HISTORY OF MATHEMATICS EDUCATION - ITALY

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Contents

Summary

Starting from the Italian unification in 1861, we describe the role of mathematics teaching in the Italian system of education giving an account of the main features of Italian mathematics education, and referring to the official syllabi, to the used textbooks, to the debates. We will see that interesting proposals in a period of about 150 years will always have to face a tendency to rigor as well as a resistance towards innovation.

1. Introduction: System of Education in the Period of the Unification of Italy.

The first law to regulate Italian schools was the Casati Law of 1859 (R.d. 13.11.1859, n. 3725). It was originally passed only for the Kingdom of Sardinia (including Piedmont) and for Lombardy, and was later gradually extended to the other Italian regions after their annexation in 1861 and 1871. The Casati Law established the general characteristics of state education.

As to secondary education, the law distinguished between a *classical education*, whose purpose was the acquisition of that literary and philosophical culture which opened the way to the State Universities (Art. 188), and a *technical education*, which aimed at providing a general culture to young people intending a job in the public services, in industry, in commerce and in agricultural management (Art. 272). Secondary education consisted of a first and a second level: to cover classical secondary education, the Casati Law introduced the Gymnasium and the Lycée (*Ginnasio-Liceo*), which were to become the point of reference for all Italian secondary education (Vita, 1986). The Technical School (*Scuola Tecnica*) and the Technical Institute (*Istituto Tecnico*) were set up for technical secondary education (Table 1.) Pupils entered the Gymnasium and the Technical School after a primary school that extended over four years. The

Technical School thus covered the age range of the present-day middle school (11-14), while the Gymnasium lasted for five years and hence included the first two years of high school. The Technical School soon lost its characteristic of being a preparatory school for the Technical Institute and was transformed into a school for general education. After the Gymnasium, pupils completed their classical education by attending the Lycée for three years. Initially, a technical education was shorter than its classical counterpart because, after the three years of Technical School, only a further three years were foreseen at the Technical Institute. However, through an 1871 reform (circ. 17.10.1871) the duration of the Technical Institute was extended to four years and, in some cases, to five.

In 1896 a Complementary Course (parallel to Technical School and to the *lower Gymnasium*, i.e. the first three years of the Gymnasium) was introduced – followed by the *Scuola Normale* - specifically for the *Istruzione Magistrale*, which was the cycle aimed at educating primary school teachers. The norms related to *Istruzione Magistrale* were included among the norms for primary instruction.

Duration	**classical instruction**	**technical instruction**	***istruzione magistrale***
4 years	Elementary School (can last from 3 to 6 years depending on the availability of the municipality; only 4 years are compulsory to access secondary education)		
3 years	Lower Gymnasium	Technical School	Complementary School
from 3 to 5 years	Upper Gymnasium (2 years) + Lycée (3 years)	Technical Institute (3/4/5 years)	Normal School (3 years)

Table 1. Instruction in Italy at the end of 19th century

2. The Role of Mathematics in the Italian Educational System

The period of the unification was without a doubt a unique period in the scientific history of Italy. Patriotism intertwined with a lay and positivist mentality born with the *Risorgimento* (the Italian Resurgence), which was widespread amongst the bourgeoisie, gave scientific research a central role that it was not to have again (Brigaglia & Masotto, 1982). Mathematical research flourished and mathematics teaching was considered relevant at all levels of education.

2.1. Mathematics in classical and technical instruction

The characteristics of the teaching of mathematics in classical and technical instruction are well defined by the Law of Education Minister Coppino in 1867 (R.d. 10.10.1867). As for the classical instruction, we read, in the first page, that

Mathematics in classical secondary schools is not to be regarded only as a set of propositions or theories, which are useful in themselves [...] to be applied to the needs of life; but primarily as a means of intellectual culture, as a gymnastics of the mind

aimed to develop the power of reasoning, and to help that right and sane criterion needed to distinguish what is true...[all translations are by the author].

The teaching of mathematics in the Gymnasium-Lycée covered initially three classes only (the second year of the upper gymnasium and the first two years of the Lycée) with a high number of hours per week (5 + 6 + 7½), then extended to all the five years of high school (4 + 4 + 7½ + 4½ + 2). The programs of 1867 reduced the previous mathematical contents in order to focus on the rational development of geometry and arithmetic (see Sections 4.1 and 4.2), but the high number of hours devoted to mathematics testifies for the credit deserved in that period to this subject. In the next decades the number of hours of mathematics in the Lycée will harshly decrease (to 2+2+3+3+2 hours per week) and the abolishment, at the end of the 19th century, of the written examination in mathematics at the end of the Gymnasium and of the Lycée shows the diminished value ascribed to mathematics.

According to the same Coppino Law of 1867, mathematics had a different role in technical education, in that:

The aim of the teaching of mathematics in technical schools is to provide young men, in a limited time, with the greatest amount of knowledge useful for applications in the arts and crafts.

The Technical Institutes had developed considerably after the unification of Italy, having undergone various reforms since 1860, all of which recognized the necessity of separate developments of humanist and technical education, with an eye to the model of the German Realschulen (Morpurgo, 1875, XXVI and on; Ulivi, 1978). At the beginning of the 20th century students in the technical instruction were about 60% of all secondary school students (Scorza, 1911). A reform of 1871 recognized the need for a general literary and scientific education in technical education and instituted a physics-mathematics section (*sezione fisico – matematica*). Since this section did not have the aim of qualifying students to go into the professions, and permitted university entrance (to the Faculty for Engineering), it could be seen as the scientific alternative to the Lycée. The first two years of the Technical Institute were common to all the sections and had a general and preparatory character. In the second "advanced" two-year phase, the syllabus of the third year was common to the physics-mathematics section and to the industrial section. In the fourth year the program was specific for the physics - mathematics section. Mathematics had 6+5 hours per week in the first two years, and 5 + 5 in the two further years of the physics-mathematics section.

2.2. Arenas for a Debate

After the unification, Italian mathematics teachers acquired a professionalization evidenced by the foundation of journals addressed to them. The first one, *Rivista di Matematica Elementare,* appeared in 1874 (Furinghetti & Somaglia, 1992). Before this year, only a few articles were published in journals addressed at professional mathematicians and students of University. This was the case of *Giornale di Matematiche,* which hosted the debate about the introduction of Euclid's Elements in the Gymnasium (see Section 4.1.). An important journal in the field of mathematics

education was *il Periodico di Matematica*, which appeared in 1886. At the beginning of the 20th century, the *Periodico* will share the stalls of mathematics teachers and educators with another journal, the *Bollettino di Matematica*, which in 1949 will change its name in *Archimede*. Both journals still survive.

In 1895 a group of mathematics teachers founded the association Mathesis (Giacardi, 2004). The aim of this association was the improvement of the scientific and didactical knowledge of the teachers. These aims had to be pursued by facing the problems concerned with school syllabi, methods of teaching, teacher training, schoolbooks, etc. The first president was the teacher Rodolfo Bettazzi. This was a period of a big interchange between school and university. May be due to the studies on the foundations of mathematics, many mathematicians showed an interest in school mathematics, not only by writing or translating manuals for the school but also by intervening in the political and cultural discussions, in teacher training, in the construction of curricula. On the other hand good teachers, as Bettazzi himself, hold lectures at the university. The association Mathesis grew rapidly and included among its members not only teachers but also university professors. Its members participated in debates about the teaching of mathematics in school, expressing their opinions in the Congresses of the Association and on its Bulletin (*Bollettino dell'Associazione Mathesis*). We will meet them again in the next sections.

Among the presidents of Mathesis we find, in 1910, Guido Castelnuovo. As a member of the Italian subcommittee of the International Commission on Mathematical Instruction (ICMI), Castelnuovo established important international contacts. The *Bollettino della "Mathesis" Società Italiana di Matematica*, published as an autonomous and rich journal in 1909-1920, hosted translations of international debates and the reports of the Italian subcommittee. In this way Mathesis participated in the discussions about the introduction of calculus in schools and about the applications of mathematics; many mathematicians, even within Mathesis, opposed the introduction of calculus as well as the utilitarian aspect of mathematics (see Fehr, 1911). When the *Bollettino* ceased the publication, *il Periodico di Matematica* became the official organ of Mathesis. Federigo Enriques was another well-known mathematician who became president of Mathesis (in 1919). Enriques succeeded in enlarging the Association such as to have a major incidence on the governments. Mathesis will in fact become a counterpart in important questions, like the problem of the standardization of the curricula in the regions that formerly belonged to the Austrian Empire, as Trento and Trieste (Zuccheri & Zudini, 2007).

2.3. Towards a Modern Lycée

Even if the physics-mathematics section of the Technical Institute could represent an alternative to the classical curriculum, the international movement towards new scientific humanities was felt also in the Italian milieu. The first signal in this direction can be noted already in 1875, when a decree of the Minister Bonghi (R.d. 7.1.1875, n. 2337) allowed students ending the Lycée to pass the school-leaving examination even if they, having a good note in mathematics, failed in Greek (or failed in mathematics having a good note in Greek). In 1899 a decree of Minister Baccelli (d.m. 3.11.1899) allowed the students of some particular Lycée of bigger towns to choose, in the last

year, between Greek and mathematics. New topics were added for those who chose mathematics, as the harmonic division, homothety, spherical triangles and the calculation of approximate values of π. This possibility was suppressed the next year, but in 1904 a new reform by Minister Orlando (R.d. 11.11.1904) again allowed to choose, in the last two years of the Classical Lycée, between Greek and mathematics. This reform was a first step towards a modern Lycée, but mathematicians themselves opposed it as contrasting the educational value of the Lycée (*Bollettino della Mathesis*, 1911).

In 1905 the Minister of Education Bianchi appointed a Royal Commission for secondary education to explore the possibility of creating a unified middle school, followed by an upper education divided into three branches. The Commission had a tormented life. Different views, including the difficulty of accepting the idea of a complete equalization of the classical and the scientific studies, led to the resignation of a part of its members and to the publication of their opinions in a volume (Galletti & Salvemini, 1908). The Commission, which had continued the work, published its final report in 1909 (Commissione Reale, 1909). Giovanni Vailati edited the part concerning mathematics. The Commission foresaw, inter alia, three Lycées: a classic, a modern and a scientific one. The programs of the *Commissione Reale* never entered into force. It is worth to mention some points concerning the scientific Lycée. In the first year, geometry is dealt with only through problems of construction, while in the second year it is introduced in a more systematic way. The third year requires the "graphical representation of the variance of the expressions of the second degree", not re-appeared until the 1980s. The concept of derivative is introduced, and its different aspects are emphasized such as the "rate of growth of a function in a given interval or in a given point." In the fifth year we find probability, besides integral calculus. Note that the "representation of statistical data" appears only in the modern Lycée. Assuming that the learning process goes from concreteness to abstraction, the comments state that pupils should not learn "theories before knowing the facts to which they refer, and from which their meaning can be obtained by abstraction".

2.3.1. The Modern Lycée: 1911 – 1923

The *Liceo Moderno* was the only partial implementation of the project presented by the *Commissione Reale*. It was founded in 1911, only in some provinces, as a section of the Lycée. The aim to prepare students for (not necessarily scientific) university studies is achieved through the study of Latin, modern languages and sciences. Mathematics is important as a language suitable to describe natural phenomena and therefore its contents, edited by Castelnuovo (1913), show significant novelties. "The renewal of mathematics in the seventeenth century is linked to the flourishing of experimental sciences. In this view the teacher will point out that the fundamental concepts of modern mathematics, that of function in particular, are suggested by observational sciences, which then exert a beneficial influence on the development of mathematics itself". The notion of a function and infinitesimal calculus were introduced, in an official way, for the first time in this school, with the suggestion to use an experimental and inductive approach - which completes the deductive method - and to harmonize it with the course of physics.

2.3.2. The Programs of the Ispettorato Centrale dell'Istruzione Media

A few years later, the Ministry of Education (1918) proposed a reorganization of the curricula of all secondary schools, edited by the general director Vittorio Fiorini, former member of the *Commissione Reale*. No changes were suggested for the *Liceo Moderno*, but the proposals of the *Commissione Reale* for the physics-mathematics section of the Technical Institute were followed. The programs took note of the fact that in many of these schools basic notions of calculus were already taught. It is recommended to introduce these topics by describing the historical process and the link with the sciences that created them. The use of derivatives in maximum problems and of the integrals in the calculation of areas and volumes will allow solving problems that previously required complicated algebraic transformations. These programs, never adopted officially, represent the most advanced vision of the role of mathematics in the school when the Gentile Reform entered into force in 1923.

3. Mathematics in Primary School and the Training of the Teachers

3.1. The Elementary School

The Casati Law also set up Elementary School. It lasted 4 years, only the two first years (*inferior course*) being compulsory. Local institutions paid for the teachers. In 1904 Elementary School was extended to two three-year courses. Pupils who wanted to enter secondary instruction could leave Elementary School after the fourth year, with a final examination. Compulsory Elementary School was extended, but depending on the number of classes existing in the municipality. Pupils could not enter Elementary School before their sixth year, and in any case had to leave the second course when becoming fifteen.

The teaching of mathematics in Elementary School was mainly devoted to arithmetic. One couldn't find geometry in the first three years, even if geometrical contents could be found in the teaching of fractions and of weights and measures (see Conti, 1911). In the second three-year course geometry teaching dealt with shapes, names and rules for the measure of the simplest geometrical figures.

It is interesting to note that teaching in Elementary Schools was influenced by psychologists and by educational sciences; this didn't happen in secondary school. We find, for instance, the interesting proposals of Anton Maria Bustelli (1889), who suggested – in the second course (two-year course at that time) - three phases for each new geometric form: 1. *presentation* of the object or of the figure (a drawing, cut paper, pieces of wood, ...); 2. *description*, which means to undo and to redo the figure analyzing the relations between its parts; 3. *definition* of the figure. Maria Montessori will take up these phases again in her geometric work (Montessori 1934); and similar phases will be described in the work of Pierre van Hiele (1958).

The programs established in 1905 (R. d. 29.1.1905) had a practical character. They suggest to avoid an excessive amount of written calculations (with respect, e.g., to the programs of 1888) and to apply mathematics to the needs of *domestic* life or of little trades. An intuitive – experimental method is suggested as well as the cyclic

development of the topics; this means that the four operations are all treated in the first class with numbers from 1 to 20, then in the second class with numbers from 1 to 100, etc. Moreover we find the suggestion to avoid technical terms (factor, quotient) and not to overcharge the children's minds. The teaching of fractions must not go beyond the needs of a popular elementary school. In the last classes calculations must be proposed in the form of problems. Examples from real life have to be "effectively real". This is particularly important in the last year, when the programs include monetary calculations (as interest, discount). The school had to furnish the class with didactic materials as: a collection of common weights and measures; a cube that can be decomposed into 8 little cubes, a cylinder, a cone, a sphere and a pyramid made of cardboard or wood; of course other materials could be added, in particular those for the teaching of geometry and for an experimental justification of rules for measure.

Starting from the Gentile reform of 1923 (see Section 5.1.), Elementary School will last five years. This reform stressed the literary and artistic aspect in education. The teacher had a certain freedom, but the programs were written under the influence of Fascist culture. The teaching of mathematics included only arithmetic.

In 1945 a Commission was set up by Italian Minister De Ruggiero with the collaboration of the Allied Countries, and chaired by the pedagogist Colonel Carlton Washburne, a follower of John Dewey. New curricula for Elementary School and Kindergarten were launched (d.m. l9.2.45 n.459 and d.l.gt 24.5.45 n.549). The teaching of arithmetic and geometry had to take into account the mental images and the intuitions of the pupil. The contents and the instructions looked back at the "pedagogical" programs of the beginning of the century; the difficult political and economic situation in Italy hindered their success.

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Biographical Sketch

Marta Menghini received her degree in Mathematics from the University of Rome "La Sapienza" on July 1976. She is associate professor in the Department of Mathematics of the same University. She produced papers in the field of Combinatoric Geometry and of History of Mathematics (in particular on the development of projective and algebraic geometry in the last century). She was a member of the commission of the Italian National Seminar of Didactics of Mathematics; from 1987 till 1995 she was in the Editorial board of "Epsilon", a magazine for high school teachers of scientific disciplines. From 1987 till 2000 she was Scientific Director of a research project about Innovation of Mathematics at School (due to an agreement between CNR and University) and edited many works of the teachers involved in the project (teaching materials, reports, conferences). From 1998 to 2000 she took part to the European Project "Modem", about innovation and production of materials for didactics of mathematics. She was in the Commission of the Italian Mathematical Union for the curricular reform of the High School. In 2008 she was in the scientific and in the organizing committee of the Congress held in Rome in the occasion of the centennial of ICMI. She is the author of numerous published works on mathematics education in secondary school and on the history of mathematics education, in particular on the teaching of geometry and on the history of teaching geometry.