STOCHASTIC MODELLING OF SPATIO-TEMPORAL PHENOMENA IN EARTH SCIENCES

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Summary

Among the different families of stochastic models for the space-time characterization of natural resources in Earth Sciences, this paper focus on those that can be defined as spatial models that incorporate a temporal component. There are as many approaches to space-time modelling as there are specificities of each case study, regarding the amount of available information and the final objectives of the study. Three types of models are discussed in the text and can be summarised and classified according to the purpose of the use of the time data:

(i) Joint space-time models where data collected in the past, in different periods of time, is used in a joint space-time framework to infer the spatial distribution of a given attribute at the present time or in a period in the very near future. A case study of air quality characterization is presented to illustrate the models.

(ii) Space models where historical data are used to build a spatial and time trend. These trends are interpreted as spatio-temporal random fields and are inferred in space for fixed periods of time. Two case studies with this type of models – air pollutant dispersion characterization and estimation of the abundance of a migratory bird, the wood pigeon – are presented .

(iii) Spatio-temporal simulation models for uncertainty assessment . Deterministic simulation models that mimic the complexity of some dynamic phenomena can be used, together with spatial stochastic simulation models, for uncertainty assessment and to visualize extreme scenarios of the attribute.

1. Introduction

The different fields in Earth Sciences involving spatio-temporal phenomena, like soil contamination, ground water quality, air pollution and geological resources, have specific models to characterize the spatio-temporal behaviour of the main attributes of those phenomena.

Pure deterministic models have been used for modelling dynamic processes in Earth Sciences: to predict, over time, the behaviour of fluids in petroleum reservoirs, to characterize the dynamic process of a contaminated soil and aquifer, to evaluate the evolution of the lava of a volcano for different time steps. Generally, the objective of such models is to obtain a credible image of the reality by mimicking the physical processes, the behaviour of the main intervenient factors, and the interaction between them. The problem is that, in most of these situations, the data are so scarce and the phenomenon so complex that interpolation in space or extrapolation in time cannot be done very far beyond the experimental observations, which makes very different images of the same reality equally credible or unreliable.

In Earth Sciences, the great majority of natural phenomena resulting from a dynamic process have two characteristics in common: the high complexity of the phenomena, usually resulting from the unknown joint interaction of known factors, and lack of information – samples, observations.

These two factors – complexity of the phenomena and lack of data – contribute to uncertainty in the knowledge of those phenomena. Deterministic models cannot include this notion of uncertainty in their conceptual frameworks. On the other hand, stochastic models succeed in dealing with the concept of uncertainty through the very simple idea of conceiving the natural phenomena as a random process. This does not, however, mean that the physical phenomena were generated by a random process. Randomness is just a way of quantifying the uncertainty of knowledge or, in other words, our ignorance about the phenomena.

Among the different families of stochastic models of natural resources in Earth Sciences, this paper will focus on those that can be defined as spatial models, usually the scope of geostatistics or spatial statistics, that incorporate a temporal component. With these models, the objective is to estimate, at a given spatial location and in a fixed period of time, the distribution of the attributes of a natural resource or a property of a spatial phenomena, or to access the uncertainty about the knowledge of that attribute or property. The characterization of a plume of contaminants in soil or water, which is sampled or monitored over several periods of time in some spatial spots, the analysis of the air quality of a region which is systematically monitored over time, the planning and control of a ecological resource observed in a given sample pattern at different periods of time, are just a few examples of problems that can be approached by such models.

There are as many approaches to space-time modelling as there are specificities of each case study, regarding the amount of available information and the final objectives of the study. The objectives of the models treated in this paper can be summarised according to the purpose of the use of the time data:

- Data collected in the past, at different periods of time, is used in a joint space-time framework to infer the spatial distribution of a given attribute at the present time or in a period in the very near future.

- Historical data are used to build a spatial and time trend. These trends are interpreted as spatio-temporal random fields and are inferred in space for fixed periods of time.

- Spatio-temporal uncertainty assessment is the aim of the third type of models presented in this paper. Deterministic models that mimic the complexity of some dynamic phenomena can be used, together with spatial stochastic simulation models, for uncertainty assessment and to visualize extreme scenarios of the attribute.

2. Joint Space - Time Models

Consider an attribute $Z(\mathbf{x},t)$ defined at a spatial location $\mathbf{x}$, $\mathbf{x} \in D$, and at an instant of time $t \in T$. Although in the joint space-time approach the observations are located in $D \times T$ referential there are substantial differences between the space and time coordinate axes: the ordering that exists in time – past, present and future – does not exist in space; the time data are collected in the past and, normally, are used in the present or future via a typical extrapolation exercise, while spatial estimation results basically from an interpolation; anisotropy – the way a given phenomenon varies in different directions of space - is a concept that exists only in space and does not make any sense in time; space and time units cannot be directly compared.

To take these limitations into account in any interpolation or extrapolation exercise is the first basic rule in the use of space-time models in Earth Sciences.

Considering the attribute *z*, two different conceptual models can be adopted, regarding the decisions of stationarity of the random function RF $Z(\mathbf{x},t)$:

i) $Z(\mathbf{x},t)$ can be considered a second order stationary RF, which means considering a constant mean in spatial and time domains:

$$\mathrm{E}\{Z(\mathbf{x}_1,t_1)\} = \mathrm{E}\{Z(\mathbf{x}_2,t_2)\} = m \qquad (1)$$

and considering the space and time covariance independent of the space-time location $(\mathbf{x},t)$:

$$\mathrm{C}\{Z(\mathbf{x}_1,t_1)\cdot Z(\mathbf{x}_2,t_2)\} = \mathrm{C}(\mathbf{h},t) \text{ where } \mathbf{h} = \mathbf{x}_1 - \mathbf{x}_2 \text{ and } t = t_1 - t_2. \qquad (2)$$

This conceptual model can be adopted when any space or time trend is detected.

ii) When a time, space or space-time trend is evident in the physical phenomenon, non-stationarity of $Z(\mathbf{x},t)$ can be assumed. The RF $Z(\mathbf{x},t)$ is decomposed into a mean component (the trend) $M(\mathbf{x},t)$ and a residual component of zero mean $Z(\mathbf{x},t)$:

$$Z(\mathbf{x},t) = M(\mathbf{x},t) + R(\mathbf{x},t). \qquad (3)$$

The mean $M(\mathbf{x},t)$ and $R(\mathbf{x},t)$ can also be decomposed into space and time components.

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Biographical Sketch

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