GEODESY AND TOPOGRAPHY

Andreas Illert
Federal Agency for Cartography and Geodesy, Germany

Keywords: Geoid, ellipsoid, map projection, control point network, global positioning system, survey, topographic map, relief representation, marginal data, DTM

Contents

Summary

The representation of features in maps is based on mathematical models of Earth. In geodesy an ellipsoid is used to approximate the figure of Earth. The geodetic datum defines the shape of the ellipsoid and its position relative to the center of Earth. Map projections provide the mathematical background for transformation of three-dimensional positions to a flat plane. The practical realization of the spatial reference system is performed by the creation and maintenance of a network of monumented control points where horizontal position and height is known to a high degree of precision. The position of the features on the surface is determined by topographic survey, photogrammetric methods, or satellite positioning systems.

1. Introduction

From their content, maps can be divided into topographic maps and thematic maps **(see *Mapping and Atlas Production*).** Topographic maps map show terrain characteristics, infrastructure, hydrography, settlement pattern, and sometimes land use. Thematic maps highlight one specific map theme such as soils or population. Maps provide a media for storage of spatial information and communicating this to others. This medium has been paper for a long time. Recently it has been replaced by digital information systems, but while the format of information storage has changed the principles of mapping remain the same.

Topographic maps are indispensable tools for administration, education, tourist orientation, recreation and military applications. They form the basis for most thematic maps. In order to fulfill these requirements, topographic maps have been constructed to a high standard of accuracy. Their content is collected by means of topographic survey, photogrammetry, or remote sensing. The positioning of topographic features needs to be founded upon a stable spatial reference system. Geodesy is the mathematics of the shape and the size of Earth on which a coordinate reference system is based. For the purpose of mapping, the location of an area must first be fixed by location in a global system and then projected from the globe to a plane for display on a sheet of paper or a computer screen.

2. The Figure of Earth

Ancient cultures took Earth for a disk, but very soon the observation of stars and planets revealed that Earth resembles a sphere. After the late 1700s measurements showed that Earth is slightly flattened at the poles. The equatorial radius is 6378 km while the polar radius is only 6357 km. Therefore, Earth's shape is best described as an ellipsoid, which is formed by rotating an ellipse around its minor axis. The rotating axis of the ellipsoid is parallel to the mean rotation axis of Earth. Mathematically, the ellipsoid model is defined by the semi-major axis a (equatorial radius) and the semi-minor axis b (polar radius). Other ellipsoid parameters such as flattening ($f = (a - b)/a$) and eccentricity are computed from these two terms.

In the times before satellite systems, geodesists determined the shape of the ellipsoid by survey of two base lines along meridians in combination with astronomical measurements. Different surveys in different parts of the world resulted in different national or regional models for the ellipsoid that usually fit best in the area where the survey was conducted. The center of these reference ellipsoids does not necessarily coincide with the mass center of Earth. Nowadays satellite techniques enable geodesists to compute the shape of the ellipsoid at global scale. The most recent ellipsoidal model GRS80 is designed for a best fit considering the whole of Earth.

The ellipsoid constitutes a mathematical description for the surface of Earth. A closer approximation to the figure of Earth is provided by the mean sea surface, which for this purpose is considered to extend under the land mass of the continents. This surface is known as the geoid. The geoid is an equipotential surface of gravity. The direction of gravity is always perpendicular to the geoid. It is a relatively smooth surface but it is also fairly irregular. This irregularity is due to anomalies in distribution of mass within Earth or changes in the materials of Earth's crust. Tidal forces and gravity differences cause even the ocean level to vary over the globe by hundreds of meters. The geoid can be described only from real measurements.

3. Coordinate Systems

Geographic positions are described by means of coordinate systems. The systems based on right-angle coordinates are referred to as cartesian coordinate systems. The systems based on angles from baselines are referred to as polar coordinate systems. The unit of length in most systems is the meter, having been defined in the late eighteenth century as 1/10 000 000 of the distance from the pole to the equator. Nowadays the definition is based on the distance that light travels in a vacuum during 1/299 792 458 of a second, which does not change the length of the meter but improves the precision of measurement.

The most popular coordinate systems are geographical coordinates. A position is described by two parameters, latitude and longitude. The geographical latitude φ is defined north and south 90 degrees from the equator, the geographical longitude λ is defined east and west 180 degrees from the meridian through Greenwich observatory near London (see Figure 1). Being based on a spherical Earth the geographical coordinates are popular for small-scale applications but less suited for precise positioning.

In geodesy a cartesian coordinate system is used. The center of the coordinate system coincides with the mass center of Earth. The Z-axis is parallel to the mean rotation axis of Earth. The X-axis is perpendicular to Z and passes through the Greenwich meridian. The Y-axis is perpendicular to both X and Z. Satellite positioning systems such as the global positioning system (GPS) are based on this coordinate system.

For description of positions on the surface of Earth it is sometimes less practical to use the X,Y,Z system. Instead, the positions are referenced to a mathematical model of the surface (i.e. the ellipsoid). A position is described by three parameters: geodetic latitude, geodetic longitude, and height above ellipsoid. The geodetic latitude is the angle from the equatorial plane to the vertical direction of a line normal to the reference ellipsoid. The geodetic longitude of a point is the angle between a reference plane and a plane passing through the point, both planes being perpendicular to the equatorial plane. The ellipsoidal height at a point is the distance from the reference ellipsoid to the point in a direction normal to the ellipsoid. It differs from the orthometric height, which is defined along the direction of gravity from the point to the geoid.

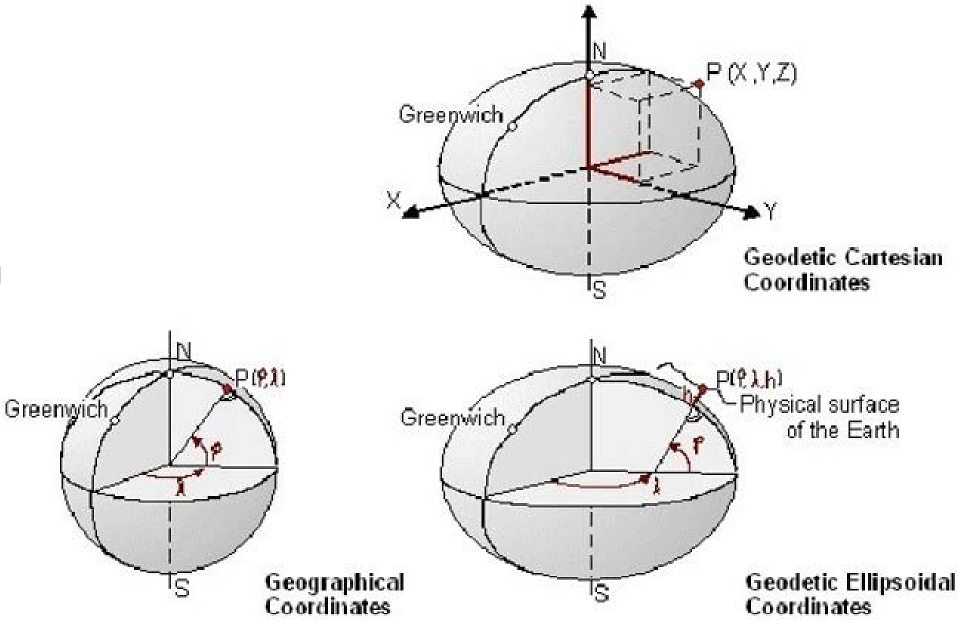

Figure 1. Coordinate systems

For transformation of positions between the cartesian coordinate system X,Y,Z and the ellipsoidal coordinate system φ,λ,h the geodetic datum is required. The geodetic datum is a set of parameters that define the shape of the ellipsoid and its position relative to the center of Earth. Complete datum conversion is based on seven parameter transformations: three translation parameters, three rotation parameters, and a scale parameter. Different nations and agencies use different data based on ellipsoids that fit best for their area of the world. Since the early 1980s the use of satellites has enabled geodesists to compute a datum that could be used anywhere on Earth. The name of the datum is WGS 84 (World Geodetic Reference System of 1984). It is based on the GRS80 ellipsoid that is determined to be the best fit for the planet as a whole, not just the best fit for one area.

Coordinates based on ellipsoidal-Earth models are required for accurate range and bearing calculations over long distances. Two-dimensional (2-d) coordinate systems are used for plane surveying, over distances short enough that Earth curvature is insignificant (less than 10 km). For conversion of geodetic cartesian coordinates X,Y,Z or geodetic ellipsoidal coordinates φ,λ,h to plane coordinates a map projection is required.

-
-
-

Bibliography

Dana P.H. (1995). *Map Projection Overview*. The Geographer's Craft Project, Department of Geography, The University of Colorado at Boulder; available on the Internet at: *http://www.colorado.edu/geography/gcraft/notes/mapproj/mapproj.html* [This material provides an introduction to map projections.]

Boehme R. (1988). Topographic cartography. *Basic Cartography, Vol. 2* (ed. R. Anson). London/New York: Elsevier Applied Science on behalf of International Cartographic Association. [This chapter describes the principles of topographic maps.]

Hake G. and Grünreich D. (1994). *Kartographie*, 599 pp. Berlin/New York: De Gruyter. [in German.] [This textbook provides an introduction to cartography and geographical information systems.]

Federal Office of Topography (2001). *Survey*. Wabern, Switzerland; available on the Internet at: http://www.swisstopo.ch.

Tortosa D. (2000). *GPS/GIS Techniques*. Lecture Notes, Saulte Ste. Marie; available on the Internet at: http://www.soonet.ca/eliris/gpsgis/gpsgis.html. [This material provides an introduction to satellite-based surveying techniques by the global positioning system.]

Biographical Sketch

Andreas Illert, born in 1959, obtained his master's in geodesy from the Technical University of Darmstadt, Germany, in 1985. Until 1992 Dr. Illert acted as a scientific assistant at the Institute for Cartography at the University of Hannover where he obtained his Ph.D. In 1993 he joined the Federal

Agency for Cartography and Geodesy at Frankfurt am Main. He has been involved in the definition of the authoritative topographic-cartographic information system for Germany and in the creation of cross-border topographic vector data for Europe. At present he is head of the unit for development of geo-information products.