MATHEMATICAL MODELS OF ENVIRONMENTAL ECONOMICS

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Contents

Summary

This chapter surveys mathematical models and methods applied in environmental economics organized along environmental problems – externalities and instruments, resource extraction, renewable resource management, the tragedy of the commons in general and in particular in the context of global warming, which is also today's prime motive for (energy) conservation incentives. To deal with these selected topics, the methods of static optimization, partial equilibrium analysis, optimal control, real option and mechanism design are introduced. A particular aspect of this survey is the consideration of thresholds across different methods. Thresholds are a way to formalize the criterion of sustainability, which gives an ecological touch to the analysis. In addition to these more elaborate expositions, further topics such as (computable)

general equilibrium models, valuation of environmental goods, corporate social responsibility, and public choice are briefly addressed.

1. Introduction

The opportunity to write about mathematical models in environmental economics suggests two different approaches: to proceed either by environmental topics or by mathematical methods. This presentation opts for the first, because mathematical models are only tools for the ultimate means of analyzing issues in environmental economics, and because interested readers are more likely to come across such a review when they have a particular environmental problem in mind.
Since the presentation proceeds along environmental topics, it seems appropriate by the title of this survey to list also the methods that are crucial for environmental economics (but not necessarily treated in this survey): The initial and still quite frequently applied tools are those of elementary micro-economics, more precisely: static optimizations subject to constraints, planning problems, including agents' reactions in competitive or non-competitive settings, partial equilibrium or general equilibrium. A natural extension within the static framework is the consideration of games. Most of these games either use continuous strategy sets or the extensive form (i.e. modeled along a game tree) for discrete strategy sets due to their tractability and the possibility to account for sequences of moves; matrix games (i.e. games described in normal form) are rare. The problem of intertemporal resource extraction was primarily responsible for the interest in dynamic optimization in environmental economics. The corresponding tools are the calculus of variations and in particular the modern extensions: control theory (applied to continuous time optimizations) and dynamic programming, which is the tool of choice for discrete time problems, dynamic games and stochastic dynamic optimizations. Mechanism design, i.e., the structuring of incentives when information is private (e.g. a polluter has better knowledge about his pollution and how to reduce it than outsiders, a pollutee knows best the true harm), is crucial for many environmental problems and is gaining more and more importance in the literature. In addition to the above addressed theoretical approaches, environmental economics is complemented by a substantial amount of empirical research, but the underlying statistical and econometric techniques are not addressed since that would go beyond the task of this survey.

The chapter is structured as follows. Section 2 discusses externalities contrasting the Pigouvian with the Coasean perspective in order to introduce simple static models and (partial) equilibrium analysis. Section 3 addresses resources (non-renewable and renewable) and within it the tools of optimal control theory and the notion of a threshold, which are crucial for ecological concerns (sustainability, resilience). Section 4 draws attention to the tragedy of the commons within a dynamic framework and thereby briefly introduces the theory of dynamic games. The issues uncertainty and irreversibility and the tools of stochastic dynamic optimization, are the content of Section 5. Global warming is addressed in sections 4 (the incentive of individuals and nations to free ride) and 5 (uncertainty and irreversibility are crucial elements of global warming) as the motivating example. This topical problem also serves as a motive to consider flexible mechanisms (joint implementation) and to provide energy conservation incentives in Section 6, which addresses the consequences and potential pitfalls from ignoring private information and shows how mechanism design can be used to derive optimal conservation incentives accounting for asymmetric information. While sections 2 – 6 apply the mentioned methods, the task of Section 7 is to list a few methods that are considered as important but that have been left out due to space limits.

2. Externalities

2.1. Pigou and Coase

The birth date of environmental economics is presumably the publication of Pigou's *Economics of Welfare* in which externalities are analyzed. The basic implications derived by Pigou – the internalization of external costs, the polluter pays principle (PPP) and pollution taxes – are part of today's public discussions and policy proposals (not only from green parties) that partially reach the status of a dogma. The basic Pigouvian argument can be sketched for a competitive 'smoke' emitting firm producing a commodity y with a neoclassical production technology f with two inputs $y = f(k,s)$ in which k aggregates all conventional inputs and s denotes pollution, "smoke"; both inputs are substitutable, $f_{ks} > 0$, so that an increase of k allows to lower the emissions for the same level of output. The factory sells the good at the market price $p = 1$ (normalized) and faces the factor price q for its inputs k.

The emissions of the firm impose costs (damage D) on a neighboring 'laundry',

$D(s),\ D_s \geq 0$,

$D_{ss} > 0,\ D(0) = 0$, and the Inada conditions, $D_s \to 0$ for $s \to 0$, and

$D_s \to \infty$ for $s \to \infty$ ensure interior solutions of the welfare optimum,

$$\max_{k,s}\ W(k,\ s) \equiv f(k,\ s) - qk\ - D(s)\,, \qquad (1)$$

from the first order conditions (the subscripted letters refer to the partial derivatives),

$$f_k(k,s) = q\,, \qquad (2)$$

$$f_s(k,s) = D_s(s)\,. \qquad (3)$$

Following Pigou, laissez faire leads to inefficient allocations since firms consider pollution as a free input that is used up to the point where its marginal product vanishes, $f_s = 0$, which violates Eq. (2) in the above characterization of an efficient allocation. However, different means – liability of the firm, pollution taxes, implement the efficient allocation in a decentralized economy. Full liability of the polluter for any damage renders the firm's profit identical to W and thus implies the welfare optimum. An alternative Pigouvian instrument is a pollution tax (τ). This modifies the firm's profit objective

$$\max_{k,s} \pi \equiv f(k,s) - qk - \tau s, \tag{4}$$

such that the first order conditions of individual firms' profit maximization (4) are identical to (2) – (3) if the tax equals the marginal damage, $\tau = D_s$. Therefore, such a 'Pigouvian' tax leads to an efficient allocation in the partial equilibrium framework stated above. In this simple example, an absolute standard, $s \le s^*$, can implement the efficient allocation too, but a relative and binding standard (the prime choice of environmental policy until recently) will not, even if set at the first best levels, either input $k/s \ge k^*/s^*$ or output oriented $y/s \ge y^*/s^*$, where the asterisk refers to the efficient standard satisfying (2) and (3). The reason is that the constraints result in a positive Kuhn-Tucker multiplier entering the first order conditions and leading to a distortion of (2) and (3).

This reasoning sounds convincing and hence it took 40 years until Ronald Coase revealed three hidden errors: First, Pigouvian analysis ignores that the parties, the polluter (smoke emitting factory) and pollutee (neighboring laundry) leave money on the sidewalk by not implementing the efficient allocation through cooperating. Therefore, no government intervention is necessary, whenever such bargaining between the involved parties is feasible; in short: *private = social value in a world without transaction costs* (known as Coase theorem). Indeed, a substantial amount of environmental and social issues falls into this category. Second, protection is ignored, i.e., the pollutee is not necessarily condemned to a passive role and may invest into protective measures (b) reducing her damage to $D(s,b),\ D_b < 0,\ D_{bb} > 0$. Hence, the efficient allocation including protection, (k^*, s^*, b^*), must satisfy,

$$f_k(k^*,\ s^*) = q, \tag{5}$$

$$f_s(k^*, s^*) = D_s(s^*, b^*), \tag{6}$$

$$D_b(s^*, b^*) = -1. \tag{7}$$

Third, Pigouvian analysis arbitrarily singles out the polluter as the bad guy when social costs do not allow for such a differentiation: who should build the fence between a cattle and a wheat farmer, do drug addicts in city centers cause the externality or is it caused by people's sensibilities, etc. Summarizing, traditionally, one-sided Pigouvian interventions can lead to inefficiency, except for the special case of no protection and infeasibility of bargaining due to transaction costs, e.g., for widely dispersed externalities. If bargaining is infeasible, efficiency dictates that the property title should be given to the party that allows for higher aggregate welfare, and hence it is not always necessarily optimal that the polluter (as, e.g., in the case of drug addicts, or beggars 'causing' the externality) pays. Instead letting the pollutee pay can be optimal (in the above example, the passers-by). Another topical example including a reallocation of property rights is smoking in restaurants: as long as the majority of visitors smoked, it was presumably efficient to let the non-smokers pay, but with less people smoking and more scientific evidence on the health risks, a reversal of the property titles as currently observed may be efficient.

2.2. Liability

The *polluter pays principle* in the form of making the polluter liable for any damage is particularly worrisome, although a dogma for many environmentalists, due to a number of reasons (some of them are given below). Although liability and taxation are equally efficient in the standard Pigou model, liability is much worse because it discourages protection by the pollutee and encourages strategic actions. If damage is private information (the issue of private information is addressed in Section 6 in more detail), as it is often the case, pollutees are induced to exaggerate damages and to skip protection in order to earn from litigation. Indeed, the possibility to litigate for any environmental harm creates potential rents at the expense of producing firms and the associated welfare loss may be huge. For example, people may acquire cheap land close to freeways, plant sensible fruits (strawberries) and then litigate for compensation of the contaminated and thus worthless harvest; or people build (weekend) homes in a farming area (again characterized by relatively cheap land) and then sue the farmers for all the noise and dirt they make. It is not too difficult to imagine many other examples such that courts proceeding along the 'polluter pays principle' lead to huge welfare losses instead to the promised gains.

2.3. Permits

With the success of the permit approach as a part of the U.S. Clean Air Act of 1990, emission trading has advanced from a textbook recommendation to an appreciated policy instrument. The positive experience with permits in the US, actual SO_2 permit prices were far below predictions, is reflected in the Kyoto Protocol that allows for emission trading and other flexible mechanisms of which some (joint implementation, clean development measures) are, however, quite problematic (see also below). And finally, following the U.S. and some companies (e.g. DuPont and BP have introduced emission trading voluntarily), emission trading has arrived in the European Union targeting CO_2 emissions. Emission trade is commonly seen as superior to the command-and-control approaches since it opens a kind of market for environmental resources, leading to the formation of certificate prices, which reflect costs better than exogenously dictated standards, regulations or taxes. A further advantage of tradable permits over taxes is that Leviathan governments, which might use environmental taxes as a fig-leaf to increase the tax revenues, are neutral to the number of permits issued and thus have no incentive to distort the choice of environmental quality.

The basic intuition about the efficiency of competitive permit trading is easy to understand, because the firm's objective is identical to (4) if τ refers to the permit price instead of the tax. And of course, in equilibrium, each firm's choice will reflect the same marginal product since otherwise arbitrage exists providing opportunities for beneficial trades (again assuming negligible transaction costs).

2.4. Examples and Classifications of Environmental Externalities

Having agreed that environmental issues are a subset of (negative) externalities, the following characterizations and differentiations are possible, emphasizing simultaneously the corresponding analytical tools.

An accounting classification is the differentiation between flow and stock externalities. In the first case only the current exposure causes damage and corresponding topical examples are noise and particulates (although in both cases, the health effects may depend on cumulative exposure). As a consequence, the tools of analysis can be confined to static optimization techniques. Stock externalities, the problem is not the flow but the accumulation, requires a dynamic analysis. Many real world environmental problems fall into this category. The most topical and threatening being global warming, where clearly the carbon dioxide emissions are not the problem (CO_2 is no poison, after all it creates the fizz in champagne and coke) but their accumulation in the atmosphere. Other examples are radiation (otherwise, X-rays would be forbidden), sulfur dioxide (a flow as well as a stock externality due to accumulation of SO_2 in the soil), the pollution of lakes and rivers, etc.

A geographical distinction is between local, national, international and global externalities. A local externality is one that is restricted to a particular space and time (again noise is a typical example, when sitting in a weekend home in the countryside, the noise in a bustling city does not affect one). International externalities include transboundary pollution with the examples of acid rain, and downstream pollution of rivers, but also of air if contingent on trade winds (e.g., in Austria, only 7% of the SO_2 immissions are from indigenous sources, the rest is 'imported'). This interdependence among parties (often very few) requires the analysis of strategic interactions, i.e., of games.

Pollution may be attributable to individuals or not, as in the case of non-point non-source pollution, which complicates the design of incentives. Clearly, the impossibility of figuring out the individual and responsible polluter complicates the application of Pigouvian instruments (not to mention Coasean bargaining). The corresponding tools are screening, sampling, monitoring, etc. using results from statistical inference.

Another classification of environmental services is the differentiation between consumptive (e.g., catching fish, logging forests, whaling) and non-consumptive services like strolling through forests, bird watching, whale watching, and the existence values, i.e. valuing existence without ever enjoying any use (e.g., in the survival of Siberian tigers). Lack of understanding how ecological balances can be preserved may lead to assigning arbitrary existence value to a species. Of course, this is an ecological issue and the (ad hoc) assumption of sustainability and conservation differentiates ecological from environmental economics that allows, at least in principle and a priori, for extinction being optimal.

The consequences of interactions with the environment are often uncertain. For example, the greenhouse effect and thus global warming is affected by substantial, and one might argue inherent uncertainty, because even a deterministic set up of the basic meteorological dynamics leads to a chaotic attractor, which rules out long-term weather forecasts. Incorporating uncertainty into environmental economics requires the inclusion of probability distributions or stochastic processes and the tools of stochastic optimization, static or even dynamic. In the latter case, applications of the real option approach, pioneered in the context of irreversible investments, can be found at an increasing rate.

Information about externalities or other relevant characteristics may be symmetric (all involved parties have the same, not necessarily deterministic information) or may be asymmetric involving private information, for example about damages. The latter case requires the tools of mechanism design.

3. Resources

The interest in the economics of finite (exhaustible, or non-renewable) resources was triggered by the oil price quadrupling in 1973 (during the Yom-Kippur war), which seemed to vindicate the Club of Rome's doomsday projections in which the good things were constrained and the bad things were not (hence no computer model is needed to figure out that the bad guys win, sooner or later). It seems surprising for theoretical economics (and the bulk of the literature is theoretical) that this interest was tied to a political event, even more so since the topic was already addressed more than forty years earlier by Harold Hotelling. The crucial feature of the study of resources is the need for dynamic or intertemporal analysis. The reason for this is that any consumption today affects the feasible consumption stream in the future. This intertemporal trade-off requires optimization in an infinite dimensional space since an entire function, in the following the exploitation path $\{x(t), t \in [0,\infty)\}$ must be determined, which constitutes an optimal control problem.

3.1. Non-renewable

The optimal exploitation of an exhaustible resource is the solution of an optimal control problem. In its most simple variant – no extraction costs, deterministic, constant discounting, efficient extraction formulated as a planning problem – it leads to the following optimization problem,

$$\max_{\{x(t)\geq 0\}} \int_0^\infty e^{-rt} u(x(t))dt, \tag{8}$$

$$\dot{R}(t) \equiv \frac{dR(t)}{dt} = -x(t), R(0) = R_0, R(t) \geq 0, \tag{9}$$

where $x(t)$ = extraction is equal to current consumption inducing (concave and non-satiating) utility u, r is the (constant) discount rate, $R(t)$ is the resource stock ('reserves') available in period t, and R_0 is the initial resource stock. The differential equation - the resource stock is reduced by the amount consumed = extracted – coupled with the associated non-negativity constraint captures the exhaustibility and irreversibility: any piece of the cake (or barrel of oil) extracted and consumed in period

t cannot be consumed in the future, requiring an intertemporal trade-off of consumption. This basic problem is extended into many directions, e.g., accounting for extraction costs declining with respect to reserves rendering the term non-renewable (since they will never be exhausted due to the high extraction costs for the last unit), uncertainty about the resource stock, exploration, backstop technologies, i.e., technologies that provide a perfect substitute albeit at high costs. The result of an optimal interior extraction policy can be obtained without variational calculus from the fact, that no arbitrage is accruable from transferring an infinitesimal unit of consumption from any period t_0 to a period t_1, i.e., the marginal utility of consumption must be constant on a net present value basis, $e^{-rt}u'(x(t))$ = constant. This implies that optimal extraction is declining over time and using the social surplus (consumer plus producer surplus) as a measure of utility u, exponentially growing prices, $p(t) = p_0 e^{-rt}$, implement the efficient allocation in a laissez faire economy; and conversely, a competitive extractive industry implies a price growing exponentially at the rate of discount.

Another class of problems addresses the issue of economic growth, or respectively, of the sustainability of consumption, when resources are an essential input into the production of goods, $y = f(k,x)$, where y denotes the perfectly malleable consumption (c) and investment good and f is a neoclassical production function (i.e. concave, with positive marginal products satisfying Inada-conditions) with the inputs capital (k depreciating at the rate $\delta > 0$) and resource (say energy in the form of fossil fuels, x). Arithmetically, the efficient (first best) allocation follows from the following optimization problem:

$$\max_{\{c(t),x(t)\}} \int_0^\infty e^{-rt} u(c(t))dt, \tag{10}$$

$$\dot{k} = f(k(t), x(t)) - c(t) - \delta k(t), k(0) = k_0, k(t) \geq 0, \tag{11}$$

$$\dot{R} = -x(t), R(0) = R_0, R(t) \geq 0. \tag{12}$$

This optimal control problem addresses in economic terms and within a parsimonious framework (recall Ockam's razor) the same problem, which the Club of Rome wanted to analyze with hundreds of equations and computer simulations ('system dynamics'). In contrast to the Club of Rome simulations, the qualitative analysis of the above optimization problem points directly at the crucial parameter for sustainable development, namely the elasticity of substitution between the inputs capital and fossil energy. This elasticity is denoted by σ, as is common in the literature (and should not be confused with the later and also usual use of σ as standard error), and defined as

$$\sigma \equiv \frac{d\ln(k/x)}{d\ln(f_x/f_k)}. \tag{13}$$

Hence, σ determines the relative change in the ratio of the factor inputs with respect to a 1% change of the ratio of marginal products and this ratio is just the marginal rate of substitution, which equals the factor price ratio of inputs in competitive markets. If $\sigma > 1$, then it is possible to accumulate sufficient amounts of capital in order to compensate for the ever smaller resource inputs. If $\sigma < 1$, then this capital accumulation is insufficient and doom cannot be and should not be avoided. Only the knife edge case of a unitary elasticity (i.e. Cobb-Douglas technology) yields the interesting case. Unfortunately, a unitary elasticity of substitution between energy and capital must be considered as (overly) optimistic, if judged by the bulk of empirical studies.

The analysis of this problem dates back to the mid-1970s. Recent extensions and investigations of this issue use, of course, the label *sustainability* coined by the Brundtland-report and the tools of the new economic growth theory developed in the mid-1980s that allow for endogenous growth for non-diminishing returns at the level of the aggregate economy. Perpetual growth requires forces – spillovers, endogenous technological change, etc. – that counter at the aggregate level, the law of diminishing returns at the level of firms, which are necessary to support a competitive equilibrium. A crucial point in most of these endogenous growth models is the existence of externalities. Hence, the competitive outcome doesn't need to be efficient.

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Biographical Sketch

Franz Wirl studied mathematics at the Technical University in Vienna (1971-76). Then he worked for six years at the OPEC Secretariat as an econometrician. He returned after completing his Ph. D. (1982) to academia, to his alma mater in 1983, first as Assistant and then as Associate Professor.

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