MATHEMATICAL MODELING OF ATMOSPHERIC PROCESSES

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Summary

The state of the atmosphere has affected human activities since time immemorial. Since the 1950swhen the first successful attempt to foresee future development of weather using the solution to the equations describing the atmosphere behavior the numerical weather prediction has become broadly used almost in all weather services around the world. Now current weather forecast is based on the solution to the system of the equations describing the behavior of the atmosphere. Our knowledge about the atmosphere is ever evolving process and possible way that helps to maintain to keep this active is the mathematical modeling of the atmospheric processes. A brief and concise overview of the basic atmospheric processes and the way how to model them is the subject of this contribution. This text could help interested readers to better understand the method of weather forecasting and interpretation of their results that are acceptable not only in the TV but also on internet. The next aim is to show the relative complexity of all atmospheric phenomena that coexist together on the wide range of all spatial scales and interact one to each other.

The purpose of this chapteris to give the reader a brief overview of the methods used in the mathematical modeling of atmospheric processes. The field of atmospheric processes comprises a relatively broad spectrum of physical problems from classical hydrodynamics or, better, aerodynamics, to quantum physics when one is speaking about Solar radiation problems, mathematics and, first of all, numerical mathematics, chemistry (mainly in connection with air-pollution problems), computer sciences and also, at least partly, such branches as hydrology, geology, geography or biology.It is necessary to emphasize that many processes in the atmosphere are of a nonlinear nature and they show, in many cases, a chaotic behavior (for example turbulence) that makes their description and modeling quite complicated. As a mathematical approach to the problem of modeling these processeswill be described, the reader is supposed to have some level of mathematical knowledge as well as knowledge of basic physics.The author of this text did notgo too far and if a reader is interested in some part of this contribution in greater details then there is a list of references containing textbooks or monographs dealing with the individual problems that are briefly mentioned here.

1. Introduction

For many centuries in the past there was a very intensive attempt to understand the weather and, of course, people tried to forecast the future state of the atmosphere. But the understanding of the atmosphere was very vague andinexact and the main ancient knowledge about the atmosphere was gathered in Aristotle's '*Meteorologica*' (Aristotle 350 B.C.E.). On the other hand, many ancient civilizations were often strongly dependent on the weather or climatic conditions and if something occurred such as, for example,to cause irregularitiesinthe usual annual events (e.g. Egyptian civilization depended on the annual Nilotic floods, for example), there could result a disaster such as a famine. Many nations living on the sea shore depended on fishery and later also on sea trade and all these activities were strongly influenced by weather conditions. Up to approximately the beginning of the 17th century the whole period was based on speculations and personal experience. This situation changed with two important inventions that also changed the approach to the information about atmosphere – the Galileo’s invention of thermometer (between 1590 and 1600) and construction of barometer by Toricelli (1643 – 44). These two instruments enabled the carrying out of real measurement of quantities such as temperature and pressure that characterize the physical state of the atmosphere. At first these meteorological observations were usually carried out at astronomical observatories. The measurements of temperature and pressure were combined with the observation of the horizontal wind velocity (speed and direction). This could give a picture of atmospheric motion close to Earth’s surface. Later on (in the 20th century) this surface observation wascomplemented with upper air soundings and the information about the current state of the atmosphere was complete.

This short historical overview can be completed with several names of outstanding people who played an important part in the development of modern meteorology. We can start in the mid 19th century. At this time the main power of ships was wind and knowledge of the wind system pattern became more and more important. It became more apparent in the era of fast sail ships, clippers,which transported mainly tea from the Far East (China) to Europe and the U.S.A.This era (the second half of the 19th century)was connected with the name of the admiral Fitz-Roy who became the head of the British Meteorological Department in 1854. In this position he was a pioneer of weather forecasting and he also pioneered the printing of a daily weather forecast in newspapers but his activities were broader and they covered not only maritime meteorology but meteorology as a whole.

Shortly before Fitz-Roy became head of the British Meteorological Department there took place the First International Meteorological Conference in Brussels in August 1853. The primary effort of this conference came from the recognition of the necessity of international co-operation to assure reliable and regular information about the weather over the sea areas of the world. Following this Conference, efforts were made to develop international co-operation in many other fields of meteorology. In this regard, a non-governmental International Meteorological Organization (IMO) was created in September 1873 during the First International Meteorological Congress held in Vienna. On the basis of this organization the present World Meteorological Organization(WMO) has developed. The Convention, by which the World Meteorological Organization was created, was adopted at the Twelfth Conference of Directors of the International Meteorological Organization (IMO) that met in Washington on 11 October 1947.

There were many milestones in the development of meteorology itself and meteorological forecasts, but we will mention here only a few of them.The important one is tied up with the Bergen school of meteorology and with such names as Vilhelm and Jacob Bjerknes,Halvor Solberg, Tor Bergeron, Carl-Gustaf Rossby, Sverre Petterssen, Erik Palmén and others. Many still valid ideas used in everyday meteorological practice come from this famous Bergen school. Vilhelm Bjerknes in 1904 argued that future state of the atmosphere can be obtained from a knowledge of the current state of the atmosphere and understanding the physical laws that describe its behavior. In fact he foresaw the future development of the method of meteorological forecasting.

The first scientist who tried to realize practically what V. Bjerknes published in 1904 was Lewis Fry Richardson. In 1922 he attempted to forecast the weather using the numerical approach (Richardson 1922). But his forecast was unsuccessful and there were several reasons for this result. It was only a pity that the majority of the meteorological community was not able to recognize his pioneering idea and rejected his way to forecast the weather. There were only individual persons that despite Richardson's results tried to continue in the way he suggested. One of them was aSoviet meteorologist,I. A. Kibel. His approach was independent ofRichardson's approach and was based on equations written and solved in so called primitive variables (components of velocity, pressure, temperature). In the system of equations he solved in the eve of the 2nd WW there was no equation for moisture.For the western world his work was a long time unknown as he published his work and results in the Russian language. He becomes more widely known when his book called "An introduction tothe hydrodynamic methods of short period weather forecasting" was published in Russian (1957) and translated into English in 1963.

An important date for numerical weather prediction is in November 1950,number 4 of volume 2 of the Swedish journal *Tellus* appeared with the pioneering and famous article of the authors J. G. Charney, R. Fjørtoft and J. von Neumann, called "Numerical Integration of the Barotropic Vorticity Equation." This article is regarded as the birth of the era of numerical weather prediction. For a period lasting sixty years from this date,mathematical meteorological models made great progress and they are now much more complicated than the model mentioned in the title and this results described in this number of *Tellus*. Of course, the outputs of contemporary forecast meteorological models are wider and cover a broad spectrum of meteorological information.

The first successfully integrated prognostic meteorological model was mentioned above:the barotropic one with the atmospheric pressure level 500 hPa as that for which the prognosis was made. In Europe the situation was a little bit more complicated as there was no electronic computer available at this time. The first attempts in European "numerical" weather prediction were based on Fjørtoft's graphical integration technique. This method was used in Scandinavian countries, Germany and also in Czechoslovakia (in this state it is connected with the name of Professor S. Brandejs of CharlesUniversity). A qualitative change happened when the first Swedish computer BESK was available. More complicated barotropic models were developed and later on the first baroclinic models were integrated. These baroclinic models resolved one important issue. They were able to describe the creation and further development of the pressure lows and highs whereas the barotropic ones weren't able to catch this evolution. But this is beyond the scope of this article.

The reason why this article starts with this concise history of weather prediction and a very brief history of numerical weather prediction is obvious. If we talk about modeling of atmospheric processes we would have to have in mind that the final goal of any atmospheric model is to increase our knowledge about atmosphere and atmospheric processes and, finally, to improve weather predictions as much as possible. One can object to this statement: And what about climate modeling? Such a question is reasonable and climate modeling,too, will be discussed later. But first it will be necessary to distinguish what the weather is and what is the climate and what do the weather forecast models accomplish and what do the climate models accomplish.

The reason why we started with a concise history of meteorological services with emphasis given to the subject of meteorological prognoses and with a brief history of numerical weather prediction is that almost all branchesof modeling of the atmospheric processes aim to improve knowledge about atmosphere and, finally, to improve methods of forecasting the future state of the atmosphere. The topic of mathematical modeling of atmospheric processes can be divided according to what process we want to describe or to learn more about. One group of phenomena that are modeled deal with the meteorological processes of various spatial and temporal scales. The next ones are those concerned with air-pollution, its transport, dispersion and also the physical and chemical processes air-pollution is subject to. An important process that affects a lot of meteorological phenomena together withair-pollution problems deals with turbulence. There exist special models aimed at the problems of cloud microphysics, precipitation and circulation especially connected with dangerous types of clouds such as Cumulonimbus (Cb). It is known that Cb can be also attached to very dangerous atmospheric phenomenon, such as tornado. In the U.S., tornadoes kill hundreds of people every year and material damages can be enormous. But tornadoes can be met with also in the European mid-latitudes and damages can be also quite high. Other dangerous phenomena are hurricanes or typhoons (how these phenomena are called differs according to the area where they appear). For many of these phenomena there is a common problem. They can hardly be simulated in laboratories and mathematical models are the suitable tools to simulate their behavior and learn more about them.

If we have to speak about modeling of the atmospheric processes it would be good to distinguish between weather and climate at first. These two subjects have one important thing incommon – the atmosphere – but each of them deals with different points of views. Weather is the current state of the atmosphere that is defined by the values of some elements such as pressure, temperature, wind speed and directions, content of air moisture and others. These quantities can be either measured or derived from the measured primary data or they can be forecast using meteorological models. Climate describes the long term regime of weather conditions in a given place or area. In fact climate encompasses long term statistics of weather elementssuch as temperature, pressure, humidity and so on. Climate can be affected with such parameters such as latitude, altitude, nearby water bodies or terrain properties. For both branches of atmospheric processes modeling—the short term “weather” processes or the long term regime ones—one has to properly describe their mechanisms as such because on this basis it is then possible to create the proper statistics for climate modeling. Here we can point out the difference between the short time or “weather” forecast or modeling and climate modeling. In the former case we want to know the future distribution of the atmospheric quantities: how they are changing and what are the most important parameters causing this change and what are the secondary ones and what are the main physical mechanisms resulting in the process of these parameters’ changing. Especially in the weather forecast we are interested in the state of the atmosphere in the future in some short time interval(up to a few tens of hours) in the area of interest.We predicta new future “current” state of the atmosphere. Climate modeling is a little bit different. The climate modeling is not a long term analogy of the above mentioned meteorological forecast. If we are interested in climate modeling then, roughly speaking, there are two possible branches. In the first onethere is the attempt to describe the future development of some atmospheric phenomenon (NAO – North Atlantic Oscillations or ENSO – El Nino Southern Oscillations) under some accepted scenario. These scenarios define the conditions which are imposed on the atmospheric system. One can, for example study the future behavior of the ENSO under the condition that all parameters (composition of the atmosphere, Solar activity and others parameters) are unchanged. The results will be the statistics of quantities such as temperature, moisture, amount of precipitation and others for some long time period and their distribution in the investigated area (one or several decades in the beginning of the next century, for example). In the second branch one is interested in the future state of the atmosphere as a whole in the investigated area provided that there will be also some well specified scenarios. An example of such activity is modeling the impact of the increasing concentration of CO_2 in the Earth’s atmosphere. There are scenarios asserting that in the future, let us say in the next twenty years, for example,the concentration of CO_2 in the atmosphere due to industrial activity increases two, three or more times in comparison with the current state. Then one is interested inwhat the statistics of quantities such as temperature, precipitation moisture and others and their spatial distribution would look like in the area of interest for one, two or more decades, for example, in the future (in the next century, for example).

Originally the climatic modelswere the global or the hemispheric ones. This means that they covered the whole globe or one hemisphere. Their resolution was too low and atthe very beginning there was a great problem to couple the atmosphere with the ocean due to the very different time scales characterizing atmospheric and oceanic processes. Now, the climate models are very complicated and complex but their resolution is still to low, especially for some special purposes. There were developed from some so called downscaling techniques that would have to improve this shortcoming.Promising way of dynamic downscalingis based on the development of regional climatic models. Thanks to the fact that the area of interest in this type of model is not the whole globe or hemisphere but only a part of one continent or even some smaller area,the resolution of the regional climatic modelcan be sufficiently high. The topic of climate modeling now is very broad and complex, and scientists are more and more interested in this topic and in the connection with the so called global changes of the Earth climate. But in the rest of this presentation we will focus on the “standard” atmospheric modeling, which means the modeling that covers short time intervals (up to several tens of hours) and that is aimed at explaining the mechanisms of atmospheric processes or their principles.

At the end of this chapter we would like to briefly mention the various possible approaches to the task of giving a short overview of the methods of the mathematical modeling of atmospheric processes. One approach can come from the scales and discuss the processes from this point of view wherein the most important processes characterizing the existing scale and relationships between them are described. Another approach consists in the dividing of the atmospheric processes into individual groups according to their physical mechanisms and describes these processes from this point of view. But here, the approach which will be adopted is based on the fulfillment of individual conservation and the structure of this text is derived from this principle.

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Bibliography

Aristotle (350 B.C.E). *Meteorologica.* Translated by E. W. Webster. (available on web: http://classics.mit.edu/Aristotle/meteorology.html. [Text describing the ancient ideas of the processes occurring inside the Earth, on the Earth and in the atmosphere.]

Betts, A. K. (1986) *A new convective adjustment scheme. Part I: Observational and theoretical basis*, Quart. J. Roy. Meteor. Soc. 112, p.677–691. [Basic parameterization of the convective processes in the atmosphere. These processes can result in the creation of the convective type of clouds and precipitation from them. This parameterization is used in the meteorological prognostic models.]

Betts, A. K. and Miller, M. J. (1986), *A new convective adjustment scheme. Part II: Single column tests using GATE wave, BOMEX, ATEX and arctic air-mass data sets*, *Quart. J. Roy. Meteor. Soc.* 112**,** p. 693–709. [Verification of the convective adjustment described in the previous reference on the data from the mentioned experiments.]

Betts, A. K. and Miller, M. J. (1993), *The Betts–Miller scheme. The Representation of Cumulus Convection in Numerical Models,* Meteor. Monogr. 46, Amer. Met. Soc., p. 107–121. [Implementation of the convective adjustment scheme described in the above references in the numerical meteorological prognostic models.]

Bohren, F. C.; Albrecht, B. A. (1998), *Atmospheric Thermodynamics*, 402 pp., New York: OxfordUniversity Press. [Textbook on the atmospheric thermodynamic.]

Chang, C. H. (1977), *Ice Generation in Clouds*, M. S. thesis, Dept. of Meteor., S. Dakota School of Mines and Technol. [Some improvements dealing with the preceding reference.]

Charney, J. G.; Fjørtoft, R.; von Neumann, J. (1950), *Numerical Integration of the Barotropic Vorticity Equation*, Tellus 2, p. 237 – 254. [An historical article describing the first used successful model foe numerical weather prediction.]

Cotton, W. R; Anthes, R. A. (1989), *Storm and Cloud Dynamics*, 880 pp., San Diego: Academic Press, Inc. [A textbook dealing with the cloud formation and storm dynamics.]

Drikakis, D.; Rider, W. (2005), High Resolution Methods for Incompressible and Low-Speed Flows, 622 pp., Berlin: Springer-Verlag. [A textbook dealing with the special numerical methods used in hydrodynamics.]

Ferziger, J. H., Peric, M. (1997), *Computational|Methodsfor Fluid Dynamics.* 364 pp., Berlin: Springer-Verlag. [A textbook dealing with the computational approaches used in hydrodynamics (aerodynamics).]

Fritsch, J. M. and Chappel, C. F. (1980), *Numerical prediction of convectively driven mesoscale pressure system. Part I; Convective parameterization.* J, Atmos. Sci.37, p. 1722-1733. [Parameterization of the convectively driven mesoscale systems and their implementation into the mesoscale numerical meteorological models.]

Garrat, J. R. (1994), *The atmospheric boundary layer*, 316 pp., Cambridge, UK: Cambridge University Press. [A useful book dealing with the atmospheric boundary layer problems.]

Geurts, B. J. (2004), *Elements of direct and large-eddy simulation*, 329 pp., Edwards. [Advanced textbook about direct and large eddy simulation of turbulence.]

Haltiner, G. J.; Williams, R. T. (1980), Numerical Prediction and Dynamic Meteorology, 496 pp., John Wiley & Sons. [Principles of dynamic meteorology and methods used in numerical weather prediction.]

Holton, J. A. (1973), *An Introduction to Dynamic Meteorology,* 319 pp., New York: Academic Press. [One of the classical textbook of dynamic meteorology.]

Houghton, D.D. (1985), *Handbook of Applied Meteorology*, 1461 pp., New York: Wiley Interscience Publications. [Handbook of routinely used approaches in meteorological everyday routine.]

Houze, Jr., R. A. (1993), *Cloud Dynamics*, 573 pp. San Diego: Academic Press, Inc. [Basic textbook of the cloud dznamic and connected processes.]

Jacobson, M. Z. (1999) *Fundamentals of Atmospheric Modelling*, 656 pp., Cambridge, UK: CambridgeUniversity Press. [More detailed textbook dealing with the fundamentals of the atmospheric processes modeling.]

Kessler, E. (1969), *On the distribution and continuity of water substance in atmosphere circulations*, Meteorol. Monogr.No. 32, 84 pp., Boston: Am. Met. Soc. [Basic parameterization of the stratiform clouds.]

Kibel, A. I. (1957), *An introduction to the hydrodynamic methods of short period weather forecasting*, translated from the original 1963, 383 pp., New York: The Macmillan Co. [Description of methods used by the Soviet meteorologist in his successful attempt to predict the future state of the atmosphere using the numerical approach. This approach dates to late thirtieth and beginning of fortieth of the last century.]

Kim, J.; Kim, D.; Choi, H. (2001), *An Immersed Boundary Finite-Volume Method for Simulations of Flow in Complex Geometries*, J. Comput. Phys. 171, p. 132 - 150. [One of the articles describing the application of the immersed boundary method using so called direct forcing.]

Knupp, P.;Steinberg, S. (1994), *Fundamentals of Grid Generation,* 285 pp., Boca Raton: CRC Press. (On the mathematical bases described methods of grid generation.)

Kuo, H. L. (1974), *Further studies of the parameterization of the influence of cumulus convection on large-scale flow,* J. Atmos. Sci. 31, p. 1232 – 1240. [Parameterization of the convective (cumulus) clouds development. The same in the subsequent four references.]

Lesieur, M.; Métais, O.; Comte, P. (2005), *Large-Eddy Simulations of Turbulence*, 219 pp., Cambridge: CambridgeUniversity Press. [Basic principles of large eddy simulation.]

Li, C. W.; Wang, L. L. (2004), *An immersed boundary finite difference method for LES of flow around bluff bodies*, Int. J. Numer. Meth. Fluids 46, p. 85 – 107.[Another way of application of the immersed boundary method.]

Nakayama, H.; Nagai, H. (2009) *Development of Local-Scale High-Resolution Atmospheric Dispersion Model Using Large-Eddy Simulation Part 1: Turbulent Flow and Plume Dispersion over a Flat Terrain,* J. Nuclear Sci. Technol. 46, p 1170-1177. [Application of the large eddy simulation approach to the air-pollution transport and dispersion.]

North, G. R.; Erukhimova, T. L. (2009), *Atmospheric Thermodynamics Elementary Physics and Chemistry*, 267 pp., Cambridge, UK: CambridgeUniversity Press. [Other textbook on the atmospheric thermodynamic.]

Pielke, R. A. (1984), *Mesoscale Meteorological Modeling*, 612 pp., Orlando: Academic Press. [Fundamentals of the mesoscale processes and their modeling.]

Pielke, R. A. (2002), *Mesoscale Meteorological Modeling,* 676 pp., 2nd edition, San Diego: Academic Press. [The second edition of the preceding book.]

Richardson, L. F. (1922). *Weather Prediction by Numerical Process,* 236 pp., Cambridge, UK: CambridgeUniversity Press. [A description of the first attempt to foresee the future development of the atmosphere using the mathematical approach.]

Rogers, R. R.; Yau, M. K. (1996), *A Short Course in Cloud Physics*, 290 pp., Butterworth-Heinemann. [Basic textbook on the cloud physics and microphysics.]

Sagaut, P. (2002), *Large Eddy Simulation for Incompressible Flows*, 592 pp., Berlin: Springer-Verlag. [A comprehensive book dealing with the all of the incompressible flows large eddy simulation topics.]

Scluenzen, K. H.; Bigalke, K.; Lenz, C. J.; Luepkes, C.; Niemeier, U.; vonSalzen K. (1996), *Concept and Realization of the Mesoscale Transport and Fluid Model 'METRAS'*, 131 pp., Meteorologisches Institut der Universit¨at Hamburg. [A detailed description of the Uni-Hamburg Mesoscale prognostic model METRAS.]

Scorer, R. (1986), *Cloud Investigation by Satellite*, 758 pp., Chichester: Ellis Horwood Limited. (Various kinds of clouds as seen from the high resolution satellite spectrometer.)

Scorer, R.; Wexler, H. (1967), *Cloud Studies in Colour*, pp.44, Oxford: Pergamon Press Ltd. [Photos of clouds.]

Stull, R. B. (2003), *An Introduction to Boundary Layer Meteorology*, 670 pp., Dordrecht: Kluwer Academic Publisher. [Textbook on the atmosphericboundary layer physics.]

Vreman, A. W. (2007), *Subgrid-modeling in large-eddy simulation of complex flows*, p. 301-304, Progress in Turbulence II, Eds. M. Oberlack et al., Heidelberg: Springer-Verlag. [Description of the sub-grid scale model of the large eddy simulation of turbulence.]

Wilcox, D. C. (2006), *Turbulence Modeling for CFD*, 522 pp., DCW Industries. [Textbook about turbulence modeling methods used in CFD application.]

WMO (1956), *International Cloud Atlas. Volume II*, pp. 224, World Meterological Organization. [Cloud atlas with detailed description of individual snaps. Professional aid for the meteorological observers.]

Yarwood, G.; Whitten, G. Z.; Rao, S. (2005), *Updates to the Carbon Bond 4 Photochemical Mechanism*, Final Report, Environ, pp. 45. [Description of the chemical mechanism for the photochemical reactions.]

You, D.; Moin, P. (2007), *A dynamic global-type coefficient sub-grid scale eddy-viscosity model for large-eddy simulation in complex geometries*, Phys. Fluids 19, p. 065110-1 – 065110-8. [Description of the same model as in Vreman but with the dynamic approach used for correction of the used coefficient.]

Zdunkowski, W. and Bott, A. (2003), Dynamics of the Atmosphere, 719 pp., Cambridge, UK: CambridgeUniversity Press. [Theory of the atmospheric dynamics.]

Zeytounian, R. Kh. (2004), *Theory and Applications of Viscous Fluid Flows*, 488 pp., Berlin: Springer-Verlag. [Textbook dealing with the theory of the viscous flow.]

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