

FAULT DIAGNOSIS FOR LINEAR SYSTEMS

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Contents

1. Introduction
 2. Model of the system, faults and uncertainties
 3. Methods of residual generation
 4. Parity space approach to Residual Generation
 5. Observer-based residual generation
 6. Fault analysis using parameter estimation
 7. Residual evaluation
 8. Conclusion and Perspectives
- Glossary
Bibliography
Biographical Sketch

Summary

This chapter outlines the basic principles and most important approaches of model-based fault detection and isolation (FDI) and, to a certain degree, fault diagnosis, using linear models. Both the parity space approach and the concept of observer-based residual generation are described in input-output format. The well established parameter estimation approach to fault analysis is briefly described in terms of using least squares estimates. It is shown how fault isolation (and robustness with respect to unknown inputs, i.e., modeling uncertainties and unmodeled disturbances) can be achieved with the fault detection filter and by decoupling in the frequency domain on the basis of observer-based residual generation. Full and approximate decoupling techniques are addressed for both structured and unstructured modeling uncertainties. It is further shown how structured residuals can be generated for sensor and actuator fault detection using the dedicated observer scheme, DOS, and the generalized observer scheme, GOS. As far as residual evaluation is concerned, we focus our consideration on the threshold test with adaptive thresholds and briefly explain how to find the threshold selector.

1. Introduction

The development of diagnosis systems in the last five decades has clearly shown that the model-based approach is by far the most powerful one. Typical for the model-based approach is to simulate on a digital computer a model of the nominal or faulty functional behavior of the system under consideration and use it as a reference for the

identification of malfunctions in the actual system. Even though all natural processes are, strictly speaking, non-linear, it is quite common and in many practical situations admissible to use linear models. This is especially true in the case of regulator problems, where the plant is controlled at a fixed operating point, around which the plant model can be linearized with satisfactory accuracy. But there are also situations in practice, where the essential behavior of the plant is intrinsically linear and the use of a linear model is the quite natural approach. Though the assumption of linearity has limitations in practice, the linear approach is of great theoretical value due to the fact that both the design and the functioning of the fault detection system becomes transparent, and the well-founded and to a high degree mature linear systems theory can be applied. These are the main reasons why the development of the model-based fault diagnosis theory has been based upon linear models from the very beginning in the early seventies.

Nevertheless, the analytical approach using fixed linear models has severe drawbacks when the non-linear character of the system under consideration is dominant or when the system is subject to substantial plant uncertainties, unmodeled disturbances, unknown parameter variations or structural changes, or when it is poorly defined. All of this is quite common in practice. In such cases, non-linear or adaptive or knowledge-based models, respectively, or, if we stay with fixed linear analytical models, *robust* fault diagnosis schemes are needed in order to reduce the number of false alarms or avoid them. Despite of these deficiencies, we base our consideration in this chapter upon the assumption, that the behavior of the plant can be represented by well-defined linear time invariant mathematical models. This assumption, though ultimately idealized, is most useful to understand the basic concepts of model-based fault diagnosis and, in particular, of fault detection and isolation (FDI), on which this chapter concentrates. It also serves as a basis for extensions towards non-linear, robust, and even knowledge-based approaches, which will be treated in detail in later contributions.

Speaking of linear mathematical models means that we take into consideration the dynamic behavior of the system in terms of linear differential equations or transfer functions (in the continuous case) or linear difference equations or z-transfer functions (in the discrete-time case). Though the computer implementation requires at any time a discrete-time representation, we base our consideration upon the continuous system representation, because our main goal is to outline only the basic ideas and concepts. Clearly, the algorithms and results obtained can easily be translated into the discrete time case, and more detailed information on the great variety of existing approaches and concepts can be found in the cited literature.

2. Model of the System, Faults and Uncertainties

In case of using *analytical* models, the system behavior may be described either in input-output or state space format. For *linear continuous* systems the state equations used for FDI are given by

$$\dot{\mathbf{x}}(t) = (\mathbf{A} + \Delta\mathbf{A}_f)\mathbf{x}(t) + (\mathbf{B} + \Delta\mathbf{B}_f)\mathbf{u}(t) + \mathbf{F}_1\mathbf{f}(t) \quad (1)$$

$$\mathbf{y}(t) = (\mathbf{C} + \Delta\mathbf{C}_f)\mathbf{x}(t) + \mathbf{F}_2\mathbf{f}(t), \quad (2)$$

where $\mathbf{x}(t) \in \mathbb{R}^n$ is the system state vector, with the system matrix \mathbf{A} , $\mathbf{u}(t) \in \mathbb{R}^p$ is the known input vector, with the input matrix \mathbf{B} , $\mathbf{y}(t) \in \mathbb{R}^q$ is the measurement vector, with the output matrix \mathbf{C} , $\Delta\mathbf{A}_f$, $\Delta\mathbf{B}_f$, $\Delta\mathbf{C}_f$ represent the effects of parametric faults, $\mathbf{f}(t) \in \mathbb{R}^s$ is the vector of (additive) actuator, sensor and component faults, with \mathbf{F}_1 and \mathbf{F}_2 known fault distribution matrices. According difference equations apply in the case of discrete-time models.

The corresponding input-output model, with p the differential operator (or shift operator if the system is discrete) is given by

$$\mathbf{y}(t) = [\mathbf{G}_u(p) + \Delta\mathbf{G}_u(p)]\mathbf{u}(t) + \mathbf{G}_f(p)\mathbf{f}(t), \quad (3)$$

where $\mathbf{G}_u(p)$ is the transfer matrix operator from \mathbf{u} to \mathbf{y} , $\mathbf{G}_f(p)$ is the fault transfer matrix operator from the additive fault vector \mathbf{f} to \mathbf{y} , $\Delta\mathbf{G}_u(p)$ denotes the deviation transfer operator caused by faults which are reflected in the parameters.

For mathematical treatment of faults it makes a big difference whether the faults are additive or multiplicative. Additive faults can be treated like external inputs. The vector $\mathbf{f}(t)$ in Eqs.(1)-(3) represents the set of additive faults such as actuator faults, sensor faults and some kinds of component faults (e.g., leaks in pipes). Faults that are reflected in system parameter variations (“parametric faults”) are characterized by $\Delta\mathbf{A}_f$, $\Delta\mathbf{B}_f$, $\Delta\mathbf{C}_f$ and $\Delta\mathbf{G}_u(p)$; we call them multiplicative, because they multiply themselves with $\mathbf{x}(t)$ or $\mathbf{u}(t)$, respectively, and are therefore not as easy to handle as additive faults. Multiplicative faults can, in principle, be approached by additive faults but then they have time-variant coefficients, and they have an effect on the dynamics of the system.

In Eqs.(1)-(3) modeling uncertainties have not been taken into account. Under modeling uncertainties in the widest sense we understand all kinds of discrepancies between the mathematical model and the fault free actual system caused by *imperfect modeling*. Typical examples are parameter variations $\Delta\mathbf{A}_d$, $\Delta\mathbf{B}_d$, $\Delta\mathbf{C}_d$ that are not mission critical like faults, unmodeled dynamics and non-linearities, neglected system disturbances, system noise, measurement noise, actuator noise. The latter are often considered in the system equations as *unknown inputs* $\mathbf{d}(t)$.

Note that since modeling uncertainties are not mission-critical, they have to be distinguished from faults in that they are tolerable with no need to be detected, but if they are misinterpreted as faults by the FDI system, this causes false alarms, and already small false alarm rates can make an FDI system totally useless.

According to the way of their mathematical treatment, the modeling uncertainties can be divided into two groups: *additive* and *multiplicative*. All kinds of unmodeled disturbances and noise act like additive external inputs. But parameter deviations multiply with state variables $\mathbf{x}(t)$ or input variables $\mathbf{u}(t)$ and are therefore *multiplicative*. **Figure 1** illustrates the difference. Consider, for the sake of simplicity, a

system consisting of a scalar gain factor a . Note that the effect of uncertainty, $\mathbf{u}(t)\Delta a$, can be interpreted as a (usually time constant) parameter variation Δa with a time variant coefficient $\mathbf{u}(t)$. Another difficulty is due to the fact that Δa affects the stability of the system.

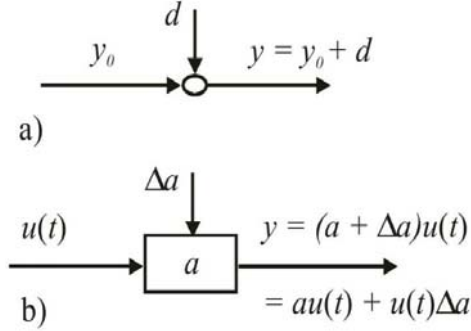


Figure 1: a) additive uncertainties, b) multiplicative uncertainties

Taking modeling uncertainties into account the state space model for residual generation reads

$$\dot{\mathbf{x}}(t) = (\mathbf{A} + \Delta\mathbf{A}_f + \Delta\mathbf{A}_d)\mathbf{x}(t) + (\mathbf{B} + \Delta\mathbf{B}_f + \Delta\mathbf{B}_d)\mathbf{u}(t) + \mathbf{F}_1\mathbf{f}(t) + \mathbf{E}_1\mathbf{d}(t) \quad (4)$$

$$\mathbf{y}(t) = (\mathbf{C} + \Delta\mathbf{C}_f + \Delta\mathbf{C}_d)\mathbf{x}(t) + \mathbf{F}_2\mathbf{f}(t) + \mathbf{E}_2\mathbf{d}(t) \quad (5)$$

where $\mathbf{d}(t)$ denotes the vector of (additive) unknown inputs, with \mathbf{E}_1 and \mathbf{E}_2 the corresponding (constant and usually known) distribution matrices, and $\Delta\mathbf{A}_d$, $\Delta\mathbf{B}_d$, and $\Delta\mathbf{C}_d$ denote the parameter uncertainties which, similar to corresponding fault-induced changes, are of multiplicative nature.

The corresponding nominal input-output model can be given as

$$\mathbf{y}(t) = [\mathbf{G}_u(p) + \Delta\mathbf{G}_u(p)]\mathbf{u}(t) + \mathbf{G}_f(p)\mathbf{f}(t) + \mathbf{G}_d(p)\mathbf{d}(t) , \quad (6)$$

where \mathbf{G}_d is the transfer matrix operator from \mathbf{d} to \mathbf{y} , and $\Delta\mathbf{G}_u = \Delta\mathbf{G}_{uf} + \Delta\mathbf{G}_{ud}$ comprises both the parametric faults and parameter uncertainties. Note that \mathbf{G}_u , \mathbf{G}_f and \mathbf{G}_d can be calculated from Eqs.(4)-(5). If the matrices \mathbf{E}_1 , \mathbf{E}_2 and $\mathbf{G}_d(p)$ are known, we speak of *structured* uncertainties; then $\Delta\mathbf{G}_{ud}$ is given, as well. But often they are unknown. Then the uncertainties are *unstructured*, but from $\Delta\mathbf{G}_u$ it is usually known that it has at least a bounded frequency response of the form

$$|\Delta\mathbf{G}_u(j\omega)| \leq \delta_u(\omega) \quad (7)$$

With these assumptions, the most general form of the residual generator can be given as a dynamic system with the input-output relation

$$\mathbf{r}(t) = \mathbf{P}(s)\mathbf{u}(t) + \mathbf{Q}(p)\mathbf{y}(t), \quad (8)$$

where \mathbf{P} and \mathbf{Q} are realizable transfer matrix operators. In order to make the residual $\mathbf{r}(t)$ become zero for the fault-free case, \mathbf{P} and \mathbf{Q} must satisfy the condition

$$\mathbf{P}(p) + \mathbf{Q}(p)\mathbf{G}_u(p) = \mathbf{0}. \quad (9)$$

Different forms of the residual generator can be obtained by using different forms of \mathbf{P} and \mathbf{Q} . Substituting $\mathbf{P}(p)$ in (8) by (9) gives the residual generator in the output equation form

$$\mathbf{r}(t) = \mathbf{Q}(p)[\mathbf{y}(t) - \mathbf{G}_u(p)\mathbf{u}(t)], \quad (10)$$

where $\mathbf{Q}(p)$ is a filter matrix operator yet free to select. By using the left coprime factorization

$$\mathbf{G}_u(s) = \hat{\mathbf{M}}_u^{-1}(p)\hat{\mathbf{N}}_u(p)$$

and choosing $\mathbf{Q}(p) = \mathbf{R}(p)\hat{\mathbf{M}}_u(p)$, the residual generator can be given in the unified, most general equation error form

$$\mathbf{r}(t) = \mathbf{R}(p)[\hat{\mathbf{M}}_u(p)\mathbf{y}(t) - \hat{\mathbf{N}}_u(p)\mathbf{u}(t)], \quad (11)$$

where $\mathbf{R}(p) = \mathbf{Q}(p)\hat{\mathbf{M}}_u^{-1}(p)$ is the so-called parameterization matrix which can be arbitrarily chosen from the set of stable systems RH_∞ .

Substituting y in Eq.(10) by Eq.(6) yields the general form of the residual relation

$$\mathbf{r}(t) = \mathbf{Q}(p)[\Delta\mathbf{G}_u(p)\mathbf{u}(t) + \mathbf{G}_f(p)\mathbf{f}(t) + \mathbf{G}_d(p)\mathbf{d}(t)], \quad (12)$$

which considers all kinds of possible model uncertainties in $\Delta\mathbf{G}_u(p)\mathbf{u}(t)$ and $\mathbf{G}_d(p)\mathbf{d}(t)$.

A key feature of any FDI system is to ensure *robustness* with respect to the model uncertainties in order to keep the false alarm rate of the FDI system zero or at least extremely small. This can be attained in both the stage of residual generation and residual evaluation. It should though be noted that this is often in conflict with the detection quality, that is to say, with the fault detection and isolation sensitivity. In terms of Eq.(12) together with (6), the robustness problem in the stage of residual generation can be stated as to find a matrix operator \mathbf{Q} such that the changes $\Delta\mathbf{G}_u\mathbf{u}$ and $\mathbf{G}_d\mathbf{d}(t)$ caused by modeling uncertainties can be distinguished from the changes $\Delta\mathbf{G}_u\mathbf{f}$ and $\mathbf{G}_f\mathbf{f}(t)$ caused by the faults.

The strategies for solving this task with analytical residual generators fall into three categories:

- 1) Perfect decoupling of the residuals from uncertainties (without making use of any know-ledge of the time or frequency characteristics of the uncertainties)
- 2) Approximate decoupling of the residuals from the uncertainties (making use of some knowledge of the time or frequency characteristics of the uncertainties).
- 3) Knowledge-based selection of those parts of the mode that reflect the faults, i. e., allow us to detect the faults while being not or minimally affected by model uncertainties.

3. Methods of Residual Generation

The most relevant analytical model-based residual generation methods developed during the last three decades have traditionally been divided into three categories:

1. Parity space approach
2. Observer-based approach (diagnostic observers)
3. Parameter estimation approach.

Though conceptually different, intensive investigations during recent years have shown that there are close relationships among these approaches. It is easy to see that the parity space approach leads to a parallel model which can be interpreted as a special class of observer, namely the so-called ‘dead-beat’ observer with all poles at the origin. This means that the residual generator resulting from the parity space approach can be subsumed, as a special case, under the group of diagnostic observers. Moreover, under certain conditions, the residuals of the parameter estimation approach can be viewed as a non-linear transformation of the residuals of the parity space approach. These relationships between the different approaches are not surprising, because all approaches exploit the same knowledge, namely the measured inputs and corresponding outputs of the system under consideration; they only process this knowledge in different ways. However, depending on the special situation, the one or other method can be more or less useful and hence the approaches are sometimes used in combination.

4. Parity Space Approach to Residual Generation

The parity space approach is based on a consistency test (‘parity check’) of parity equations; these are properly modified system equations in which the inputs and outputs are replaced by the actual process measurements. The reason for the modification of the system equations is to decouple the residuals from the system states and from the effects of disturbances, and the effect of the faults under consideration from the other faults for the purpose of isolation. The results of inconsistency, i.e. the residuals of the parity equations, are used as indicators of the faults. The parity equations can be derived from the state space model of the system or from the transfer functions (or operators). Leading to a special type of observer, it has turned out that the parity space approach is usually easier to carry out than the observer-based one, because it has less design

freedom that has to be managed by the designer.

For linear systems, the basic idea of the parity space approach can be most simply outlined in input output format in the frequency domain. Let $\mathbf{u}(t)$ be the input vector, $\mathbf{y}(t)$ the output vector, and $\mathbf{G}_u(s)$ the transfer function matrix (“the model”) of the system, then the basic configuration of the residual generator of the parity space approach in input-output format is as shown in **Figure 2**.

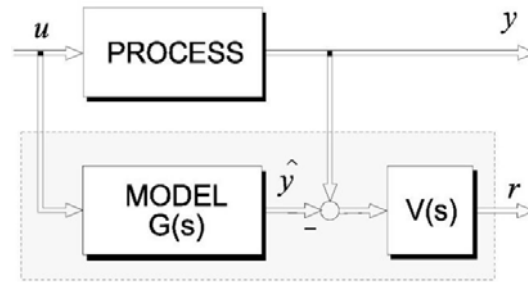


Figure 2: Basic configuration of the residual generator in the parity space approach

The residual vector $\mathbf{r}(t)$ can be calculated in terms of the Laplace transform ($\mathbf{R}(s) = \mathcal{L}\{\mathbf{r}(t)\}$ etc.) as

$$\mathbf{R}(s) = \mathbf{V}(s)[\mathbf{Y}(s) - \mathbf{G}_u(s)\mathbf{U}(s)] , \quad (13)$$

where $\mathbf{V}(s)$ represents the transfer matrix of a filter yet free to select in order to reach, for example, decoupling of the effect of a fault from the other faults or from the unknown inputs.

The decoupling being achieved by $\mathbf{V}(s)$ finally means a restriction to that subset of the system relations $\mathbf{Y}(s) - \mathbf{G}_u(s)\mathbf{U}(s)$ which are independent of or at least only weakly dependent upon the other faults (for fault isolation), or upon the critical modeling errors and/or unmodeled disturbances (for robustness). Note that for the special choice of $\mathbf{V}(s)$ as

$$\mathbf{V}(s) = \mathbf{Q}(s)\hat{\mathbf{M}}_u(s) , \quad (14)$$

where $\hat{\mathbf{M}}_u(s)$ is defined by the factorization $\mathbf{G}_u(s) = \hat{\mathbf{M}}_u^{-1}(s)\hat{\mathbf{N}}_u(s)$, and $\mathbf{Q}(s)$ is free to select, the structure of Figure 2 becomes equivalent to the structure of the diagnostic observer (Figure 3). This proves the close relationship between the parity-space and the observer-based approach, which will be described in more detail in the next paragraph.

5. Observer-based Residual Generation

The basic concept of the observer-based residual generation consists in the reconstruction of the output vector of the system of interest from partial sets of measured output variables with the aid of an observer or Kalman filter, where the

estimation error (or innovation, resp.) or a function or functional of it is used as the residual. It is important to note that contrary to the *state observer* that is used for state feedback in the case of incomplete measurement of the system states, a diagnostic observer is an *output observer* aiming at the creation of output redundancy. Hence, in the case of linear systems, diagnostic observers can efficiently be designed in the frequency domain without use of state space theory.

The standard structure of a linear diagnostic observer of full order is shown in **Figure 3**. The actual output vector, \mathbf{y} , is compared with the output vector, $\hat{\mathbf{y}}$, of the nominal model and the difference, $\mathbf{r} = \mathbf{y} - \hat{\mathbf{y}}$, is fed back with the feedback gain matrix \mathbf{H} . The feedback is necessary to compensate for unmatched initial conditions and to stabilize the observer if the system is unstable. It also provides design freedom in order to reach fault isolation by decoupling the effects of faults from other faults or robustness by decoupling the effects of faults from the effects of unknown inputs. In a similar way, one can use reduced order observers or Kalman filters (in the case of noisy measurements), or even non-linear or knowledge-based observers if the system is essentially non-linear or no analytical model is available.

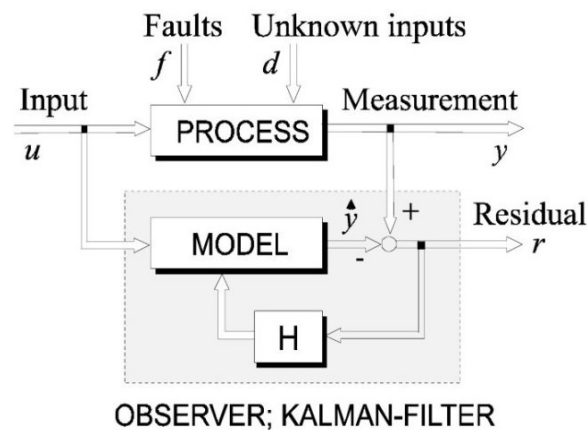


Figure 3: Basic configuration of an observer-based residual generator: output observer of full order

Under ideal conditions, the feedback gain matrix \mathbf{H} has to be chosen such that the residual vector \mathbf{r} or a measure $J(\mathbf{r})$ becomes

$$\mathbf{r} = \mathbf{0} \text{ or } J(\mathbf{r}) = 0 \text{ for } \mathbf{f} = \mathbf{0} \quad (15)$$

$$\mathbf{r} \neq \mathbf{0} \text{ or } J(\mathbf{r}) \neq 0 \text{ for } \mathbf{f} \neq \mathbf{0} \quad (16)$$

independent of the unknown input \mathbf{d} . If it is not possible to decouple \mathbf{r} perfectly from \mathbf{d} , so that \mathbf{r} or $J(\mathbf{r})$ take values different from zero at $\mathbf{f} = \mathbf{0}$, one has to utilize the *increment* of \mathbf{r} caused by the fault. A fault is then declared if \mathbf{r} or the measure $J(\mathbf{r})$ surpasses a certain threshold \mathbf{T} or J_{th} , respectively, that is assigned larger than the effect of the unknown inputs in the fault-free case:

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Biographical Sketch

Paul M. Frank received the degrees of Dipl.-Ing. in Electrical Engineering in 1959, Doctor Ing. in 1966 and Habilitation in 1973, all from the University of Karlsruhe, Germany. From 1959 – 1976 he has been an Assistant Professor and Associate Professor at the University of Karlsruhe. 1974 – 1975 he spent a year as a scholar and guest professor at the University of Washington, Seattle, U.S.A. From 1976 – 1999 he has been a full professor and head of the department of Measurement and Control at the Gerhard-Mercator-University of Duisburg, 1980/81 chairman of the faculty of Electrical Engineering. From 1977 - 2000 he has been a permanent guest lecturer at the Ecole Nationale Supérieure de Physique de Strasbourg, ENSPS, France. Since 1999 he is a professor emeritus. A co-founder of the German-French Institute of Automation and Robotics IAR 1986, he holds the position of a honorary president since 2000. Prof. Frank was president of the European Union Control Association, EUCA, from 1999 – 2001. Prof. Frank holds three honorary doctor degrees, from the University of Iasi, Romania 1994, the Université de Haute Alsace, Mulhouse, France, 1997, and the Technical University of Cluj-Napoca, Romania, 1998, and he has received medals of merit from several universities. He is a member of VDI/VDE-GMA and a Fellow of IEEE.

Prof. Frank's main interests are in automatic control with focus on fault diagnosis and fault tolerant

control systems, analysis and design of robust control systems, sensitivity theory, fuzzy and neural network techniques in control and system supervision. He has published or edited seven books and published more than 460 papers in technical journals and international conferences, organized the European Control Conference 1999, and he is co-editor of several technical journals.