

## CONTROL OF CHAOS AND BIFURCATIONS

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## Summary

The research field of chaotic systems control has seen rapid development since the late 1980s. This state-of-the-art technology in the beginning of the 21st century is presented in this article. Preliminaries are given to the notion and properties of chaotic systems, models of controlled plants, as well as control goals. Several major approaches in this research field are discussed in detail: feedforward or “nonfeedback” control (via periodic excitation input); OGY method (based on linearization of Poincaré maps); Pyragas method (using time-delay feedback); traditional (linear and nonlinear) engineering control methods; adaptive and intelligent control strategies including neural networks and fuzzy control. Bifurcation control, on the other hand, has also been developed rapidly, for which feedback control, washout-filter, normal forms, harmonic balance approximation, and frequency-domain control methods all work to their advantages. A number of existing and potential applications of controlling chaos and bifurcations in science and engineering are described briefly. Some directions of current research and prospects of the field are outlined.

## 1. Introduction

*Chaos control* refers to purposefully manipulating chaotic dynamical behaviors of some complex nonlinear systems. *Bifurcation control*, on the other hand, refers to the task of designing a controller that can modify the bifurcating properties of a given nonlinear system, so as to achieve some desirable dynamical behaviors. Bifurcation control not only is important in its own right, but also suggests an effective strategy for chaos control, since bifurcation and chaos are usually “twins;” in particular, period-doubling bifurcation is a typical route to chaos in many nonlinear dynamical systems.

As a new and young discipline, chaos and bifurcations control has in effect come into play with many traditional scientific and technological advances today. Automatic control theory and practice, on the other hand, is a traditional and long-lasting engineering discipline. It has recently rapidly evolved and expanded, to overlap with and sometimes completely encompass many new and emerging technical areas of developments, and chaos and bifurcations control is one of them.

Both chaos control and bifurcation control technologies promise to have a major impact on many novel, perhaps not-so-traditional, time- and energy-critical engineering applications. Examples include such as data traffic congestion control in the Internet, encryption and secure communication at different levels of communications, high-performance circuits and devices (e.g., delta-sigma modulators and power converters), liquid mixing, chemical reactions, power systems collapse prediction and protection, oscillators design, biological systems modeling and analysis (e.g., the brain and the heart), crisis management (e.g., jet-engine surge and stall), nonlinear computing and information processing, and critical decision-making in political, economic as well as military events. In fact, this new and challenging research and development area has become an attractive scientific inter-discipline involving control and systems engineers, theoretical and experimental physicists, applied mathematicians, and physiologists alike.

There are many practical reasons for controlling or ordering chaos and bifurcations. In a system where chaotic and bifurcating responses are undesired or harmful, they should be reduced as much as possible, or totally suppressed. Examples of this include avoiding fatal voltage collapse in power networks, eliminating deadly cardiac arrhythmias, guiding disordered circuit arrays (e.g., multi-coupled oscillators and cellular neural networks) to reach a certain level of desirable pattern formation, regulating dynamical responses of mechanical and electronic devices (e.g., diodes, laser machines, and machine tools), and organizing a multi-agency corporation to achieve optimal performance.

Ironically, recent research has shown that chaos and bifurcations can actually be quite useful under certain circumstances, and there is growing interest in utilizing the very nature of them, particularly in some novel time- and/or energy-critical applications. A salient observation about this possibility is that chaos enables a system to explore its every dynamical possibility due to its ergodicity. When chaos is controllable, it can provide the system designer with an exciting variety of properties, richness of flexibility, and a cornucopia of opportunities. Oftentimes, conventional engineering design tried to completely eliminate such “irregular” dynamical behaviors of a system. However, such over-design is usually accomplished at the high price of loss of flexibility in achieving optimal performance near the stability boundaries, or at the expense of radically modifying the original system dynamics which in many cases is undesirable or unnecessary. Likewise, a prominent feature of bifurcation is its close relation with various vibrations (periodic oscillations or limit cycles), which sometimes are not only desirable but may actually be necessary. Mechanical vibrations and some material and liquid mixing processes are good examples in which bifurcations (and chaos) are very desirable. A further idea, suggested as useful in power systems, is to use the onset of a small oscillation as an indicator for proximity to collapse. In control systems engineering, the deliberate use of nonlinear oscillations has been applied effectively for system identification.

It has been shown that the sensitivity of chaotic systems to small perturbations can be used to rapidly direct system trajectories to a desired target using minimal control energy. This may be crucial, for example, in inter-planetary space navigation. A suitable manipulation of chaotic dynamics, such as stability conversion or bifurcation delay can significantly extend the operational range of machine tools and aircraft engines, enhance the artificial intelligence of neural networks, and increase coding/decoding efficiency in signal and image encryption and communications.

It has been demonstrated that data traffic through the Internet is likely to be chaotic. Special chaos control strategies may help network designers in better congestion control, thereby further benefiting the rapidly evolving and expanding Internet, to handle the exponentially increasing demands from the industry and the commercial market.

Fluid mixing is another good example in which chaos is not only useful but actually very desirable. The objective here is to thoroughly mix together two or more fluids of different kinds, while minimizing the control energy required. For this purpose, fluid mixing turns out to be much simpler to achieve if the particle motion dynamics are

strongly chaotic. Otherwise, it is difficult to obtain rigorous mixing properties due to the possibility of invariant two-tori in the flow. This has been one of the main subjects in fluid-mixing processing, known as *chaotic advection*. Chaotic mixing is also important in engineering applications involving heat transfer. One example is in plasma heating within a nuclear fusion reactor, where heat waves are injected into the reactor, for which the best result is obtained when the convection inside the reactor is chaotic.

Within the context of biological systems, chaos and bifurcations control seems to be a crucial mechanism employed by the human brain in carrying out many of its tasks such as learning, perception, memorization and particularly conceptualization. Additionally, some recent laboratory studies reveal that the complex dynamical variability in a variety of normal-functioning physiological systems demonstrates features reminiscent of chaos and bifurcations. Some medical evidence lends support to the idea that control of certain chaotic cardiac arrhythmias may soon lead to the design of a safe and highly effective intelligent pacemaker. In fact, chaos and bifurcation have become public focal points in various research areas of life sciences, medicine research, and biomedical engineering.

Motivated by many potential real-world applications, current research on controlling chaos and bifurcations has proliferated in recent years. With respect to theoretical considerations, chaos and bifurcations control poses a substantial challenge to system analysts. This is due to the extreme complexity and sensitivity of chaotic and bifurcating dynamics, which in turn is associated with the reduction in long-term predictability and short-term controllability of chaotic systems.

A controlled chaotic or bifurcating system is inherently non-autonomous. In most cases, it cannot be converted to an autonomous system since the required time-dependent controller has yet to be designed and therefore cannot be defined as a new system state variable. Possible time-delay, noise, and coupling influences often make a controlled chaotic or bifurcating system extremely complex topologically. As a result, many existing theories and methodologies for autonomous systems are no longer applicable to the analysis and control of many chaotic and bifurcating systems. On the other hand, chaos and bifurcations control poses new challenges to controller designers and automation engineers. A successful controller implementation in a chaotic or bifurcating environment is generally difficult to achieve due to the extreme sensitivity of chaos to parameter variations and noise perturbations, and to the non-robustness of chaos with respect to the structural stability of the physical devices involved.

Notwithstanding many technical obstacles, both theoretical and practical developments in this area have experienced remarkable progress in the last decade. It is now known that both chaos and bifurcations can be controlled via various methods, and yet there are still many challenging control-theory-oriented problems remaining to be solved and there are many potential engineering applications of this new technology to be further explored.

This article aims at presenting some current achievements in this challenging field at the forefront of research, with emphasis on the engineering perspectives, methodologies, and potential applications, as well as some further research outlooks in these exciting and promising new research areas of chaos and bifurcations control.

## 2. Features of Chaos

Chaotic system is a deterministic dynamical system exhibiting irregular, seemingly random behaviors. Such behaviors can be observed if any two trajectories, starting close to each other, will diverge after some time (the so-called “sensitive dependence on initial conditions”)

It means that even if one knows the state of the system with high accuracy, one cannot give an accurate prediction of its future behavior.

There exist different mathematical definitions of chaotic systems and chaotic behaviors. Most definitions are based on appropriate formalization of the stability concept and on the notion of *attractor*: minimal set of points in the state space of the system, attracting all the nearby trajectories starting from its vicinity.

Standard examples of attractors are stable equilibrium and stable limit cycle. Stability in these examples means that if two trajectories start on the attractor close to each other, they remain close with the growth of time (see *Stability Concepts*).

In the middle of the XXth century some counterintuitive examples of dynamical systems were found where two trajectories, starting close to each other, diverge after some time to a finite distance. Such a behavior is nothing but instability of trajectories within attractor.

If, in addition, the attractor is bounded, its trajectories should exhibit irregular, seemingly random behavior which corresponds to an intuitive sense of the term “chaos.” Therefore, it is natural to call an attractor *chaotic* if it is bounded and all the trajectories starting from it are unstable in the sense that it does not converge to a fixed point, a limit cycle, or a limit set (and this instability is characterized by a positive Lyapunov exponent).

A system is called *chaotic* if it possesses at least one chaotic attractor. That is, chaotic systems are characterized by local instability and global boundedness of the trajectories.

Since local instability of a linear system implies unboundedness (infinite growth) of its solutions, chaotic system are necessarily nonlinear, i.e., described by a nonlinear mathematical model. Perhaps the most popular example of chaos is given by the Lorenz system

$$\begin{cases} \dot{x} = \sigma(y - x), \\ \dot{y} = rx - y - xz, \\ \dot{z} = bz + xy. \end{cases} \quad (1)$$

Solutions of (1) for some parameter values look like irregular oscillations (see, e.g. Figure. 1 and Figure. 2 for the case of  $\sigma = 10, r = 97, b = 8/3$ ).

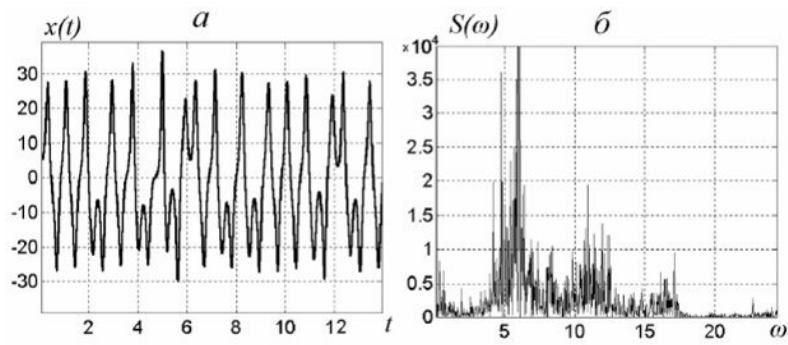


Figure 1: Chaotic trajectory  $x(t)$  of the Lorenz system and its spectrum.

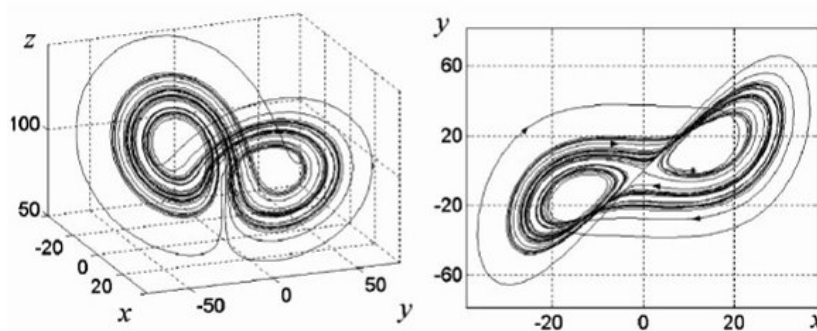


Figure 2: Chaotic attractor of the Lorenz system in the 3D space and its projection onto the plane  $x - y$ .

Sometimes, instead of “chaotic attractor,” the term “strange attractor” is used, which was introduced by D. Ruelle and F. Takens in 1971 to mean that the attractor is a porous (fractal) set that cannot be represented as a piece of a smooth manifold and therefore has a non-integer dimension (see the discussion on fractal dimension in *Analysis of chaotic systems*).

Another notion of chaos is inspired by the statistical approach to the study of dynamical systems. In statistical (ergodic) theory, a dynamical system is understood as a transformation of sets or measures rather than a transformation of points. Such an approach allows us to describe some integrated and typical properties of system trajectories, eliminating exceptional and atypical ones. The corresponding definition of chaotic attractor was introduced by Ya. Sinai in 1979. It emphasizes the mixing property of chaotic systems, which means, loosely speaking that any bounded set of initial conditions will be spread over the whole attractor as time goes.

Coexistence of different definitions of chaos reflects the complexity of this scientific notion, encompassing a variety of mathematical and physical views of it. Theoretical studies are often based only on some properties and features of chaotic systems, without emphasizing on a specific and rigorous definition.

Study of chaotic systems has been based on a number of concepts of nonlinear systems theory, introduced for qualitative and quantitative analysis: Lyapunov exponents,

Poincaré maps, delayed coordinates, fractal dimensions, entropies, etc. This active field of research is commonly called *Chaos Theory* (see *Analysis of chaotic systems*).

An important property of chaotic trajectories is the *recurrence*: they return to any vicinity of any past value of the system trajectory. A recurrent motion necessarily returns to any vicinity of any of its previous piece at least once and, therefore, infinitely many times. Recurrence implies another important feature of chaos: chaotic attractor is the closure of all the *unstable periodic trajectories* (UPO) contained in it.

Another feature of chaotic systems, related to sensitive dependence on initial conditions, is the high sensitivity with respect to the changes of their parameters or some input variables (controlling actions). It means that small changes in a controlling variable may produce large variations in system's behaviors. Such a phenomenon and its implications in physics were described in the seminal paper of 1990 by E. Ott, C. Grebogi and J. Yorke, which triggered an explosion of activities and enormous publications during the following decade.

(see *Analysis of chaotic systems*)

### 3. Methods of Chaos Control

A typical goal for controlling a chaotic system is to transform its chaotic trajectory into a periodic one. In terms of control theory, this means stabilization of an unstable periodic orbit or unstable equilibrium. A specific feature of this problem is the possibility of achieving the goal by means of an arbitrarily small control action. Other control goals like synchronization (concordance or concurrent change of the states of two or more systems) and chaotization (purposeful generation of a chaotic motion by means of control) can also be achieved by small control in many cases.

More subtle objectives can also be specified and achieved by control, for example, to modify a chaotic attractor of the free system in the sense of changing some of its characteristics (Lyapunov exponents, entropy, fractal dimension), or delay its occurrence, or change its locations, etc.

#### 3.1. Feedforward Control by Periodic Signal

Methods of *feedforward* control (also called *non-feedback* or *open-loop* control) change the behavior of a nonlinear system by applying a properly chosen input function  $u(t)$  – external excitation – usually a periodic signal. Excitation can reflect influence of some physical actions, e.g. external force or field, or it can be some parameter perturbation (modulation). In these cases, the value  $u(t)$  depends only on time and does not depend on the current measurements of the system variables. Such an approach is attractive because of its simplicity: no measurements or executions are needed for state information. It is especially advantageous for ultra-fast processes at the molecular or atomic level, or open flow control, where no possibility of system variable online measurements exists.

The possibility of significant changes to system dynamics by periodic excitation has been known, perhaps, since the beginning of the 20th century: A. Stephenson discovered in 1908 that a high frequency excitation can stabilize the unstable equilibrium of a pendulum.

Analysis of general nonlinear systems affected by high frequency excitation is commonly based on the Krylov-Bogoljubov averaging method. According to the averaging method, stability analysis of a periodically excited system is reduced to the analysis of a simplified averaged system. The method provides conditions guaranteeing approximate stabilization of a given equilibrium or a desired (target) trajectory.

The possibility of transforming a periodic motion into a chaotic one, or vice versa, by means of periodic excitation of a medium level was demonstrated by V. Alexeev and A. Loskutov in 1985 for a fourth-order system describing dynamics of two interacting populations. K. Matsumoto and I. Tsuda demonstrated the possibility of suppressing chaos in a Belousov-Zhabotinsky reaction by adding a white noise disturbance in 1983.

These results were based on computer simulations. In 1990, R. Lima and M. Pettini, and later R. Chácon, studied suppression of chaos in one degree-of-freedom nonlinear oscillators analytically, applying the Melnikov method. Since Melnikov method leads to intractable calculations for state dimensions greater than two, analytical results are known only for periodically excited systems with one degree of freedom. For higher-dimensional cases, computer simulations are used instead.

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## Biographical Sketches

**Alexander Lvovich Fradkov** born May 22, 1948; received the Diploma degree in mathematics from the

Faculty of Mathematics and Mechanics of St. Petersburg State University (Dept. of Theoretical Cybernetics) in 1971; Candidate of Sciences (Ph.D.) degree in Engineering Cybernetics from St. Petersburg Mechanical Institute (now - Baltic State Technical University - BSTU) in 1975 and Doctor of Sciences degree in Control Engineering in 1986 from St. Petersburg Electrotechnical Institute.

From 1971 to 1987 he occupied different research positions and in 1987 became Professor of Computer Science with BSTU. Since 1990 he has been the Head of the Laboratory for Control of Complex Systems of the Institute for the Problems of Mechanical Engineering of Russian Academy of Sciences. He is also a part time professor with the Faculty of Mathematics and Mechanics of St. Petersburg State University (Dept. of Theoretical Cybernetics).

His research interests are in fields of nonlinear and adaptive control, control of oscillatory and chaotic systems and mathematical modeling. He is also working in the borderland field between Physics and Control (Cybernetical Physics). Dr. Fradkov is coauthor of 350 journal and conference papers, 10 patents, 15 books and textbooks.

Dr. Fradkov is the Vice-President of the St. Petersburg Informatics and Control Society since 1991, Member of the Russian National Committee of Automatic Control, IEEE Fellow since 2004. Dr. Fradkov was Co-Chairman of 1st-10th International Baltic Student Olympiades on Automatic Control in 1991-2002; NOC Chairman of the 1st and 2nd International IEEE-IUTAM Conference "Control of Oscillations and Chaos" in 1997 and 2000; NOC Chairman of the 5th IFAC Symposium on Nonlinear Control Systems (NOLCOS'01). He was an associate editor of European Journal of Control (1998-2001); a member of IEEE Control Systems Society Conference Editorial Board (1998-2004); a member of the IFAC Technical Committees on Education and Nonlinear Control.

Dr. Fradkov was awarded William Girling Watson Scholarship in Electrical Engineering (University of Sydney, 1995) and JSPS Fellowship for Research in Japan in 1998-1999. During 1991-2003 he visited and gave invited lectures in 65 Universities of more than 20 countries.

**Guanrong Chen** received the M.Sc. degree in Computer Science from Zhongshan University, China, and the Ph.D. degree in Applied Mathematics from Texas A&M University, USA. He was a tenured Full Professor in the University of Houston, Texas, USA before he took up the Chair Professor position in the City University of Hong Kong in 2000, where he is now also the Founding Director of the Centre for Chaos and Complex Networks. He is a Fellow of the IEEE since 1996, for his fundamental contributions to the theory and applications of chaos control and bifurcation analysis. Prof. Chen has (co)authored 15 research monographs and advanced textbooks, about 300 SCI journal papers, and about 200 refereed conference papers, published since 1981, in the field of nonlinear systems dynamics and controls. Prof. Chen served and is serving as Deputy Chief Editor, Advisory Editor, Features Editor, and Associate Editor for 8 international journals including the IEEE Transactions on Circuits and Systems and the International Journal of Bifurcation and Chaos. He received the 1998 Harden-Simons Prize for the Outstanding Journal Paper Award from the American Society of Engineering Education, the 2001 M. Barry Carlton Best Annual Transactions Paper Award from the IEEE Aerospace and Electronic Systems Society, and the 2005 IEEE Guillemin-Cauer Best Annual Transactions Paper Award from the IEEE Circuits and Systems Society. He is an Honorary Professor of the Central Queensland University, Australia, and Honorary Guest-Chair Professor of more than ten universities in China.