WAVES IN THE OCEANS

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Contents

Summary

A description of the dynamical properties of oceanic wave is presented, which comprises wind waves at the sea surface, internal waves and inertial oscillations and inertial waves, Kelvin waves and Rossby waves. The dispersion relations and wave guide properties are discussed with the aid of free waves. In order to illustrate the different roles played by the various wave types in shaping the oceanic responses to wind forcing, we consider examples of forcing a coastal ocean. Processes such as geostrophic adjustment by radiating away inertial waves, establishment of an undercurrent by propagating Kelvin waves, and the dispersion of coastal currents by Rossby waves are discussed. The specific properties of equatorial trapped waves and

their dynamical role are also described.

1. Introduction

The most familiar example of oceanic waves is that of the wind waves at the sea surface. But this is only one example of a variety of oceanic waves. The spectrum of oceanic motions covers a broad range of scales, seconds to years and centimeters to thousands of kilometers. This range is also reflected in the various types of oceanic waves. Physical mechanisms of waves can be characterized by the restoring forces, which are for example gravity, density variations, and rotation of the earth, and topography in conjunction with rotation.

Gravity waves occur as disturbances of the sea surface or as internal waves at density gradients in the interior of the sea. Wind waves and the high frequency part of the internal wave spectrum are independent of the effects of earth rotation.

The Coriolis force due to earth rotation controls the properties of inertial waves, topographically or coastally trapped waves, which are related to changes in the bottom topography, and Rossby waves, which owe their existence the change of inertial frequency with latitude.

Waves communicate information of forcing events through the ocean. There are two important aspects: (1) The dispersive properties of waves and the existence of wave guides. These properties tell us how the waves are controlled by the restoring forces and how they propagate in space and time. (2) The generation of waves by external forcing and their role in shaping oceanic responses and dynamical balances.

An example of a simple type of waves is that of plane waves which can be used to characterize wave properties

$$\zeta(x,t) = \zeta_0 \exp(i\omega t - ikx), \tag{1}$$

where ζ_0 is the amplitude, ω the frequency, $\omega = \frac{2\pi}{T}$ (T- wave period), and k the wavenumber, $k = \frac{2\pi}{L}$ (L- wavelength). The phase speed c and the group speed c_{gr} are

$$c = \omega / k \qquad \text{and} \qquad c_{gr} = \frac{d}{dk}\omega . \tag{2}$$

Waves are dispersive if the phase speed is different from the group speed.

Although simple sinusoidal waves are not typically found in the sea, they are useful as a mathematical expression because real oceanic waves can be represented as a superposition of very many sinusoidal waves with different frequencies and wavenumber by means of Fourier integrals. Such packages of sinusoidal waves can propagate without changing their shape if they are non-dispersive waves. For dispersive waves the different Fourier components propagate with different phase speeds and hence the initial wave package would disperse into various parts.

2. Basic equations

The dynamics of waves are governed by the basic hydrodynamic and thermodynamic equations. In the following we need the basic equations in different approximations which reflect different scales and the role of earth rotation. For the surface waves we need the Euler's-equation

$$\frac{\partial}{\partial t}\vec{v} + \vec{v}\cdot\nabla\vec{v} = -\frac{1}{\rho_0}\nabla p + \vec{g}\,, \qquad (3)$$

where $\vec{v} = (u, v, w)$ are the components in x, y and z directions respectively, p is the pressure, and $\vec{g} = (0, 0, -g)$ is the gravity acceleration of the earth with the magnitude g. The density of the water, ρ_0, can be assumed constant. The incompressibility of water implies

$$\nabla\cdot\vec{v} = 0. \qquad (4)$$

Let $z = \zeta(t, x, y)$ and $z = -H(x, y)$ be the equations for the free sea surface and for the bottom. Then the boundary conditions for the vertical current component are

$$w = \zeta_t + u\zeta_x + v\zeta_y, \quad \text{for} \quad z = \zeta(x, y, t)\,, \qquad (5)$$

and

$$-w = uH_x + vH_y, \quad \text{for} \quad z = -H(x, y) \qquad (6)$$

We denote partial derivatives also by the subscripts $x, y, z,$ or t. For horizontal flows the condition that the motion cannot penetrate through walls is given by the dot product

$$\vec{v}\cdot\vec{n} = 0, \qquad (7)$$

where $\vec{n}$ is the normal unit vector of a lateral boundary. In an infinite ocean the dependent variables such as $\vec{v}$ and p are required to remain bounded.

The governing equations for wave processes affected by earth rotation are linearized Boussinesq equations which follow after several traditional approximations.

$$u_t - fv + p_x = X, \qquad (8)$$

$$v_t + fu + p_y = Y, \qquad (9)$$

$$w_{tt} + p_{zt} + N^2 w = 0, \qquad (10)$$

$$u_x + v_y + w_z = 0, \qquad (11)$$

where the inertial frequency *f*, is defined as Coriolis parameter at a given latitude, ϑ,

$$f = 2\Omega \sin\vartheta, \tag{12}$$

with Ω being the magnitude of the angular frequency of the earth rotation $\Omega = \frac{2\pi}{1day}$. Further, N^2, is the Brunt-Väisälä- Frequency.

$$N^2 = -\frac{g}{\rho}\frac{d\rho}{dz} \tag{13}$$

which is a measure of the density stratification. The pressure *p* in Eqs.(8-10) the dynamical pressure is in the Boussinesq approximation, i.e., the hydrostatic contribution was subtracted.

3. Gravity Waves at the Sea surface

3.1. Free Waves

For simplicity we consider waves which propagate in x-direction and are independent of y. These waves are non-rotational and this implies the existence of a potential function, ϕ, i.e.,

$$\vec{v} = \nabla\phi \tag{14}$$

Since the acceleration of gravity can also be expressed as the gradient of a scalar $\vec{g} = \nabla(-gz)$ the equation of motion can be reduced to a Bernoulli equation for the potential function, ϕ.

$$p = -\rho g z - \rho\phi_t - \frac{\rho}{2}(\nabla\phi)^2 \tag{15}$$

From (4) we find

$$\Delta\phi = 0. \tag{16}$$

Moreover we consider a flat bottom, $\nabla H = 0$, where (6) reduces to

$$w(x,-H) = 0 \text{ or } \phi_z(-H) = 0. \tag{17}$$

At the sea surface, $z = \zeta(x,t)$ we have

$$\phi_z(\zeta) = \zeta_t + \phi_x\zeta_x. \tag{18}$$

For small amplitude motion the nonlinear terms can be neglected and the conditions at *z* = 0 are

$$\zeta_t = \phi_z \quad \text{and} \quad g\zeta = -\phi_t \tag{19}$$

and the bottom coordination for $z = -H$ is

$$\phi_z = 0. \tag{20}$$

The set (16), (19), and (20) forms a boundary value problem, which has a periodic solution with respect to x and t. The solution is

$$\phi(x,z,t) = \zeta_0 \frac{\omega}{k} \frac{\cosh(k(z+H))}{\sinh(kH)} \cos(kx - \omega t). \tag{21}$$

The corresponding eigenvalue condition determines the dispersion relation

$$\omega^2 = gk \tanh(kH) \tag{22}$$

For the remaining dependent variables it follows

$$u = -\zeta_0 \omega \frac{\cosh(k(z+H))}{\sinh(kH)} \sin(kx - \omega t), \tag{23}$$

$$w = \zeta_0 \omega \frac{\cosh(k(z+H))}{\sinh(kH)} \cos(kx - \omega t), \tag{24}$$

$$\zeta = -\zeta_0 \sin(kx - \omega t). \tag{25}$$

From the dispersion relation (22) we find the phase and group velocity as

$$c = \frac{\omega}{k} = \sqrt{\tfrac{g}{k} \tanh(kH)}. \tag{26}$$

and

$$c_{gr} = \frac{d\omega}{dk} = \frac{c}{2}\left(1 + \frac{2kH}{\sinh(2kH)}\right). \tag{27}$$

The behavior of the phase and group velocity is shown in terms of functions of the wavelength in Figure 1, (solid line).

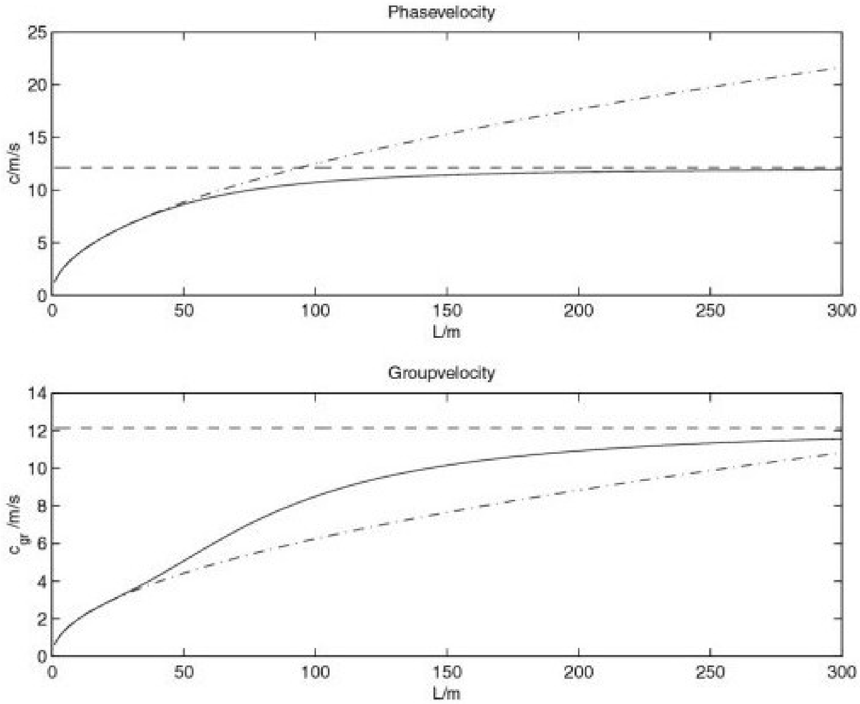

Figure 1: Phase- and group velocity of wind wave versus wavelength, shown for water depth 15 m. Shown are the full relations (solid), the deep water relation (dash-dot), and the shallow water relations (broken).

3.2 Energy Considerations

Multiplication of $\rho u_t + p_x = 0$, and $\rho w_t + p_z = 0$ by u and w respectively, and addition of the results gives an equation for the kinetic energy,

$$\frac{\partial}{\partial t}\frac{\rho}{2}(u^2 + w^2) + \nabla \cdot \mathbf{v}p = 0. \tag{28}$$

The change of kinetic energy in time is balanced by the divergence of the energy flux, $p\vec{v}$. The kinetic energy per unit horizontal area is found by averaging over a wavelength and by vertical integration

$$E_{kin} = \frac{1}{L}\int_0^L dx \int_{-H}^0 dz \frac{\rho}{2}(u^2 + w^2) = \frac{g\rho}{4}\zeta_0^2 \tag{29}$$

The potential energy is given by the work required to deform the sea surface

$$E_{pot} = \frac{\rho g}{L}\int_0^L dx \int_0^\zeta dz\, z = \frac{g\rho}{4}\zeta_0^2 \tag{30}$$

The average kinetic and potential energies are equal. This fact is known as the principle of equipartition of energy. Introducing the mean squared displacement of the sea surface, $\overline{\zeta^2}$,

$$\overline{\zeta^2} = \frac{1}{\lambda}\int_0^{\lambda} dx\zeta_0 \sin^2(kx-\omega t) = \frac{\zeta_0^2}{2} \tag{31}$$

we find

$$E = E_{kin} + E_{pot} = \rho g\overline{\zeta^2} = \frac{\rho g}{2}\zeta_0^2 \tag{32}$$

The average energy flux through a vertical plane, e.g. $x=0$, is given by the integral over one wave period, τ ,

$$F_u = \frac{1}{\tau}\int_0^{\tau} dt\int_{-H}^{0} pudz = c_{gr}E \tag{33}$$

The energy is transported with the group speed.

3.3 Deep Water Waves

For deep water waves the wavelength is smaller than the water depth, $kH \gg 1$. Since $\tanh(kH) \to 1$ for large arguments we find from (22)

$$\omega = \sqrt{gk}, \tag{34}$$

and the phase and group speeds follow as

$$c = \sqrt{\frac{g}{k}} \qquad \text{and} \qquad c_{gr} = \frac{1}{2}\sqrt{\frac{g}{k}} = \frac{c}{2}. \tag{35}$$

For water deeper than half the wavelength, the wavelength is the only variable which affects the wave speed. The behavior of the phase and group velocity of deep water waves are shown as functions of the wavelength in Figure 1, (dash-dot line)

Noting that for $kH \gg 1$,

$$\frac{\cosh(k(z+H))}{\sinh(kH)} \simeq \frac{\sinh(k(z+H))}{\sinh(kH)} \simeq e^{kz}, \tag{36}$$

applies, we obtain for the dependent variables

$$u = -\zeta_o\omega e^{kz}\sin(kx-\omega t), \tag{37}$$

$$w = \zeta_o\omega e^{kz}\cos(kx-\omega t), \tag{38}$$

$$\zeta = -\zeta_o \sin(kx - \omega t). \tag{39}$$

3.4 Shallow Water Waves

For shallow water waves the wavelength, *L*, is by definition much larger than the water depth, *H*, i.e., $kH \ll 1$. Then an expansion of the dispersion relation (22) gives

$$\omega^2 = kg(kH - \frac{(kH)^3}{3} + ...), \tag{40}$$

or

$$\omega = k\sqrt{gH}. \tag{41}$$

The shallow water phase speed and the group speed are equal, $c = c_{gr} = \frac{\omega}{k} = \sqrt{gH}$ and the water depth is the only variable which affects the wave speed. Shallow water waves show no dispersion. The phase and group speeds of shallow water waves are straight lines in Figure 1, (dashes).

The dependent variables are

$$u = -c_o \frac{\zeta_o}{H} \sin(kx - \omega t)\,, \tag{42}$$

$$w = \zeta_o \frac{(z + H)}{H} \cos(kx - \omega t), \tag{43}$$

$$\zeta = -\zeta_o \sin(kx - \omega t). \tag{44}$$

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Biographical Sketch

Wolfgang Fennel received his PhD in theoretical physics from the University of Rostock in 1973. After an initial career in statistical physics he turned his scientific interests to theoretical oceanography and since 1976 he worked in the Institute for Marine Sciences in Warnemünde. His habilitation work was on the theory of turbulent diffusion in the sea. Later he expanded his inerests to oceanic waves and coupling of physical and biological models. Since 1992 he is head of the department of physical oceanography and instrumentation at the Baltic Sea Research Institue and since 1994 he is also professor at the University of Rostock. He is author of numerous scientific papers covering turbulent diffusion, responses of the ocean to wind and buoyancy forcing, coupling of physics and biology in models and other topics of oceanography. Wolfgang Fennel was a vice-president of SCOR from 1998 to 2002.