SELF-TUNING CONTROL

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Contents

Summary

Self-tuning control is a simple method for controlling systems with uncertainty which has a long history of successful application.

1. Introduction

There are many approaches to the adaptive control of unknown systems *(see Adaptive control*). Within this broad area, Self-tuning control provides a pragmatic approach to the control of unknown systems which combines two well-established technologies:

1. the *design* of a controller for a *known* dynamical system and

2. the recursive *identification* (see *Identification of Linear Systems in Time Domain*) of *unknown* system parameters from measured system input and output data.

Fig 1 shows the basic idea of self-tuning control. There are four boxes in this diagram which can be interpreted as follows:

System: represents the dynamical system to be controlled with input u and output y.
Ident: represents the recursive *system* identification algorithm used to estimate unknown system parameters θ.
Design: represents the design method which chooses the *controller* parameters Θ in terms of the system parameters θ.
Controller: represents the feedback controller which manipulates the system input u to drive the system output y towards the desired system output, or setpoint w. The controller has a set of adjustable parameters Θ.

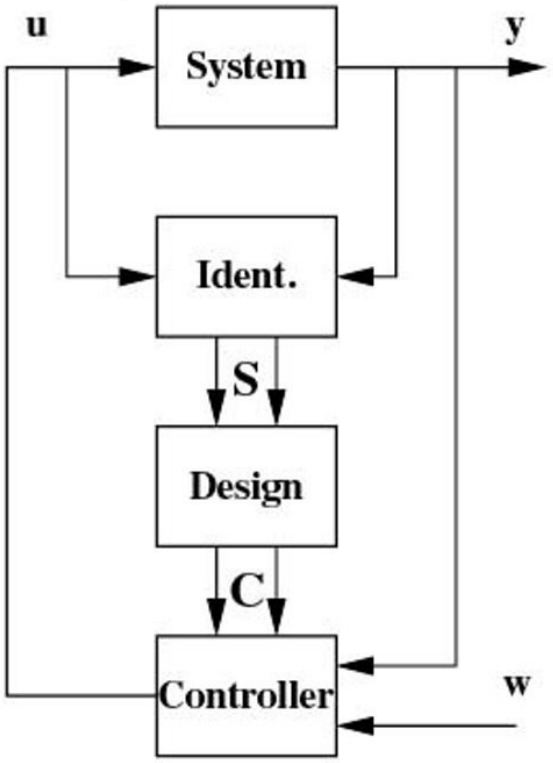

Figure 1: Explicit self-tuning control

The self-tuning approach is simple insofar as it takes the *certainty equivalent* approach. In particular the fundamental simplifying assumptions in the creation of the algorithm are that:

1. The controller design does *not* account for errors in the estimated parameters

2. The controller design does *not* explicitly generate control signals leading to better system identification

These are simplifying assumptions insofar as the design of the controller and the system identification are independent.

However, the user should be aware that this simplification potentially degrades performance. In particular, a more sophisticated adaptive controller would have three components in the design of the control signal:

1. the *certainty equivalent* component as in the self-tuning approach,

2. a *caution* component reflecting the fact that the controller is based on uncertain system parameters and

3. a *probing* component to produce system behavior leading to better system identification.

The optimal design of such adaptive controllers is sometimes called *dual* control (see. *Adaptive Dual Control*). Nevertheless, there are many situations were the gain in simplicity outweighs any potential loss in performance; and many more cases where the heuristic addition of caution and probing can improve the situation.

In this article, attention is restricted to linear, single-input, single-output systems of the form (see *General models of dynamic systems***)**

$$A(s)y(t) = B(s)u(t) + C(s)\zeta(t), \tag{2}$$

where $y(t), u(t)$, and $\zeta(t)$ are the system output, input and disturbance process at the continuous-time t. $A(s), B(s)$ and $C(s)$ are polynomials in the Laplace operator s. Despite the fact that $C(s)$ appears in the system equation, it will be treated as a design parameter: this can clearly be done by redefining ζ_i appropriately.

Such systems can be discretized (see *Discrete-time, sampled-data, digital control systems, quantization effects*) to give

$$A(z)y_i = B(z)u_i + C(z)\zeta_i, \tag{3}$$

where y_i, u_i and ζ_i are the system output, input and disturbance process at the discrete-time i. $A(z), B(z)$ and $C(z)$ are polynomials in the forward shift operator z.

An important feature of such models is that they can be rewritten in *linear-in-the parameters* form as:

$$\frac{A(s)}{C(s)}y(t) - \frac{B(s)}{C(s)}u(t) = \zeta(t) \tag{4}$$

$$\frac{A(z)}{C(z)}y_i - \frac{B(z)}{C(z)}u_i = \zeta_i. \tag{5}$$

Because $C(s)$ (in the discrete-time case $C(z)$) assumed known, such models have the property that all unknown parameters-the parameters of $A(s)$ and $B(s)$ (in the discrete-time case $A(z)$ and $B(z)$)-appear in the transfer function *numerators*.

The linear-in-the parameters model of Equation 4 can be written in the *state-variable filter* form:

$$\frac{d}{dt}X(t) = \mathfrak{C}X(t) + \begin{pmatrix} y(t) \\ u(t) \end{pmatrix} \tag{6}$$

$$\theta^T X(t) = \zeta(t), \tag{7}$$

where θ is the *system parameter vector*

$$\theta = (a_0 \;\; a_1 \;\; \ldots \;\; a_n; \;\; b_1 \;\; \ldots \;\; b_n)^T \tag{8}$$

and $X(t)$ contains the filtered measured data corresponding to the linear-in-the parameters model of Equation 4. In Laplace operator terms

$$X(t) = \frac{1}{C(s)}(s^n y(t) \;\; s^{n-1} y(t) \;\; \ldots \;\; y(t); \;\; s^{n-1} u(t) \;\; \ldots \;\; u(t))^T \tag{9}$$

Notice that usually at least one parameter (element of θ) is known a-priori. The vector of the *unknown* parameters will be called $\overline{\theta}$, and the corresponding data vector $\overline{X}(t)$.

In discrete-time form the linear-in-the parameters model of Equation 5 becomes

$$X_d(i) = \mathfrak{C}_d X_d(i-1) + \begin{pmatrix} y_i \\ u_i \end{pmatrix} \tag{10}$$

$$\theta_d^T X_d(i) = \zeta_i \tag{11}$$

Based on these linear-in-the-parameters models a simple *recursive parameter identification* algorithm is:

$$\frac{d}{dt}\hat{\overline{\theta}}(t) = -K(t)X^T(t)\hat{\theta}(t), \tag{12}$$

where $\hat{\theta}(t)$ is the *estimate* of θ, $\hat{\overline{\theta}}(t)$ is the *estimate* of $\overline{\theta}$, and $K(t)$ is the *identification gain vector*. Note that $X^T(t)\hat{\theta}(t)$ is a scalar product yielding a real number; all other terms in the equation are vectors. Possible choices of $K(t)$ are given in section 2.4.

The corresponding discrete-time version is

$$\hat{\bar{\theta}}_i = \hat{\bar{\theta}}_{i-1} - K_i X_i^T \hat{\theta}_{i-1} \qquad (13)$$

2. Categorization of Self-Tuning Controllers.

Section 1 delineates self-tuning controllers from the wider field: self-tuning control is based on the simplifying certainly-equivalence assumption. However, there are many subdivisions within the self-tuning field and, for this reason, this section provides a framework for categorising self-tuning controllers. The four main issues here are:

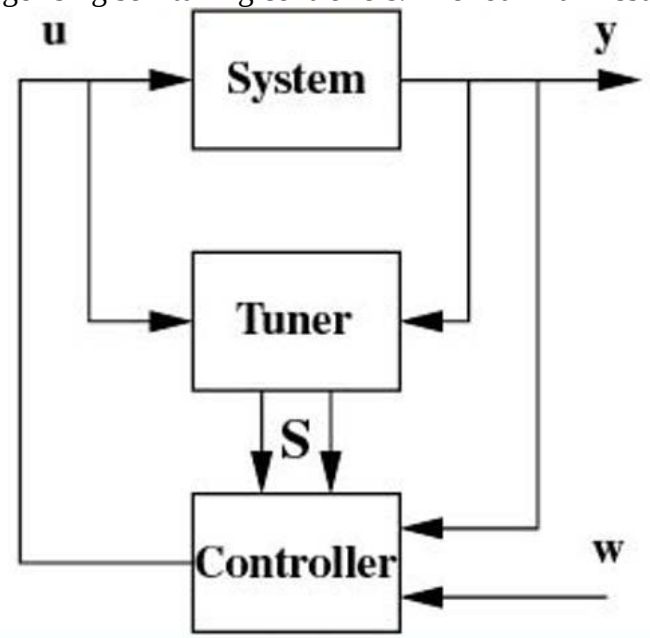

Figure 2: Implicit self-tuning control

1. *Explicit* or *implicit* self-tuning controller (Note that "indirect" is sometimes used in place of "explicit" and "direct" is sometimes used in place of "implicit").
2. *Continuous-time* or *discrete-time* formulation.
3. Choice of controller design method.
4. Choice of identification method.

These issues are expanded in the following subsections.

2.1 Explicit or implicit

Figure 1 outlines the explicit approach. The name arises because the controller parameters Θ are *explicitly* computed (by the block labelled "Design" in terms of the system parameters θ. This has the advantage that many identification and control design approaches can be combined in this fashion.

Figure 2 outlines the implicit approach. The name arises because the two blocks in Figure 1 labelled "Ident." and "Design" are collapsed into a single block labelled "Turner"; the block labelled "Tuner" *implicitly* calculates the controller parameters Θ *without* computing the system parameters θ as an intermediate step.

The implicit approach has the advantage that:

1. it is simpler in that the controller parameters Θ are computed directly by the block labelled "Tuner"
2. it cannot suffer from the potential problem with the explicit method that there may be some values of the system parameters θ for which the design method gives no solution for Θ

The implicit approach has the disadvantage that

1. some design methods cannot be put into implicit form.

2.2 Continuous-time or discrete-time

The first self-tuning controller called by that name was developed at the same time as the early microprocessors and the adoption of digital control by industry. It is therefore not surprising that it was developed in a discrete-time context as that is how digital computers see the world.

$$\text{CT} \xrightarrow[\text{Discretise}]{} \text{Discrete model} \xrightarrow[\text{Design}]{} \text{Discrete controller}$$

$$\text{CT} \xrightarrow[\text{Design}]{} \text{Continuous controller} \xrightarrow[\text{Discretise}]{} \text{Discrete controller}$$

where CT = Continuous System

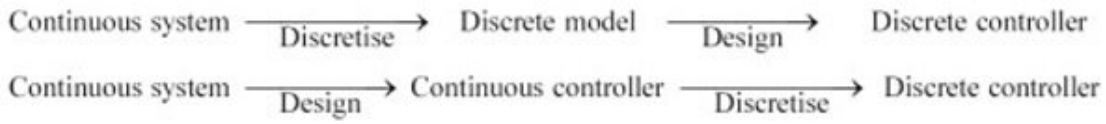

Figure 3: Two design routes

However, digital implementation does not imply discrete-time design. In fact, there are two possible design roots as indicated in Figure 3.

Discrete-time design as indicated in the *upper* part of Figure 3 and

Continuous-time design as indicated in the *lower* part of Figure 3

The two approaches have the same end points (Continuous system and Discrete controller); but the sequence of discretize and design operations are reversed leading to different domains for the control design.

The Continuous-time approach has the advantage that:

1. it is based on the physical system where the parameters have direct physical interpretation

2. it retains the physical significance of properties such as relative degree

3. it avoids artifacts of sampling such as non-minimum phase zeros
4. the sampling rate can be chosen *after the controller design*

It has the disadvantage that

1. discretization has to be explicitly performed to design the controller

2. $C(s)$ must be chosen so that the linear-in-the parameters model of Equation 4 contains proper transfer functions to avoid practical implementation problems.

2.3 Choice of controller design method

There are many controller design methods that can be used in the context of self-tuning control. There are two methods that will be discussed in detail here; other related methods are given elsewhere (see *Minimum Variance Control*).

1. Generalised minimum-variance control methods (see *Minimum Variance Control*).

2. Pole-placement methods (see *Pole placement control*).

The Generalised minimum-variance approach has the advantage that

1. It is simpler

2. It has many interpretations including a form of model-reference control

3. Implicit versions are readily available

4. It has no problems with systems with common factors in the numerator and denominator.

It has the disadvantage that

1. Systems with unstable inverses may lead to unstable responses

2.4 Choice of identification method

The continuous-time identification methods considered here are of the form of Equation 12. The difference lies in the choice of $K(t)$. Three versions will be noted here which are all of the form:

$$K(t) = S^{-1}(t)\bar{X}(t) \qquad (14)$$

Least mean square $S(t) = \bar{X}^T(t)\bar{X}(t)$ (a scalar)

Stochastic approximation $\frac{d}{dt}S(t) = \bar{X}^T(t)\bar{X}(t)$ (a scalar)

Least squares $\frac{d}{dt}S(t) = \bar{X}(t)\bar{X}^T(t)$ (a matrix)

Least squares with exponential forgetting $\frac{d}{dt}S(t) + \beta Sc = \bar{X}(t)\bar{X}^T(t)$ (a matrix) where β is the *exponential forgetting factor*.

The discrete-time equivalents are:

Least mean square $S_i = \bar{X}_i^T\bar{X}_i$ (a scalar)

Stochastic approximation $S_i = S_{i-1} + \bar{X}_i^T\bar{X}_i$ (a scalar)

Least squares $S_i = S_{i-1} + \bar{X}_i\bar{X}_i^T$ (a matrix)

Least squares with exponential forgetting $S_i = \beta_d S_{i-1} + \bar{X}_i\bar{X}_i^T$ (a matrix) where β_d is the discrete-time *exponential forgetting factor*.

There are numerous numerical tricks to effectively solve such recursive least-square algorithms which are beyond the scope of this chapter (see *Identification of Linear Systems in the Time Domain*).

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Biographical Sketch

Peter J. Gawthrop was born in Seascale, Cumberland, in 1952. He obtained his BA (first class honours), D.Phil. and MA degrees in Engineering Science from Oxford University in 1973, 1977, and 1979 respectively. Following a period as a Research Assistant with the Department of Engineering Science at Oxford University, he became W. W. Spooner Research Fellow at New College, Oxford. He then moved to the University of Sussex as a Lecturer, and later a Reader in control engineering. Since 1987, he has held the Wylie Chair of Control Engineering in the Department of Mechanical Engineering at Glasgow University. He was involved in founding the Centre for Systems and Control – a cross-departmental research grouping at Glasgow with about twelve full time academic staff including four professors.

His research interests include self-tuning control, continuous-time system identification and system modelling - particularly using bond graphs in the context of partially-known systems. He is interested in applying control techniques to a number of areas, including process control, robotics aerospace systems and anaesthesia. He has coauthored and authored some 120 conference and journal articles and three

books in these areas. He was an associate editor of Automatica and an honorary editor of IEE Proceedings Pt. D, and serves on the editorial boards of a number of journals including the IMechE Journal of Systems and Control, Journal of Process Control, IMA Journal of Mathematical Control and Information and the International Journal of Adaptive Control and Signal processing and the European Journal of Control. In 1994 he was awarded the Honeywell International Medal by the Institute of Measurement and Control. In 1999 he spent a year in Australia at the Universities of Newcastle and Sydney.