ELECTRIC FIELD OF THE EARTH

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Summary

The article consists of three parts. The first one gives the average amplitudes of the earth currents in mV/km for different periods calculated from the apparent resistivity values of the magnetotelluric sounding curves measured in the Pannonian Basin at middle latitude. After that, it characterizes the different types of earth current variations.

The second part is devoted to the electromagnetic (EM) induction technique with which the distribution of the resistivity of rocks and minerals can be determined underground, with different penetration depths depending on the period of the electromagnetic field (magnetotellurics, geomagnetic deep sounding, and tellurics).

In the third part, there is a short summary on the resistivity of the rocks and minerals on the surface, and their dependence on the temperature, pressure, water content, and partial melting. A description of the main conductivity anomalies (CA) follows, with their causes determined by EM induction techniques in Earth’s crust and the upper mantle of a young orogen (Carpatho-Pannonian region) and of a very old cratogene (Fennoscandian shield). This description gives a possibility for comparison of the resistivity distribution in two extreme continental cases of Earth’s history, and limits the resistivity in Earth’s interior in connection with its physical conditions, therefore determining the earth current amplitudes of different penetration depths.

1. Introduction

The connection between the geomagnetic and geoelectric fields is described by the Maxwell equations, which result in quite simple equations for a homogeneous half-space and for a vertically incident plane electromagnetic wave. According to these equations, the direction of the geoelectric field vector is perpendicular to the
geomagnetic field vector. The ratio of the two fields is the impedance, which depends on the period of the variation and on the conductivity of the underlying half-space. This rather strong simplification gets less and less valid for variations with longer and longer periods. The inhomogeneity of the underlying layers may distort both the amplitude ratio and the direction of the electric field vector. The exploration of such inhomogeneities is exactly the task of geoelectric exploration methods, and the detection of the inhomogeneity is possible because of the deviation from the field in a homogeneous half-space.

Thus in the following, the geoelectric field is described in outline for a geomagnetic source field supposed to be a vertically incident plane wave and for a homogeneous half-space under the station. Then, more sophisticated formulas for the geoelectric field are summarized, and information concerning the distribution of the conductivity depending on the tectonic setting of the area is given, taking as examples the Carpatho-Pannonian orogen region and of the cratogene Baltic Shield.

2. The Electric Field as it Appears in Telluric and Magnetotelluric Studies

The electric field \( E \) corresponding to known geomagnetic variations \( H \) of period \( T \) is connected to these variations by the following relation (see also Section 3, Eq. (4)):

\[
\begin{align*}
E_x &= Z_{xx}(T).H_x + Z_{xy}(T).H_y \\
E_y &= Z_{yx}(T).H_x + Z_{yy}(T).H_y
\end{align*}
\] (1)

where \( Z \)-s are components of the complex impedance tensor depending on the period of the variation, \( T \). These equations are simplified in the case of a homogeneous half-space to:

\[
\begin{align*}
E_x &= Z.H_y \\
E_y &= Z.H_x
\end{align*}
\] (2)

The apparent resistivity, \( \rho \), and the electric field vector \( E \) are:

\[
\rho = 0.2TZ^2; \ E = H\left(\frac{5\rho}{T}\right)^{1/2}
\] (3)

(The same formula applies if \( E \) is in the direction of an axis of the telluric absolute ellipse and this direction is not distorted, that is, it remains perpendicular to \( H \)). Using this formula, the electric field can be computed for an arbitrary distribution of the resistivity for a given spectrum of the magnetic variations.

Figure 1 represents a magnetotelluric (MT) sounding curve above a homogenous half-space of \( \rho = 20 \, \Omega \, m \), and three (\( \rho_{\text{min}} \)) MT sounding curves from the Carpathian Basin: that for the Nagycenk Observatory is about the average of the basin, and the two curves
at the uppermost and lowermost resistivity levels are both at stations with sedimentary cover.

Figure 2 shows the supposed spectrum of the magnetic variations at (geomagnetic) latitudes around 45° to 50°, with an exponential connection between amplitude and period. This approximation is a very crude one: the upper line indicates the maximum amplitudes that can be expected in intervals without geomagnetic storms (the geomagnetic field may be rather inhomogeneous in geomagnetic storms, therefore it is not advisable to use such intervals in MT studies). Amplitudes well below the bottom line may also be experienced in geomagnetically quiet times. Moreover, geomagnetic amplitudes increase, at least in certain frequency ranges, toward higher latitudes.

Figure 1. Some typical ($\rho_{\text{min}}$) MT sounding curves from the Carpathian Basin

Figure 2. Supposed “average” and “maximum” amplitudes in $H$
Using the data of Figures 1 and 2, electric amplitudes can be computed for different geological situations. Figure 3 is based on the “average” spectrum from Figure 2: different geomagnetic amplitudes—for example, those of the “maximum” curve in Figure 2—can be logarithmically transferred into this plot (same ratio of the $H$ and $E$ amplitudes).

Table 1 summarizes the expected amplitudes, occurrence, and noise sources in the different period ranges (GM means in the Table 1 “geomagnetic”).

<table>
<thead>
<tr>
<th>$T$, s</th>
<th>Type</th>
<th>Occurrence</th>
<th>$H$ amplitude</th>
<th>Typical noise</th>
<th>Recommended time of measurements</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.01</td>
<td>ELF</td>
<td>always</td>
<td>tens of pT</td>
<td>electric network</td>
<td></td>
</tr>
<tr>
<td>0.03</td>
<td>ELF</td>
<td>always</td>
<td>tens of pT</td>
<td>electric network and harmonics</td>
<td></td>
</tr>
<tr>
<td>0.1</td>
<td>Schumann</td>
<td>nearly always, strong Q</td>
<td>tens of pT, in Q events up to 100 pT or more</td>
<td>impulses from the electric network, lightning</td>
<td></td>
</tr>
<tr>
<td></td>
<td>resonance</td>
<td>events about once an hour</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.3</td>
<td>noise</td>
<td>random</td>
<td>tens of pT</td>
<td>impulses from the electric network, lightning</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>Pc1 (pearl)</td>
<td>not too often during night and winter, solar minimum</td>
<td>“pearls” with duration of about 1 min, amplitudes of a tenth to a few nT</td>
<td>impulses from the electric network</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>night (and autumn or winter)</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>Pi1</td>
<td>during night, high GM and solar activities</td>
<td>a tenth of a nT</td>
<td>impulses from the electric network</td>
<td>night</td>
</tr>
<tr>
<td>10</td>
<td>Pc2</td>
<td>high GM and solar activities</td>
<td>seldom, extremely variable, in GM storms may be a few nT</td>
<td></td>
<td>early morning</td>
</tr>
</tbody>
</table>
Concerning the electric field, it should be remarked that among the roughly two hundred stations measured by the Sopron group in the Carpathian Basin, in the Alps, and in Finland, the number of exceptional situations was about 2%. In two cases, the minimum electric field was nearly zero due to a ratio of about two orders of magnitude between maximum and minimum resistivity, one of them at an Alpine earthquake zone at Murau, and one at the Eastern boundary fault of the Alps near Sopron. In two cases,
the distortions in the electric field were strong enough to make computations impossible, one of them in the center of the Transdanubian anomaly where storm-time variations at the Nagycenk Observatory were filtered into Pc2 signals, and one in the Gail Valley, at the Alpine suture zone with exceptionally high conductivity. In the fifth case, inexplicable electric variations were present at a station in the volcanic, mountainous area of northeast Hungary, which were not correlated with the magnetic field and had no apparent source. All of these points are in mountainous areas with thin or non-existent sedimentary cover. From the point of view of MT soundings they represent very small areas, thus the real occurrence of these exceptional cases is less than the 2% indicated above.

3. Methods for the Determination of the Geoelectric Structure(s) of the Earth

3.1 About the Physical Properties of Earth Materials

The electromagnetic study of the subsurface structure(s) of Earth is based on measurable differences in electromagnetic properties (electrical conductivity $\sigma$, magnetic permeability $\mu$, dielectric permittivity $\varepsilon$) of different earth materials. The object of electromagnetic methods is to determine the spatial distribution of these electromagnetic properties below Earth’s surface. Mainly the electrical conductivity (or its reciprocal function, the electrical resistivity $\rho = 1/\sigma$) is considered, since the magnetic permeability is important only at sites with ferromagnetic minerals (otherwise the free space value, $\mu_0 = 4\pi 10^{-7}$ V s/(A m) can be used). The permittivity plays role only in case of very rapid field variations. $\sigma$, $\mu$, and $\varepsilon$ depend not only on position, but also on the temperature, pressure, time, and frequency of the electromagnetic field. They may even be (and in reality they are mostly) tensorial. Anyway, the minimum requirement is that they should be linear quantities.

3.2 Theory of Electromagnetic Methods

The Maxwell equations can be formally transformed into wave equations (the so-called Helmholtz equations), but the high-frequency electromagnetic waves—because of the conductivity of earth materials—cannot penetrate under Earth’s surface into the depth ranges that are the most interesting for geophysical research and exploration. The electromagnetic field at such subsurface depths (that is, at a few tens, hundreds, thousands, or tens of thousands of meters) may only vary at a small rate. Thus, the displacement currents $\partial D/\partial t$ become negligible in comparison with the conduction currents $j$. In other words, in a typical geophysical problem, the Helmholtz equations become diffusion equations.

Consequently, in electromagnetic geophysics, instead of the general Maxwell equations, the so-called quasi-stationary field equations ($\partial D/\partial t << j$ and $\partial D/\partial t \neq 0$), or the stationary field equations ($\partial D/\partial t = 0$) are usually used. In Table 2, all these three cases are shown, together with two other (from our point of view, extreme) situations.
<table>
<thead>
<tr>
<th>Classification of electrodynamics</th>
<th>Electromagnetic waves in dielectrics</th>
<th>General electrodynamics (Full Maxwell equations)</th>
<th>Quasi-stationary field</th>
<th>Stationary field</th>
<th>Static fields</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( \frac{\partial D}{\partial t} \gg j )</td>
<td>( \frac{\partial D}{\partial t} \ll j ) and ( \frac{\partial D}{\partial t} \neq 0 )</td>
<td>( j \neq 0, \ \frac{\partial j}{\partial t} = 0 )</td>
<td>( j = 0, \ \frac{\partial j}{\partial t} = 0 )</td>
<td>( j = 0, \ \frac{\partial j}{\partial t} = 0 )</td>
</tr>
<tr>
<td>Notations:</td>
<td>( \text{rot } H \equiv \frac{\partial D}{\partial t} )</td>
<td>( \text{rot } H = j + \frac{\partial D}{\partial t} )</td>
<td>( \text{rot } H \equiv j )</td>
<td>( \text{rot } H = j )</td>
<td>( \text{rot } H = j )</td>
</tr>
<tr>
<td>( \text{div } B = 0 )</td>
<td>( \text{div } B = 0 )</td>
<td>( \text{div } B = 0 )</td>
<td>( \text{div } B = 0 )</td>
<td>( \text{div } B = 0 )</td>
<td>( \text{div } B = 0 )</td>
</tr>
<tr>
<td>( B = \mu H )</td>
<td>( B = \mu H )</td>
<td>( B = \mu H )</td>
<td>( B = \mu H )</td>
<td>( B = \mu H )</td>
<td>( B = \mu H )</td>
</tr>
<tr>
<td>( \text{rot } E = -\frac{\partial B}{\partial t} )</td>
<td>( \text{rot } E = -\frac{\partial B}{\partial t} )</td>
<td>( \text{rot } E = -\frac{\partial B}{\partial t} )</td>
<td>( \text{rot } E = 0 )</td>
<td>( \text{rot } E = 0 )</td>
<td>( \text{rot } E = 0 )</td>
</tr>
<tr>
<td>( \text{div } D = 0 )</td>
<td>( \delta = 0 )</td>
<td>( \delta = 0 )</td>
<td>( \delta = \text{constant} )</td>
<td>( \delta = \text{constant} )</td>
<td>( \delta = \text{constant} )</td>
</tr>
<tr>
<td>( D = \varepsilon E )</td>
<td>( D = \varepsilon E )</td>
<td>( D = \varepsilon E )</td>
<td>( D = \varepsilon E )</td>
<td>( D = \varepsilon E )</td>
<td>( D = \varepsilon E )</td>
</tr>
<tr>
<td>( \frac{\partial j}{\partial t} = 0 )</td>
<td>( \frac{\partial \delta}{\partial t} = 0 )</td>
<td>( \frac{\partial j}{\partial t} = 0 )</td>
<td>( \frac{\partial \delta}{\partial t} = 0 )</td>
<td>( \frac{\partial j}{\partial t} = 0 )</td>
<td>( \frac{\partial \delta}{\partial t} = 0 )</td>
</tr>
<tr>
<td>( j = 0, \ \sigma = 0 )</td>
<td>( j = \sigma E )</td>
<td>( j = \sigma E )</td>
<td>( j = 0, \ \sigma = 0 )</td>
<td>Magnetic methods and electrostatic analogy of geoelectric methods</td>
<td></td>
</tr>
</tbody>
</table>

Application in geophysics: Atmospheric propagation

Geoelectrics:
- sounding, profiling, and mapping techniques

Geophysical methods:
- Electromagnetic induction:
  1. By using natural source: magnetotellurics, geomagnetic deep sounding, tellurics, audio-magnetotellurics
  2. By using artificial source: sounding, profiling, and mapping techniques

Table 2. Place of electromagnetic geophysical methods (electromagnetic induction and geoelectrics) in classical electrodynamics, and application of different forms of the Maxwell equations in geophysics.
Electromagnetic methods may be either frequency-domain or time-domain ones. Table 3 illustrates the quasi-stationary electromagnetic field components in a homogeneous conducting half-space (which is considered as the simplest model of the subsurface) due to an infinitely large plane source. The characteristic depth parameters provide a useful estimation of the depth of investigation at a given frequency and half-space conductivity.

An illustration for the determination of the skindepth (a widely known parameter in the frequency domain) is given in Figure 4.

In the case of a non-plane source, the depth of investigation is significantly influenced by the geometry of the configuration, and mainly by the transmitter–receiver distance. In order to have larger and larger depths of investigation, larger and more distant sources are needed. Fortunately, Earth produces electromagnetic field variations in just the period range that is the most suitable for crustal, lithospheric, and asthenospheric studies.

| The diffusion equations | Direction of propagation: \( z \)  
|-------------------------|-----------------|
| Components: \( E_x \) and \( H_y \)  
| \[ \frac{\partial^2 E_x}{\partial z^2} - \mu\sigma \frac{\partial E_x}{\partial t} = 0 \] and \[ \frac{\partial H_y}{\partial z^2} - \mu\sigma \frac{\partial H_y}{\partial t} = 0 \]  |

| The two domains  
| are Fourier-transform pairs  
| Frequency domain  
| Time domain  |
|----------------|----------------|
| Field generation  
| harmonic \( (e^{iat}) \), \( \omega = 2\pi f = 2\pi / T \)  
| step-like \([u(t)]\) magnetic field  |
| \( E_x(z, \omega) = E_0 e^{-\frac{\mu\sigma}{2}z} e^{i(\omega t - \mu z)} \)  
| \( E_x(z, t) = \frac{2}{\sqrt{\pi}} e^{-\left(\frac{\mu\sigma}{4}\right)^{1/2} t} \)  
| \( H_y(z, \omega) = -\frac{\sigma}{\omega\mu} E_0 e^{-\frac{\mu\sigma}{2}z} \)  
| \( H_y(z, t) = \frac{1}{2} - \text{erf}\left(\frac{\mu\sigma}{2t}\right) \)  |

| Characteristic depth parameters  
| The depth where the amplitudes are \( e (e \approx 2.7) \) times less than at \( z = 0 \) is  
| \( \delta = \left(\frac{2}{\omega\mu\sigma}\right)^{1/2} \), where \( \delta \) skindepth  
| The depth where for a fixed time the field reaches a maximum is  
| \( z_{\text{max}} = \left(\frac{2t}{\mu\sigma}\right)^{1/2} \)  |

Table 3. Quasi-stationary \( \left(\frac{\partial D}{\partial t} \ll j\right) \) electromagnetic fields in the frequency and in the time domains in conducting half-space due to a plane wave source.
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**Biographical Sketches**

**Antal Ádám** was born in Szekszárd (Hungary) on September 13, 1929. He graduated in mining engineering at the University for Heavy Industry, Sopron, in 1952. His field of interest includes geoelectric and electromagnetic methods, instruments, and electric resistivity distribution in Earth’s interior. Qualifications include: university doctor degree, 1962; candidate of technical sciences, 1965; doctor of technical sciences, 1970. He was research fellow at the Geodetic and Geophysical Co-operative (1952–1955), and at the Geophysical Research Laboratory of the Hungarian Academy of Sciences (1955–1957), head of department at this laboratory (between 1957 and 1972), and deputy director (and head of the Geophysical Department) of the Geodetic and Geophysical Research Institute of the Hungarian Academy of Sciences (1972–1999). At present, he is research professor and titular professor at the Geophysical Department of the Loránd Eötvös University (from 1974). He was co-chairman, then chairman of the electromagnetic working group of the IAGA (1975–1983); chairman of the National Committee of the International Lithosphere Program (from 1985); correspondent of the European Geophysical Society (1982–1986). He is a member of the Hungarian Academy of Sciences (corresponding member: 1990, ordinary member: 1993), Associate of the Royal Astronomical Society (from 1985), honorary doctor (1989) of the Oulu University; corr. member of the Austrian Academy of Sciences (1995). Awards include Academy Award (1962, 1970), Loránd Eötvös Memorial Award (1980), and Széchenyi Award (1996). He is a member of the editorial board of the Acta Geodaetica et Geophysica Hungarica.

**László Szarka** (1954, Derecske) graduated as a geophysical engineer at the Miskolc University in 1977. He has done research work in the field of electromagnetic geophysics at the Geodetic and Geophysical Research Institute of the Hungarian Academy of Sciences, in Sopron, since 1977. He is now scientific adviser and the head of the electromagnetic department. He got the degree “Doctor of the Academy in earth science” since 1997. In 1999, he habilitated at the Miskolc University, and then he was nominated to head of the Geoscience Institute of Sopron University (now: University of Western Hungary). After several short-term invitations, between 1997 and 1999 he worked part-time at the University of Orsay (Université Paris Sud XI) as “professeur associé temporaire.”
József Verő, born in Sopron, July 23, 1933. He graduated from the Sopron University as an engineer of geophysics. After a short time with the Uranium Exploration Company, he joined the present Geodetic and Geophysical Research Institute in 1957, where he is at present deputy director. He also lectures at the University of Western Hungary in Physics, and was elected corresponding member of the Hungarian Academy of Sciences in 1995. His interests are geomagnetic pulsations and electromagnetic induction studies. He is author of about 200 papers, many of them in international cooperations.