GROWTH AND DEVELOPMENT WITH INCOME AND WEALTH DISTRIBUTION

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Contents

Summary

The chapter discusses some of the major developments in growth economics over the last twenty years or so. Instead of summarizing a large number of papers in the area, it attempts to concentrate on a few contributions and brings out their essence. The idea is to familiarize the reader with the general trend of the subject in a manner that will help him/her to pursue the subject on his/her own. Since the subject is technically demanding, this chapter develops most of the mathematical preliminaries that will prove helpful to a reader who intends to study the subject seriously. However, the mathematics is developed in an intuitive manner. Only a minimal level of prior training in mathematical techniques is necessary to read the chapter. The chapter though addresses an audience with a minimal training in economics.

1. Introduction

The minimal requirement for enhancing economic welfare is a rise in the productivity of an economic system. Such rises make it possible for the population to enjoy more consumption, both in terms of variety as well as in terms of quantity. Normally, these improvements are quantified by means of a rise in the per capita output of the society. However, even though a rise in the annual growth rate of an economy's real per capita GDP suggests a rise in the standard of living enjoyed by the inhabitants of the society, per capita GDP growth alone is not a perfect indicator of welfare improvement. Distribution of the GDP across the population matters a great deal also. A large value of per capita GDP, accompanied by severe inequality of distribution, may sometimes signify a deterioration of human well being rather than advancement. A proper understanding of the manner in which an economy develops calls therefore for a study of both the growth rate as well as the manner in which a society's inhabitants access the improvements in the standards of living that growth promises. In what follows, we shall summarize theoretical work on both these aspects of the problem.

We shall start with the factors that explain the growth rate itself. This is procedurally important, since one must identify the average prior to discussing dispersions around it. Most nations which are reasonably advanced in terms of a wide variety of social indicators do exhibit significantly large values of real per capita GDP also. Consequently, attempts to maintain, and if possible raise, the sustainable growth rate of an economy's per capita GDP continue to occupy policy planners, economists and governments in power. As with most of economics, the explanations offered center around two of the most basic tools of economic analysis, supply and demand. If demand could be stimulated to rise over time, producers will produce more, the supply of goods and services will go up and along with it the GDP. Moreover, if the population does not grow faster than the GDP, then per capita GDP will register an increase also. On the other hand, if demand keeps growing without commensurate increases in supply, then GDP can rise at best in nominal terms without a matching rise in the supply of real goods and services. Consequently, to explain the growth or decay of economies over time, economists are led to study the causes underlying the growth of demand and supply over time.

Clearly, a temporary rise in demand cannot be an interesting phenomenon to study when the subject of analysis is growth over time, which must refer at the least to a positive growth rate sustained over several years. As already noted, however, growth in demand alone would not suffice unless it is accompanied by a corresponding increase in supply. Further, when demand induced GDP grows steadily, it is not adequate to view the corresponding supply responses as movements along a supply curve. Indeed, sooner or later, it would be infeasible for supply to respond to demand rises unless the production capacity is also raised. This means that a study of sustainable growth of per capita real GDP calls for an inquiry into the factors responsible for a growth in the *capacity* to produce, or *shifts* in the supply curve.

2. The Neoclassical Model of Economic Growth

It is not possible to appreciate recent advances in growth economics without recalling the seminal contribution of Solow (1956). We shall begin therefore by presenting the basic tenets of his work.

At each instant of time t, the economy produces a single aggregative commodity Y means of capital and labor. Denoting the capital stock at t by $K(t)$, the economy faces an instantaneous constraint on aggregate consumption and capital accumulation or investment ($Z(t)$) given by

$$C(t) + Z(t) = Y(t) \tag{1}$$

where $Y(t)$ stands for the flow of output at t. No borrowing against future is permitted to enhance current expenditure. It is standard practice to refer to $Z(t)$ as gross investment. A fraction δ of the capital stock depreciates through use per instant of time. Hence, $Z(t)$ leads to a *net* addition of

$$\dot{K}(t) = Z(t) - \delta K(t) \tag{2}$$

to the capital stock at each t. Accordingly, $\dot{K}(t)$ stands for net investment. Equation (1) is rewritten as

$$C(t) + \dot{K}(t) = Y(t) - \delta K(t) \tag{3}$$

The population at time t, assumed to be identical with the labor force, is denoted $L(t)$. Both K and L are fully employed and the labor force grows at the exogenously given rate n determined by demographic forces. The technology for producing Y is represented by an aggregate production function

$$Y(t) = F(K(t), A(t)L(t)), \tag{4}$$

where $K(t)$ and $L(t)$ are the flows of capital and labor services entering the production process at t. We may think of the economy as being populated by a large number of identical firms, each using the technology F. In this case, (4) stands for the aggregate output they produce by using same quantities of labor and capital. Observe that the use of the same notations for capital stock and services as well as for population size and labor services implies that the stock-flow ratios for both factors are assumed to be constants (normalized to unity).

The coefficient $A(t)$ of $L(t)$ represents technological progress and satisfies

Assumption T $\dot{A}/A = \mu > 0$.

There are different ways in which the notion of technical progress may be formalized. Equation (4) in particular captures it by introducing a distinction between the apparent size of the labor force and its effective size. An improvement in work efficiency is tantamount to a reduction in the time taken to complete a given job. Alternatively, a worker finishes two jobs (say) as opposed to one during any fixed interval of time as her/his efficiency improves. Consequently, from the point of view of work performed, the efficient worker may be treated as two less efficient ones. Viewed this way, technical progress is referred to as labor augmenting. When labor augmentation assumes the form $A(t)\ L(t)$, technical progress is called Harrod-neutral. Technical progress is neutral when the share of wages and profits in national income are unaffected by improved productivity of factors(s). Under Harrod-neutrality, the shares are unchanging along growth paths which leave the capital-output ratio unaffected. Such paths will be called balanced growth paths in the sequel. For a useful discussion of technical change see Burmeister and Dobell (1970), Chapter 3. See also Barro & Sala-i-Martin (2003). **Assumptions 1** and **2** imply together that the isoquants relating K and AL are strictly convex to the origin. **Assumption T** says that the labor force is effectively augmented at the rate μ on account of a rise in work efficiency generated by technological change.

Particular values attained by variables at a time point such as *t* will be represented by C_{t_0}, K_{t_0} etc. Nonetheless, the time index will often be dropped to achieve notational simplicity, unless essential for the argument. The function $F(.,.)$ satisfies the standard neoclassical properties, viz.

Assumption F1 $F(.,.)$ is continuous and differentiable. $F(0, AL) = F(K, 0) = 0$ and F displays constant returns to scale in $K\ AL$.

Assumption F2 $F_1, F_2, F_{11} < 0, F_{22} < 0.$

Assumptions $F_{11} < 0, F_{22} < 0$ imply the law of diminishing returns.

In a static environment, the **Assumption F1** predicts diminishing marginal product of a factor as increasing doses of the factor are combined with constant quantities of the other factor. For a growing economy, however, the factor services K and AL would be increasing simultaneously. In this case, what diminishing returns impliy is that the marginal product of capital (say) will rise if AL grows faster than K. In particular, a faster growth in A relative to that in K (as of given L) causes the marginal product of K to increase. In other words, it is the direction of change in K/AL that will determine how marginal products of factors respond to improvements in the size of labor augmenting technical change.

We shall denote per capita variables by small case letters. Thus, $y = Y/L,\ c = C/L,\ k = K/L$ and $z = Z/L$. Also, it will be useful to rewrite (4) in terms of quantities per unit of *effective labor* (i.e. AL). Thus, denoting Y/AL and K/AL by $\hat{y}$ and $\hat{k}$ respectively and using **Assumption F1**, we obtain

$$\hat{y} = F(\hat{k}, 1) = f(\hat{k}), f(0) = 0, \tag{5}$$

f is continuous and differentiable.

Assumption F2 implies

Property f1 f is a strictly concave function with $f'(\hat{k}) > 0$.

In addition to **Assumptions F1 and F2**, we will impose

Assumption F3 $f'(\hat{k}) \to \infty$ as $\hat{k} \to 0$ and $f'(\hat{k}) \to 0$ as $\hat{k} \to \infty$.

It is easy to verify (using the assumption of constant returns to scale) that $\partial F / \partial K = f'(\hat{k})$ and $\partial F / \partial L = f(\hat{k}) - \hat{k}f'(\hat{k})$. Thus, **Assumption F3** implies that the marginal product of capital increases without bound as capital becomes indefinitely scarce relative to the factor AL. On the other hand, using Euler's theorem, we see that

$$\frac{\hat{k}f(\hat{k})}{\hat{y}} + \frac{f(\hat{k}) - \hat{k}f'(\hat{k})}{\hat{y}} = 1.$$

This means that $0 \le (f(\hat{k}) - \hat{k}f'(\hat{k}))/\hat{y} = 1 - \hat{k}f'(\hat{k})/\hat{y} \le 1$. Hence,

$\partial F / \partial(AL) = f(\hat{k}) - \hat{k}f'(\hat{k}) = f(\hat{k})\{1 - \hat{k}f'(\hat{k})/\hat{y}\} \to 0$ as $\hat{k} \to 0$. In other words, as $\hat{k} \to 0$, the marginal product of effective labor turns indefinitely small. Put differently, the marginal product of capital\index{capital, marginal product of} rises and that of effective labor falls boundlessly as K *relative to* AL becomes indefinitely scarce (irrespective of the *absolute* values of K and AL).

Assumption F3 is referred to as an Inada condition and constitutes a regularity requirement. It guarantees that the model has mathematically meaningful solutions. Using (4), we rewrite (1) as

$$C(t) + Z(t) = F(K(t), A_t L_t). \tag{6}$$

At each t, this equation can be viewed as the transformation frontier between $C(t)$ and $Z(t)$ given $K(t)$. Deflating both sides by $A_t L_t$, (6) reduces to

$$\frac{c(t)}{A_t} + \hat{z}(t) = f(\hat{k}(t)), \tag{7}$$

where $\hat{z}(t) = Z(t)/A_t L_t$. Since

$$\begin{aligned}
\hat{z} &= \frac{\dot{K} + \partial K}{AL} \\
&= \frac{\dot{K}}{K}\frac{K}{AL} + \delta\hat{k} \\
&= \left(\frac{\dot{K}}{K} - (\mu + n)\right)\hat{k} + (\mu + n + \delta)\hat{k} \qquad (8) \\
&= \frac{\dot{\hat{k}}}{\hat{k}}\hat{k} + (\mu + n + \delta)\hat{k} \\
&= (\dot{\hat{k}} + (\mu + n + \delta)\hat{k},
\end{aligned}$$

an alternative representation of (7) is

$$\frac{c(t)}{A_t} + \dot{\hat{k}}(t) + (\mu + n + \delta)\hat{k}(t) = f(\hat{k}(t)),$$

or,

$$c(t) + A_t \dot{\hat{k}}(t) = A_t\{f(\hat{k}(t)) - (\mu + n + \delta)\hat{k}(t)\}, \qquad (9)$$

where, according to (8), $[(\mu + n + \delta)\hat{k}(t)]$ represents the minimum level of gross investment per unit of effective labor, (i.e., $\hat{z}$), that leaves $\hat{k}$ unchanged.To keep the discussion simple, we shall assume that investment is reversible. The assumption of reversible investment implies that capital, once installed, can be dismantled and used for consumption.

The economy's growth path is determined in the following manner. First of all, each firm operates in a perfectly competitive market. Similarly, the factor markets are perfectly competitive also. At each instant of time, the existing quantities of capital and labor services areoffered inelastically in the factor markets. The intersection of the marginal productivity curve of the factor, i.e., the demand curve for the factor and its inelastic supply determines the equilibrium factor returns. For capital, this is the equilibrium rate of interest, r. For labor, it is the equilibrium real wage rate. The aggregate output that is produced by means of these factors is divided up into consumption and savings. The saved part is reinvested to determine the capital stock at the next instant of time and the exercise just described is repeated. Thus, if we know the division of output between savings and investment then we can use (9) to predict the growth path of capital and output for the economy, starting out from any given pair (K_0, L_0) at the historically given starting point for the economy.

The larger the saving, the larger is the capital stock and, correspondingly, the larger is future output. Thus, a large saving, hence small consumption today, makes it feasible for tomorrow's consumption and welfare to be large. But a small consumption today reduces today's welfare. Consequently, the level of consumption a society chooses at any point of time needs to be optimally decided by balancing off current losses against future gains. We are ready now to search for the optimal growth path.

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Biographical Sketch

Dipankar Dasgupta was Professor of Economics at the Indian Statistical Institute, Delhi and Kolkata. He received his PhD from the University of Rochester in 1972 and has taught in Canada, India and Japan. He is currently a Visiting Scholar at the Centre for Studies in Social Sciences, Calcutta.