CONTROLLER DESIGN FOR DISTRIBUTED PARAMETER SYSTEMS

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Contents

Summary

Control design for distributed parameter systems is discussed. Instead of trying to give an overview, which would necessarily be incomplete, emphasis is put on so-called direct methods, which means design methods not based on finite dimensional approximations. Two direct design methods are presented, an internal model control approach and a flatness-based one.

The first is illustrated by a heat exchange laboratory experiment and the second is explained on an example based on the linear heat equation. The underlying mathematical tools are state space and semigroup methods for the first and series

expansions for the second method.

1. Introduction

Control of distributed parameter systems (DPS) is a very wide field, which is of increasing importance in many engineering disciplines. For these (infinite dimensional) systems, which are described by partial differential equations, there is an even larger variety of problems and concepts than in finite dimension (based on ordinary differential equations), and the mathematics required in DPS control design is more involved.

Of course, in view of the many different control problems in DPS control, even more design approaches exist. In the present chapter emphasis is put on two boundary control methods used to address tracking and motion planning problems. They are known as the internal model approach and the flatness-based approach. Each of them is illustrated by an example, a laboratory heat exchange process for the internal model control and a heat equation for the flatness-based control.

The chapter is structured as follows. In order to provide an overview of the kind of problems encountered in DPS control, problems and methods are briefly discussed in the next section. Section three introduces a direct state space approach using semigroup theory, which is illustrated through a heat exchange process. Internal model control of the heat exchange process is then discussed in Section four. Finally, the flatness-based approach, a promising new method for motion planning and open loop control design is described in Section five.

2. Control Problems and Control Design Methods

Distributed parameter systems are described by partial differential equations, often in conjuction with ordinary differential equations and non-differential equations. These models may be very complex, involve multiple dependent variables and independent space coordinates in complex domains, sometimes with free boundaries, and they may be time dependent and non-linear. However, in many technological applications rather simple models that are precise enough for control design purposes can be used. Often one can restrict to a few variables with major variations in only one space dimension. Moreover, as in finite dimension, linearization about stationary regimes is reasonable in many situations. Therefore, variants of rather few typical equations cover a wide range of processes. Typically, one encounters parabolic equations like the “heat equation” or “diffusion equation” in heat transfer and chemical processes, hyperbolic equations describing transport and wave propagation phenomena and plate and beam equations in elasticity.

Even in systems of distributed nature, control inputs (and measurements) are most often lumped quantities, i.e. they act at particular points, often on the boundary. Important control problems comprise motion planning, steering along trajectories and stabilization, for example.

As for the design methods, two major classes can be distinguished, namely indirect and direct methods. Indirect methods (also called early lumping methods) are based on approximations of either the equations or the solutions by finite dimensional systems. For instance, replacing spatial differential operators by finite difference schemes leads to a finite set of ordinary differential equations. On the other hand, classical methods of approximation of solutions are spectral (also called modal) methods, based on solution of eigenvalue problems and the method of weighted residuals. All these indirect methods are important for simulation purposes and can also be used in controller design, although the approximate models obtained that way are often of high dimension. On the other hand, direct methods (also called late lumping methods) use the partial differential equations for the control design. An approximation is done only for implementation purposes. Both, the internal model control and the flatness-based control considered here fall into this latter category.

A rough classification of problems and methods may distinguish

- different types of models:

- Models describing the time evolution of a single or of multiple spatially dependent variables (scalar fields): For instance, modeling the reaction process in a tubular chemical or biotechnological reactor may require the differential balance of enthalpy together with molar balances of several reactants. This leads to coupled (nonlinear) partial differential equations (PDE) for the temperature and several concentrations.

- according to the dimension of the space of independent variables: Continuing the previous example, in a long tubular reactor one may often neglect the spatial dependence in the cross sections which leads to a model depending on the axial variable only.

- linear (respectively linearized) and nonlinear models.

- models in which the major phenomenon is oscillation, diffusion, transport, etc. give rise to different mathematical properties: hyperbolic and parabolic second order systems for example.

- different types of control inputs:

- distributed inputs, i.e. inputs depending on a space variable; these occur very seldom in practice. However, sometimes a modeling of inputs as a product of a function of time and a function of space is useful depending on the mathematical approach used.

- lumped boundary inputs, which are very frequent in applications. Examples are the torque applied to a flexible motor shaft or the feed concentration of a tubular reactor.

- parametric inputs, which are lumped quantities occurring as a parameter in the model equation, like the velocity of the forced conductive flow in a tubular reactor.

- different control problems:

- -stabilization problems: Of course stabilization, or often just enhancement of convergence, is a major control problem. A very large field of DPS control design is devoted to this subject, namely the design of elastic mechanical structures like large robot arms or buildings like towers and bridges.

- -tracking problems, which are typical for start-up situations: How can a transition from rest to rest can be achieved, possibly in a finite time? As an example, one may think of turning a flexible robot arm by a prescribed angle.

- different classes of control design methods:

- -indirect methods (early lumping), which means the approximation of the PDE model by a set of ordinary differential equations, which is often of high dimension. This lumping may be achieved via approximation of the equations by discretization (either physically motivated or by approximating differential operators with difference schemes) or via approximation of the solution. In both cases the finite dimensional techniques are used for the finite dimensional approximation.

- -direct methods (late lumping), which means working with the PDE model for the control design and using approximations only for the implementation of the control or for the simulation.

- different mathematical tools employed for the system analysis and the control design: examples are state-space methods, semigroups, frequency domain techniques or operational calculus.

- different control design approaches: As an example, energy based methods are most useful for design of stabilizing controllers. These are combined with tracking controllers defined by one of the methods proposed in this chapter to achieve stable tracking.

3. State Space and Semigroup Approach

The semigroup approach is well-suited for the state-space representation of infinite dimensional systems. In this section, the involved operators are explicitly given for a specific boundary control problem representing a heat exchange process. This enables one to formally use well-known concepts like open loop, closed loop, stability, stabilization, etc. However, as will be detailed in this section, with such generalizations some care must be taken.

3.2. Mathematical Model of a Heat Exchange Process

The semigroup approach is illustrated with a heat exchange process comprising a cascade of two coupled heat exchangers (see Figure 1). The first is a parallel flow exchanger, the second a counterflow one. Before entering the second exchanger, the fluid of the internal tube, which is common to both exchangers and which is to be heated, is in contact with the environment.

The deviations of the temperature fields around a stationary profile are used to describe the process: $\theta(z,t)$ in the inner tube, $\theta_1(z,t)$ and $\theta_3(z,t)$ in the outer tubes. The environmental temperature is assumed constant. Accordingly, θ_2, its deviation from the stationary value, is zero.

The velocities v_0, v_1 and v_3 in the three tubes may all be different, while the other physical parameters are supposed to be equal for all fluids, and constant. They are the heat capacity c_p, the density ρ, and the thermal conductivity λ. The mathematical model is obtained through energy balances.

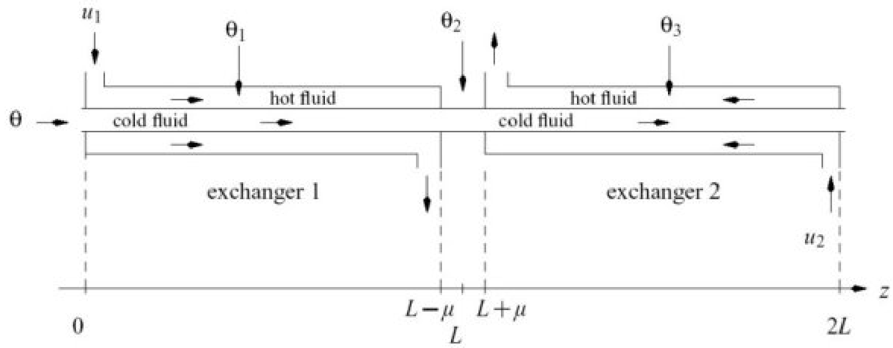

Figure 1: Tubular heat exchangers

Considering sufficiently small variations one obtains the linear mathematical model

$$\frac{\partial\theta}{\partial t} = -v_0\frac{\partial\theta}{\partial z} + \frac{\lambda}{\rho c_\mathrm{p}}\frac{\partial^2\theta}{\partial z^2} + \sum_{i=1}^{3} h_i\left(\theta_i - \theta\right), \qquad z \in \Omega_1 = \left(0, 2L\right)$$

$$\frac{\partial\theta_1}{\partial t} = -v_1\frac{\partial\theta_1}{\partial z} + \frac{\lambda}{\rho c_\mathrm{p}}\frac{\partial^2\theta_1}{\partial z^2} + h_1\left(\theta - \theta_1\right), \qquad z \in \Omega_2 = \left(0, L-\mu\right)$$

$$\theta_2 = 0\,, \qquad z \in \left[L-\mu, L+\mu\right]$$

$$\frac{\partial\theta_3}{\partial t} = v_3\frac{\partial\theta_3}{\partial z} + \frac{\lambda}{\rho c_\mathrm{p}}\frac{\partial^2\theta_3}{\partial z^2} + h_3\left(\theta - \theta_3\right), \qquad z \in \Omega_3 = \left(L+\mu, 2L\right)$$

with $h_i \equiv 0$ on $\Omega_{j\neq i}, i, j = 1,2,3$. (Here the arguments *z* and *t* are omitted for the sake of readability.)

The initial conditions are

$$\theta\left(z,0\right) = 0, \quad \theta_1\left(z,0\right) = 0, \quad \theta_3\left(z,0\right) = 0$$

The inflow temperatures of the outer tubes are the two boundary control inputs u_1 and u_2:

$$\theta(0,t) = 0, \qquad \left.\frac{\partial \theta}{\partial z}\right|_{2L} = 0$$

$$\theta_1(0,t) = u_1(t), \qquad \left.\frac{\partial \theta_1}{\partial z}\right|_{L-\mu} = 0$$

$$\theta_3(2L,t) = u_2(t), \qquad \left.\frac{\partial \theta_3}{\partial z}\right|_{L+\mu} = 0$$

The control objective concerns the inner tube temperature between the exchangers, $y_1(t) = \theta(L,t)$, and its outflow temperature $y_2(t) = \theta(2L,t)$. These two temperatures should follow prescribed trajectories, such as to achieve a new stationary regime, for instance.

3.2. Representation in State Space

A state space form

$$\dot{x}(t) = A_{\mathrm{d}} x(t) \quad \text{on} \quad \Omega, \quad \text{for} \quad t > 0$$

of the heat exchanger model can be introduced with

$$x(t) = \begin{pmatrix} \theta(z,t) \\ \theta_1(z,t) \\ \theta_3(z,t) \end{pmatrix},$$

$$A_{\mathrm{d}} = \begin{pmatrix} -v_0 \frac{\partial}{\partial z} + \alpha \frac{\partial^2}{\partial z^2} - h_1 - h_2 - h_3 & h_1 & h_3 \\ h_1 & -v_1 \frac{\partial}{\partial z} + \alpha \frac{\partial^2}{\partial z^2} - h_1 & 0 \\ h_3 & 0 & v_3 \frac{\partial}{\partial z} + \alpha \frac{\partial^2}{\partial z^2} - h_3 \end{pmatrix}$$

where $\alpha = \lambda / (\rho c_{\mathrm{p}})$ and $\Omega = \Omega_1 \oplus \Omega_2 \oplus \Omega_3$. Similarly, the control is described by

$$F_b x(t) = B_b u(t)$$

with $F_b x(t) = \frac{1}{2\mathrm{v}} \int_{\Omega} \begin{pmatrix} \mathbf{1}_{[-v,+v]} \theta_1(z,t) \\ \mathbf{1}_{[2L-v,2L+v]} \theta_3(z,t) \end{pmatrix} dz, \quad B_b u(t) = u(t) = \begin{pmatrix} u_1(t) \\ u_2(t) \end{pmatrix}$

valid on $\Gamma = \partial\Omega$, the boundary of Ω. The output is defined by

$$y(t) = \begin{pmatrix} y_1(t) \\ y_2(t) \end{pmatrix} = Cx(t) = \frac{1}{2\mathrm{v}} \int_\Omega \begin{pmatrix} \mathbf{1}_{[L-\mu-v, L-\mu+v]} \theta(z,t) \\ \mathbf{1}_{[2L-v, 2L+v]} \theta(z,t) \end{pmatrix} dz$$

Here

$$\mathbf{1}_{[z_0 - v, z_0 + v]} = \begin{cases} 1 & \text{for } z \in [z_0 - v, z_0 + v] \\ 0 & \text{for } z \notin [z_0 - v, z_0 + v], v > 0 \end{cases}$$

where *v* is a small positive constant.

It is worth noting that the input *u* and the output *y* are lumped quantities, with values in R^2, while the state evolves in an (infinite dimensional) function space *X*. As most often, *X* is taken to be the space of square integrable functions, which is a Hilbert space. Here

$$X = L_2(\Omega_1) \oplus L_2(\Omega_2) \oplus L_2(\Omega_3) = X_1 \oplus X_2 \oplus X_3$$

and for $\varphi^T = (\varphi_1, \varphi_2, \varphi_3), \varphi \in X$ the inner product is $\langle \varphi_l, \varphi_k \rangle_{X_i} = \int_{\Omega_i} \varphi_l \varphi_k d\Omega_i$ and the norm is

$$\|\varphi\|_X^2 = \langle \varphi, \varphi \rangle_X = \langle \varphi_1, \varphi_1 \rangle_{X_1} + \langle \varphi_2, \varphi_2 \rangle_{X_2} + \langle \varphi_3, \varphi_3 \rangle_{X_3} \varphi$$

Finally, to complete the definition of A_d, its domain of definition is specified as

$$\mathcal{D}(A_\mathrm{d}) = \{\varphi \in X \mid \varphi_i, \varphi_i' \text{ are absolutely continuous, } \varphi_i'' \in L_2(\Omega_i);$$

$$\varphi_1(0) = 0, \varphi_i'(2L) = 0, \varphi_2'(L-\mu) = 0, \varphi_3'(L+\mu) = 0\}$$

3.3. Abstract Boundary Control System

The evolution equation on a Hilbert space *X*, given by

$$\dot{x}(t) = Ax(t) + f(t), \quad x(t) \in X, t > 0, \quad x(0) = x_0, \tag{1}$$

where *A* is a closed linear operator, admits a formal solution

$$x(t) = T_A(t) x_0 + \int_0^t T_A(t-s) f(s) ds.$$

Here, T_A is the semigroup associated with the operator *A* (i.e., *A* is the infinitesimal generator of T_A) and the domain of A is defined as $\mathcal{D}(A)=\{\xi\in X \mid \lim_{t\to 0+}\frac{1}{t}(T_A(t)-I)\xi \text{ exists}\}$.

The state space representation of the heat exchange process introduced in the previous section slightly differs from (1), because the control acts on the boundary. Following an approach initiated by Fattorini in 1968, a change of variables and operators allows a change of representation:

$$\dot{\zeta}(t)=A\zeta(t)-D\dot{u}(t),\quad \zeta(t)\in\mathcal{D}(A_{\mathrm{d}}),t>0,\qquad \zeta(0)=x(0)-Du(0) \tag{2}$$

where

- A is the "extension operator" of A_{d}, which means $A\zeta(t)=A_{\mathrm{d}}\zeta(t)$ for all $\zeta\in\mathcal{D}(A)$ and $\mathcal{D}(A)=\{\zeta\in\mathcal{D}(A_{\mathrm{d}}) \mid F_{\mathrm{b}}\zeta=0\}$, A is assumed closed and densely defined in X.
- D is the bounded "distribution operator" describing the action of the boundary control on the state: $D\in\mathcal{L}(U,X)$, the set of bounded operators from $U=R^2$ in X such that $Du\in\mathcal{D}(A_{\mathrm{d}}), F_{\mathrm{b}}(Du)=B_{\mathrm{d}}u, \forall u\in U$, and D leaves the operator A_d unchanged (i.e., $\mathrm{im}(D)\subset\ker(A_d)$).

System (1) is called an abstract boundary control system. If the semigroup T_A exists, its formal solution can be written as

$$\zeta(t)=T_A(t)\zeta(0)-\int_0^t T_A(t-s)D\dot{u}(s)ds$$

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Biographical Sketches

After his "Doctorat" in 1990, **Dr. Touré** was an Assistant Professor at the University of Lyon (France). In 1997 he got the "Diplôme d'Habilitation ă Diriger des Recherches" with work on control of distributed parameter systems and applications in chemical engineering. He is the manager of a network group in this domain for the "Groupement de Recherches en Automatique" of French C.N.R.S. since 1998. At present he is professor at the University of Orleans (France) and the director of the "Laboratoire Vision et Robotique". His research concerns the controller design for infinite dimensional systems and nonlinear optimization, with applications in chemical engineering.

Dr. Rudolph got an engineering diploma in technical cybernetics from the "Universitat Stuttgart",Germany in 1989 and a "Doctorat" from the "Université Paris XI, Orsay", France in 1991 as well as the "Dr.-Ing. Habil." Degree from Technische Universität Dresden (Germany) in 2003. He has been a fellow in the postdoctoral programme and the "Habilitandenprogram" of the "Deutsche Forschungsgemeinschaft" and an invited researcher at several research institutions in France. Currently he is a "Privatdozent" and a senior researcher ("oberassistent") at the "Institut für Regelungs- und Steuerungstheorie" of the "Technische Universiat Dresden", Germany. His main research interests are in controller and observer design for nonlinear systems, algebraic systems theory, infinite dimensional systems, and applications.