *try to find* a feedback control $u(x)$ that renders $\dot{V}(x,u(x))$ negative definite. With an arbitrary choice of $V(x)$ this attempt may fail, but if $V(x)$ is a CLF, we can find a stabilizing control law $u(x)$. For the nonlinear system

$$\dot{x} = f(x) + g(x)u, \qquad (1)$$

$V(x)$ is a CLF if, for all $x \neq 0$,

$$L_g V(x) = 0 \;\; \Rightarrow \;\; L_f V(x) < 0, \tag{2}$$

where $L_g V(x) := \frac{\partial V}{\partial x} g(x)$. By standard converse theorems, if (1) is stabilizable, a CLF exists. From (2), we see that the set where $L_g V(x) = 0$ is significant, because in this set the uncontrolled system has the property $L_f V(x) < 0$.

However, if $L_f V(x) > 0$ when $L_g V(x) = 0$, then $V(x)$ is not a CLF and cannot be used for a feedback stabilization design (an observation that helps eliminate bad CLF candidates).

The CLF concept is extended to systems

$$\dot{x} = f(x, w) + g(x, w)u, \tag{3}$$

where w is a disturbance known to be bounded by $|w| \le \Delta$, where Δ may depend on x. $V(x)$ is an RCLF (a *robust* CLF), if for all $|x| > c$, a control law $u(x)$ can be found to render $\dot{V}$ negative for any w such that $|w| \le \Delta$. The value of c depends on Δ and on the chosen $u(x)$. For systems jointly affine in u and w,

$$\dot{x} = f(x) + g(x)u + p(x)w, \tag{4}$$

an RCLF is a $V(x)$ for which a class-$\mathcal{K}_\infty$ function $\rho(\cdot)$ exists such that

$$|x| > \rho(|w|) \;\; \Rightarrow \;\; \exists u : L_f V(x) + L_p V(x) w + L_g V(x) u < 0. \tag{5}$$

Again, the set $L_g V(x) = 0$ is critical because in it we require that

$$L_f V(x) + |L_p V(x)| \, \rho^{-1}(|x|) < 0, \tag{6}$$

which means that $L_f V(x)$ must be negative enough to overcome the effect of disturbances bounded by $|w| < \rho^{-1}(|x|)$.

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Biographical Sketches

Petar V. Kokotovic has been active for more than thirty years as control engineer, researcher and educator, first in his native Yugoslavia and then, from 1966 through 1990, at the University of Illinois, where he held the endowed Grainger Chair. In 1991 he joined the University of California, Santa Barbara where he directs the Center for Control Engineering and Computation. He has co-authored eight books and numerous articles contributing to sensitivity analysis, singular perturbation methods, and robust adaptive and nonlinear control. Professor Kokotovic is also active in industrial applications of control theory. As a consultant to Ford he was involved in the development of the first series of automotive computer controls and at General Electric he participated in large scale systems studies. Professor Kokotovic is a Fellow of IEEE, and a member of National Academy of Engineering, USA. He received the 1983 and 1993 Outstanding IEEE Transactions Paper Awards and presented the 1991 Bode Prize Lecture. He is the recipient of the 1990 IFAC Quazza Medal, 1995 IEEE Control Systems Award, and the 2002 Richard Bellman Heritage Award.

Murat Arcak was born in Istanbul, Turkey in 1973. He received the B.S. degree in Electrical and Electronics Engineering from the Bogazici University, Istanbul in 1996, and his M.S. and Ph.D. degrees in Electrical and Computer Engineering from the University of California, Santa Barbara under the direction of Petar Kokotovic in 1997 and 2000, respectively. In 2001 he joined Rensselaer Polytechnic Institute, Troy, NY, as an assistant professor in the Electrical, Computer and Systems Engineering Department. His research interests are in nonlinear control theory and applications. Dr. Arcak is a member of IEEE and SIAM, an associate editor on the Conference Editorial Board of IEEE Control Systems Society, and serves as a consultant for United Technologies Research Center, Hartford, CT.