EARTHQUAKE GROUND MOTION

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Contents

Summary

Seismology is based on the collection, analysis, and interpretation of seismograms, namely the records of the motion of a given point on Earth induced by the passage of seismic waves that can be caused both by artificial (e.g., explosions) and natural (e.g., earthquakes) processes. Even focusing only on earthquake phenomena, the range of studies is broad, extending from global seismology, thus dealing with the study of the structure of the whole planet, to the definition of the seismic input for structural engineers. This paper tries to present a general perspective on this extremely wide phenomenological scenario, but it will focus mainly on the seismic ground motion synthesis. Modern computational techniques yield solutions to the forward problem of seismic wave propagation, specifically, calculation of the seismic wave field as a function of position and time, starting from a given model, which is as realistic as possible, of the seismic source and of the propagation medium. Such calculations not only provide the theoretical framework for solving the inverse problem for the structural and source parameters, but they can be used to estimate the strong ground motions in the vicinity of an anticipated earthquake. Such realistic simulations (e.g., synthetic seismograms) could allow predisaster surveys that can be usefully employed to mitigate the effects of the next earthquake, using all available technologies. This paper intends to

present the three main elements that influence the ground motion at a particular site. The first describes the generation of seismic waves in terms of the properties (e.g., the size) of the earthquake source. The second describes the effects of the Earth system on the propagation of such waves to the considered location, and the third describes how the local geological and topographical conditions may influence the resulting motion at the given site.

1. Introduction

The conceptual framework for the studies on earthquake ground motion historically started (since the late 1600s) from the description of the mechanical behavior of simple linear elastic bodies, and the body of the theory developed significantly during the late 1800s and early 1900s, thus providing a powerful base of knowledge for the analysis of seismic waves. Simple models (e.g., layered half-spaces, characterized by only one direction of heterogeneity), have been and are extremely useful in providing initial insight to the general elastic properties of Earth. However, the contribution of seismology to the knowledge of Earth's interior has dramatically improved starting in the 1960s when high-quality instrumental data became available and computers enabled fast data processing. From the analysis of high-quality seismograms, it became evident that large lateral heterogeneities are not confined to the transition areas between oceans and continents, or just to the superficial geology, but characterize the whole structure of Earth with the possible exception of the outer core. In fact, one of the major goals of studies on global seismology is the determination of the three-dimensional structure of Earth by different methods—direct and inverse—that can be grouped under the common name of seismic tomography (see *Continental Crust*; *Oceanic Crust*; and *Mantle and Core of the Earth*).

When an earthquake fault ruptures, it causes two types of deformation: static and dynamic. Static deformation is the permanent displacement of the ground due to the event, whereas dynamic motions are the seismic waves that may represent up to 10% of the driving energy of plate tectonics. More specifically, seismic waves can be represented as elastic perturbations propagating within a medium, originated by a transient disequilibrium in the stress field.

The transient seismic waves generated by any substantial earthquake propagate all around and entirely through Earth, and a sensitive enough receiver can record the seismic waves from even minor events occurring anywhere in the world. The resulting ground motion can be divided in two levels: weak motion, characterized by small amplitudes generated by distant or small events, and strong motion, characterized by large amplitudes generated by near or large events. Strong motion is the context where seismic hazard estimates and engineering analysis are defined.

1.2. Measuring Ground Motion

The instrumental record in seismology is the basis of the quantitative analysis and documentation of seismicity, namely the occurrence of earthquakes in space and time. Most of the recording systems are called seismographs, which allow accurate relationships to be developed between the seismic record and the amplitude and frequency content of the causal ground motion. Generally, they are designed to magnify weak motions generated by small or distant seismic events, allowing earthquakes to be located and, analyses to be conducted of seismic sources, and studies to be conducted of the parts of Earth through which seismic waves travel. The core of a seismograph is called seismometer: it is the actual sensor of the ground motion, and it is primarily based on a damped inertial pendulum system. Transitory disturbances, like seismic waves, make the planet move and cause the pendulum to return to the initial equilibrium position because of its inertial properties. The relative movement between the inertial pendulum and the moving Earth is converted to a signal that is the record of the passing seismic waves, namely the seismogram.

The dynamical equation governing the pendulum in an inertial reference system has to include the forcing ground motion and the deviation of the pendulum mass from its equilibrium state. In the case of a simple vertical pendulum (see Figure 1) the so-called indicator equation is as follows:

$$\ddot{x}(t) + 2\gamma\omega_0\dot{x}(t) + \omega_0^2 x(t) = -GU(t) \qquad (1)$$

where the dot indicates the time derivative, x is the actual seismogram, U is the ground displacement, γ is the damping factor, ω_0 is the resonant frequency of the undamped system, and G is a magnification coefficient.

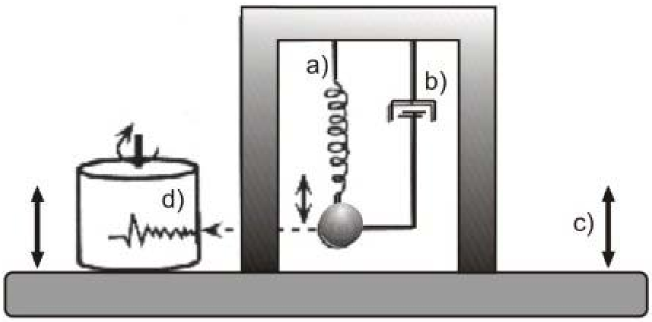

Figure 1.Scheme of inertial-pendulum vertical seismograph
(a) Inertial pendulum; (b) damper; (c) vertical ground motion; (d) seismogram

The solutions to Eq. (1) for prescribed $U(t)$ characterize the system response, and they can easily found through Fourier transformation and harmonic analysis. If $U(t)$ is a simple harmonic (i.e., $U(t) = e^{i\omega t}$), the Fourier transform of $x(t)$ is $X(\omega)e^{i\phi}$, and one has the following response curves:

$$X(\omega) = \frac{G\omega^2}{\left(\left(\omega^2 - \omega_0^2\right) + 4\omega^2\omega_0^2\gamma^2\right)^{1/2}}$$
$$\phi = -\tan^{-1}\left(\frac{2\omega\omega_0\gamma}{\omega^2 - \omega_0^2}\right) + \pi \qquad (2)$$

and from (2) some conclusions may be immediately drawn: (i) if the system is undamped, the solutions have greatest response at frequencies near the natural frequency (resonance); (ii) if the damping factor is 1, the system is critically damped and oscillation is minimized; (iii) if $\omega << \omega_0$, the amplitude response is proportional to ω^2, and the system records ground acceleration (accelerographs); and (iv) if $\omega >> \omega_0$, recorded displacement is proportional to that of the ground. While original seismometers were mechanical and gave an analog output record, modern instruments are designed to be sensitive to velocity in order to obtain an electrical output signal (voltage) that can be easily digitized and analyzed. The most modern versions (since 1975), the so-called force-feedback instruments, use a force proportional to the inertial mass to cancel the relative mass motion, and the electrical equipment outputs an electrical signal to assess how much feedback force (corresponding to ground acceleration) is needed. Such a strategy has dramatically improved the bandwidth and linearity of the instrument response, since mass excursion is very small, allowing modern broadband recordings.

1.2. Intensity and Magnitude Scales

Prior to the instrumental seismology revolution (i.e., before 1950), the necessity of comparing earthquakes, and of achieving better-defined macroseismic (i.e., directly observable) earthquake effects due to different seismic events, led to the definition of seismic intensity scales. Intensity scales are numerical scales, based on the definition of different discrete levels of severity for earthquake effects, each step providing a qualitative description of such effects. Intensity scales can be used to define the areas of common damage from ground shaking. Usually, the highest intensities values are grouped in the area near to the fault that generates the seismic event, but the proximity of the source region to anthropic areas, the quality of existing buildings, construction practices, and site effects (see Section 3.3) can deeply influence the intensity measurements. However, even if intensity values can be of a subjective and qualitative nature, macroseismic data are necessary for the study of historical seismicity, based only on historical records.

Instrumental recordings allow an almost direct estimate of earthquake size, called earthquake magnitude. Although a rough single-value estimator of source strength, it is one of the most used parameters due to its simple characteristics. The main concept is that of a relative-size scale based on the measurements of seismic phase amplitudes, and the basic assumption is that, source geometry and travel path aside, larger events generate larger amplitudes.

$$M = \log\left(\frac{A}{T}\right) + f(r,h) + C_{\mathrm{S}} + C_{\mathrm{R}} \qquad (3)$$

where A is the ground displacement, T the period at which it is measured, the function f accounts for epicentral distance (r) and focal depth (h), and C_{S} and C_{R} are station and regional corrections, respectively. The logarithmic scale is used because the amplitude term, measured after instrumental response has been removed, may vary enormously. Many magnitude scales have been developed (since the original Richter "local" scale), but those most commonly used are the surface-wave magnitude, M_{S}, and body-wave magnitude, m_b. The former is determined from the amplitude of Rayleigh waves (see Section 3.2.2) at a period of 20 s that dominate seismograms from distant or shallow events, while the latter is determined by the maximum amplitude of the first few cycles of P-waves (see Section 3.2.1) that usually occur at periods of approximately 1 s and best characterize deep events.

A unified scale, first introduced in the mid-1970s, encompasses the limitation of saturation suffered by other scales and that is based on the concept of seismic moment, M_0 (see Section 3.1). Such a scale has the great advantage of being directly connected to earthquake size, while being independent of other parameters like event depth and epicentral distance. It has been named the moment magnitude, measured in dyne-centimeters, and defined as follows:

$$M_W = \log\left(\frac{M_0}{1.5}\right) - 10.73 \qquad (4)$$

in order to be generally consistent with other scales over a wide range of magnitudes, until the others begin to saturate (i.e., when they stop increasing with increasing earthquake size).

2. Theoretical Basis

The properties of seismic waves are ruled by the physics of elastic bodies, namely bodies that tend to revert to the natural stress-free state when the applied forces are removed. The analytical framework is represented by the formalisms of the elastodynamic theory and, in order to take into account macroscopic phenomena, it is assumed that the material is a continuum (i.e., that the matter is distributed continuously in space). In this manner, it is possible to define the mathematical functions that describe the fields associated with displacement, stress, and deformation. Furthermore, being interested in the motion of the elastic body under consideration, the Lagrangian description is used, where the motion of each particle is analyzed in space and time, and the vector field $\mathbf{u}(\mathbf{x},t)$, associated with the displacement at the time t from the position $\mathbf{x}$ assumed at a given reference time, can be defined at any point of the body.

2.1. Equations of Elastic Motion

Considering the balance of forces, including inertia, body forces and surface forces acting on a cubic element within the continuum, and applying the Newton's law, we obtain the system of equations of motion (5):

$$\begin{aligned}
\rho\frac{\partial^2 u_x}{\partial t^2} &= \rho X + \frac{\partial\sigma_{xx}}{\partial x} + \frac{\partial\sigma_{yx}}{\partial y} + \frac{\partial\sigma_{zx}}{\partial z}\\
\rho\frac{\partial^2 u_y}{\partial t^2} &= \rho Y + \frac{\partial\sigma_{xy}}{\partial x} + \frac{\partial\sigma_{yy}}{\partial y} + \frac{\partial\sigma_{zy}}{\partial z}\\
\rho\frac{\partial^2 u_z}{\partial t^2} &= \rho Z + \frac{\partial\sigma_{xz}}{\partial x} + \frac{\partial\sigma_{yz}}{\partial y} + \frac{\partial\sigma_{zz}}{\partial z}
\end{aligned} \qquad (5)$$

where a Cartesian coordinate system (x, y, z) is adopted and ρ is the density of the material. The quantity $\sigma_{ij}(\mathbf{x},t)$ (i = x, y, z; j = x, y, z) indicates the second order stress tensor (i.e., the j-th component of the traction acting across the plane normal to the i-th axis). Its spatial derivatives represent the forces internal to the medium due to the contact between the adjacent particles. The quantities X, Y, and Z are the components of body forces, for a unit mass; these can be due to forces deriving from processes external to the medium or to forces between particles that are not adjacent (e.g., the gravitational force).

The deformations inside the medium can be represented by the second order strain tensor $e_{kl}(\mathbf{x},t)$ (k = x, y, z; l = x, y, z) are assumed infinitesimal and can be written as a function of displacements:

$$\begin{aligned}
e_{xx}+e_{yy}+e_{zz} &= \frac{\partial u_x}{\partial x}+\frac{\partial u_y}{\partial y}+\frac{\partial u_z}{\partial z}\\
e_{xy}=e_{yx} &= \frac{1}{2}\left(\frac{\partial u_x}{\partial y}+\frac{\partial u_y}{\partial x}\right)\\
e_{yz}=e_{zy} &= \frac{1}{2}\left(\frac{\partial u_z}{\partial y}+\frac{\partial u_y}{\partial z}\right)\\
e_{xz}=e_{zx} &= \frac{1}{2}\left(\frac{\partial u_z}{\partial x}+\frac{\partial u_x}{\partial z}\right)
\end{aligned} \tag{6}$$

In a general elastic body, the relation between stress and strain (i.e., the constitutive relation) can take a very complex form, since it has to include the effects of parameters including pressure, temperature, and the amount and variability of stress. Nevertheless, considering small deformations and stresses of short duration (conditions satisfied in most seismological problems), we can assume that the solid behaves linearly, and Hooke's Law appropriately links stresses and deformation through the constitutive relation as follows:

$$\sigma_{ij}=C_{ijkl}e_{kl} \tag{7}$$

where the convention of repeated indices is used; C_{ijkl} are the components of a fourth-order symmetric tensor, whose 81 elements are the elastic moduli.

It has to be mentioned that Eq. (7) can be obtained assuming small perturbations to a stress-free reference state, but this is not the case for the actual Earth, due to self-gravitation. The problem of the strain of an elastic solid from a condition of great initial stress cannot be treated by ordinary elasticity equations, since Eq. (7) assumes proportionality between strain and actual stress. A number of studies conducted early in the twentieth century on the strain of a gravitating sphere approached the problem as if the body in question were deformed starting from an initial configuration in which no gravity exists. If applied to a body of planetary dimensions, this procedure produces deformations far beyond elastic the limits of known materials. This method produces valid solutions only if it is also assumed that the material under consideration is incompressible. In this case, the effect of gravity alone is to create in the sphere a condition of hydrostatic stress with zero strain. A solid body, however, may reasonably be supposed to behave elastically for small deformations, even if initially in a condition of great stress strain. In this case, it seems reasonable to apply the ordinary laws of elasticity with the following modification: the relation "the actual stress is proportional to the strain" is replaced by "the incremental stress is proportional to the strain." The prestressed state then serves as the reference state.

If the solid is isotropic (as mostly occurs for the Earth), the components of the C_{ijkl} tensor assume the same value for all the axes; and, since e_{ij} are the components of a symmetric tensor, Eq. (7) becomes:

$$\sigma_{ij} = \lambda e_{kk}\delta_{ij} + 2\mu e_{ij} \tag{8}$$

where δ_{ij} is the Kronecker delta, and the scalar quantities λ and μ are called Lamè parameters.

2.2. Representation Theorem

The system of equations in Eq. (5) in conjunction with the constitutive relation of Eq. (7) represent the most fundamental framework in the theory of seismology, connecting the forces acting on an elastic medium to the displacement measurable within it. It should be noted that the specification of the body forces over the volume V of the elastic medium and of the tractions on its surface S determine univocally ("uniqueness theorem"), given the initial conditions, the displacement field throughout the medium. Another important property of elastodynamic systems is the possibility of representing the displacement field developing in an elastic body in terms of quantities that have generated the disturbance. This property, known as the "representation theorem," may be formally expressed in several ways, but the common physical principle is that the displacements developing from realistic seismic sources can be expressed in terms of the field developed by the simplest set of displacements. Such an elementary displacement field is termed elastodynamic and is described by a Green's function, which is a second-order tensor with components G_{ij}. These represent the solutions to the i^{th} component of motion of Eq. (1) in the case where the body forces are replaced by a single force acting in the n^{th} direction (i.e., $\mathbf{f} = \delta_{in}\delta(\mathbf{x} - \mathbf{x}_0)\delta(t - t_0)$).

The representation theorem represents the natural framework to incorporate the process of slip on a buried fault and the related process of seismic waves generation. Considering an elastic medium of volume V bounded by the external surface S with a fault B, the slip is supposed to act between the two adjacent surfaces, B_+ and B_-. Thus, the seismic source can be introduced into the elastic body schematizing the fault, which is assumed planar, with a discontinuity in the displacement field through the plane B; normal stresses are assumed continuous. The displacement at a point of the medium can be written as follows:

$$u_i(\mathbf{x},t) = \iint_B m_{kq} * G_{ik,q}\, dB \tag{9}$$

where * indicates the convolution symbol, and

$$m_{kq} = [u_n]\nu_j C_{njkq} \tag{10}$$

are the components of the moment density tensor, representing the strength of a (k,q) couple with arm pointed in the q direction, and $\boldsymbol{\nu}$ is the unitary vector normal to plane B. If the slip, namely the displacement discontinuity across the fracture plane, is contained the plane B, it will be always perpendicular to $\boldsymbol{\nu}$; this corresponds to a shear dislocation model, and Eq. (10) becomes

$$m_{kq} = \mu\left(\nu_k [u_q] + \nu_q [u_k]\right) \tag{11}$$

If it is assumed that the periods and wavelengths of interest are large in comparison, respectively, with the rise time (the time necessary for the slip to occur at a point on the fault plane) and with the dimensions of the finite fault, the source acts as a point source. Thus, the contributions from the different surface elements dB can be considered in phase, and Eq. (9) becomes

$$u_i(\mathbf{x},t) = M_{kq} * G_{ik,q} \tag{12}$$

where

$$M_{kq} = \iint_B m_{kq}\, dB = \iint_B \mu\left(\nu_k [u_q] + \nu_q [u_k]\right) dB \tag{13}$$

are the components of the moment tensor, whose scalar related quantity

$$M_0 = \mu \text{ (averageslip fault area) } = \mu\bar{u}A \tag{14}$$

is called the seismic moment and is the most fundamental parameter used at present to measure the energy radiated by an earthquake caused by a fault slip. Thus, the representation theorem shows the rigorous dynamical equivalence of the faulted medium and an unfaulted medium with a set of coupled forces distributed on the fault plane B. Eq. (9) shows that the seismic motion throughout an elastic body can be represented by an integration over the region where the seismic source acts, and the motion interacts with the displacement field associated with the proper Green's function that represents the propagation medium.

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Bibliography

Aki K. and Richards P.G. (1980). *Quantitative Seismology: Theory and Methods*, 736 pp. University Science Books. [This book presents the general aspects and approaches to quantitative and theoretical seismology in a comprehensive and deep way; useful at intermediate and advanced knowledge levels.]

Bard P.-Y. (1997). Local effects on strong ground motion: basic physical phenomena and estimation methods for microzoning studies. *SERINA- Seismic Risk: An Integrated Seismological, Geotechnical and Structural Approach*, pp. 229–300, Thessaloniki: ITSAK Ed. [This paper presents an extended review of theoretical and empirical methods for the site effects estimation.]

Ben-Menhaem A. and Singh S.J. (2000). *Seismic Waves and Sources*, 1136 pp. Dover Publications. [This book presents in a very detailed way the aspects of the processes of generation and propagation of seismic waves]

Bolt B.A. (1998). *Earthquakes*, 377 pp. San Francisco: Freeman. [This textbook presents a revised and updated general review on earthquakes and their effects; useful at an introductory knowledge level.]

Bullen K.E. and Bolt B.A. (1985). *An Introduction to the Theory of Seismology*, 499 pp. Cambridge, UK: Cambridge University Press. [This book represents a comprehensive reference of general seismology; useful at an introductory knowledge level.]

Dahlen F.A. and Tromp J. (1998). *Theoretical Global Seismology*, 944 pp. Princeton: Princeton University Press. [This book contains a comprehensive and detailed treatment of global seismology describing the normal-mode body-wave, and surface-wave theory and application methods.]

Lay T. and Wallace T.C. (1995). *Modern Global Seismology*, 521 pp. San Diego: Academic Press. [This book presents all the essential aspects of quantitative seismology in a very comprehensive way; useful at an intermediate knowledge level.]

Panza G.F., Romanelli F., and Vaccari, F. (2000). Seismic wave propagation in laterally heterogeneous anelastic media: Theory and applications to seismic zonation. *Advances in Geophysics* Vol. 43, pp. 1–95, San Diego: Academic Press. [This paper presents a review of methods and applications for the deterministic seismic hazard assessment at regional and local scales.]

Reiter L. (1990). *Earthquake Hazard Analysis*, 254 pp. New York: Columbia University Press. [This book describes the application of deterministic and probabilistic seismic hazard analysis from a perspective useful to earth scientists, engineers, and decision makers.]

Biographical Sketch

Dr. **Fabio Romanelli,** received his Laurea in physics (1993) and doctorate in geophysics (1998) from the University of Trieste, Italy. After being a researcher of the GNDT (Italian National Group for the Defense from Earthquakes), he is now a researcher of the Department of Earth Sciences at the University of Trieste. His research involves the study of the excitation and the propagation of seismic and Tsunami waves in laterally heterogenous media. The recent focus of his work has been the theoretical estimation of the seismic site effects.The seismology group at DST (*Dipartimento di Scienze della Terra*) of Trieste is a leader in the high-frequency modal summation method to compute synthetic seismograms. The research interests cover a wide variety of topics in seismology, and they are performed in the framework of numerous national and international projects, also in collaboration with the Structure and Non-linear Dynamics (SAND) Group of the Abdus Salam International Centre for Theoretical Physics.