ENTROPY SYSTEMS THEORY

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Keywords: Entropy, thermodynamic entropy, Boltzmann’s entropy, information-theory entropy, social entropy, social entropy theory, equilibrium, macrostate, microstate, globals, mutables, immutables, PILOTS

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Summary

Entropy is a concept that was derived in the nineteenth century during the study of thermodynamic systems. Entropy exists in all systems, nonliving and living, that possess free energy for doing work. As system energy declines, entropy increases. Entropy has precise mathematical and statistical definitions, but can be approximately defined as the degree of disorder or uncertainty in a system. If a system is isolated, or cut off from all inputs of matter and energy, it will tend toward a maximum, according to the Second Law of Thermodynamics. Entropy is important for the study of life support systems because it offers a valuable way of defining and measuring sustainable systems. A sustainable system must, by definition, ensure that its entropy level does not rise to the maximum, as maximum entropy is tantamount to system death. To be sustainable, the system must have life support subsystems that can ensure supplies of matter, free energy, and information sufficient to maintain entropy levels below the maximum.

In addition to thermodynamic entropy, statistical equations for entropy have been derived by Boltzmann in physics and by Shannon in the field of information theory. Entropy has also been applied in social science in the form of Social Entropy Theory (SET). Since entropy is a complex concept and has been widely applied in a variety of disciplines, it is helpful to identify the principal dimensions of the entropy concept, and to use these as criteria for the evaluation and interpretation of specific entropy
applications.

1. Introduction

A life support system is any natural or human-engineered system that furthers the life of the biosphere in a sustainable fashion. This implies not only that the system can attain an adequate level of life support, but also that it is reasonable to assume that this level can be maintained for the foreseeable future without undue harm to the environment.

It is important to recognize entropy as an indicator of the state (or level) of the system. Entropy can be viewed an important way of defining and measuring sustainability. As a preliminary definition, entropy can be described as the degree of disorder or uncertainty in a system. If the degree of disorder is too great (entropy is high), then the system lacks sustainability. If entropy is low, sustainability is easier. If entropy is increasing, future sustainability is threatened. The change in entropy is inversely related to the change in free energy. As energy is expended to do work, entropy decreases. If no energy is available in the system, its entropy level will remain constant or increase.

2. History

The entropy concept originated in thermodynamics in the mid-nineteenth century. It has been widely applied since in a variety of fields, including physics, chemistry, engineering, biology, economics, anthropology, general systems theory, information theory, psychotherapy, sociology, urban planning, and others (see History and Philosophy of the Systems Sciences: The Road towards Uncertainty).

2.1 Thermodynamic Entropy

The term “entropy” was coined by Clausius in nineteenth-century thermodynamics, and is the subject of the Second Law of Thermodynamics, which states that in an isolated thermodynamic system, entropy will either remain constant or increase toward its maximum, but cannot decrease. This means that in an isolated system, the system will gradually become more and more disordered, until it reaches maximum entropy. This is a complete state of rest or dissolution, with an absence of available energy for doing work.

The phenomenon of entropy was originally observed in thermodynamic systems such as heat baths. An isolated system is one that is closed to inputs of both matter and energy. This means that since no new heat energy can be added, the system can never become hotter, but can only remain the same temperature, or become colder. As it loses heat over time, its entropy increases, until finally it reaches its maximum.

This state of maximum entropy is called thermodynamic equilibrium. This is sometimes described as the state of “system death.” Such thermodynamic systems, as found in nature, are “irreversible systems,” where heat cannot flow from colder to hotter parts of
the system, but only from hotter to colder areas. Thermodynamic entropy is denoted by the symbol $S$, and the formula for change in entropy is:

$$dS > \frac{dQ}{T}$$  \hspace{1cm} (1)

where $S$ is entropy, $Q$ is heat, and $T$ is the temperature of the system. The difference in two entropy states, $S_1$ and $S_2$ is:

$$S_2 - S_1 > \int_{1}^{2} \frac{dQ}{T} \quad \text{(irreversible process)}$$  \hspace{1cm} (2)

Entropy ($S$) can only remain constant or increase, until it reaches a maximum. When the system is in thermodynamic equilibrium, then:

$$dS = 0$$  \hspace{1cm} (3)

In physics, work and entropy are inversely related. The principal way to decrease entropy is to do work through the expenditure of free energy. If free energy is available, and is expended to do work, then the system becomes more orderly and entropy decreases. But if all available energy has been expended, then no more work can be done, and entropy will either remain constant or increase.

### 2.2 Boltzmann’s Entropy

In addition to thermodynamic (or heat-change) entropy, physicists also study entropy statistically. The statistical or probabilistic study of entropy is presented in Boltzmann’s law:

$$S = k \ln w$$  \hspace{1cm} (4)

Where $S$ is entropy, $k$ is Boltzmann’s constant, and $w$ is the probability that the system will exist in the state it is in relative to all the possible states it could be in. Boltzmann’s equation is somewhat different from the original Clausius (thermodynamic) formulation of entropy. First, the Boltzmann formulation is structured in terms of probabilities, while the thermodynamic formulation does not entail the calculation of probabilities. The thermodynamic formulation can be characterized as a mathematical formulation, while the Boltzmann formulation is statistical. Second, the Boltzmann equation yields a value of entropy ($S$) while the thermodynamic formulation yields only a value for the change in entropy ($dS$). Third, there is a shift in content, as the Boltzmann equation was developed for research on gas molecules, rather than thermodynamics. Fourth, by incorporating probabilities, the Boltzmann equation focuses on microstates, and thus explicitly introduces the question of the relationship between macrostates (the larger context) and microstates (the context for the probabilities). Macrostate/microstate relations are specified in theoretical physics. However, they become quite problematic when the notion of entropy is extended beyond physics, and researchers may not be certain how to specify and measure the macrostate/microstate relations.

### 2.3 Information Theory Entropy
Unlike the first two entropy approaches (thermodynamic entropy and Boltzmann’s entropy), the third major form of entropy was not derived within the field of physics, but was developed instead in a new field known as information theory (also known as communication theory). This newer specification of entropy is denoted by a measure known as Shannon’s $H$. For a sample of $N$ objects or persons, and a variable containing $K$ categories:

$$H = -\sum p_i \ln p_i$$  \hspace{1cm} (5)

where $H$ is entropy, and $p_i$ is the probability of being in category $i$ out of the $K$ possible categories. Examination of $H$ reveals that the measure varies from a minimum value of zero to a maximum value of $\log K$ (where $K$ is the number of categories). For a sample of $N$ objects (such as persons), the minimum value of zero is attained when all $N$ objects occupy a single category. The maximum value of $H$ ($\log K$) is attained when all $N$ objects are evenly distributed among the $K$ categories, so that each category contains exactly $1/N$ of the total sample.

There are a few points to note about the $H$ measure, and also about its comparisons to Boltzmann’s entropy (Eq. (4)). The maximum value of $H$ is not a fixed value, but is dependent upon the number of categories, $K$. The degree to which this is a problem depends upon the particular variable or subject matter to which $K$ is applied. For example, if $H$ is applied to sex, and only two values (female and male) are recognized, then sex is seen to be a natural dichotomy, and its maximum value will be fixed. However, if one wishes to compute $H$ for social class, then there may be disagreement, for example, about whether social class comprises three categories (lower class, middle class, and upper class), or five categories (lower class, lower-middle class, middle class, upper-middle class, and upper class). While minimum $H$ is fixed at zero in both cases, maximum $H$ varies, depending upon whether the number of categories chosen is three or five.

Another point to note when comparing $H$ with Boltzmann’s equation is that the formula for $H$ specifically includes both population or sample size ($N$) and category size ($K$), and that both of these are used in computing the probabilities $p_i$. In contrast, Equation (4), Boltzmann’s equation, does not identify the population size and categories, but is instead written in terms of the probability that an event may occur out of all the ways it could occur. These represent two different ways of approaching the macrostate/microstate problem.

A further issue with $H$ is the question of how it should be interpreted. Consider the application of $H$ for a situation where $K = 5$, as in the case of social class as given above. Here, $H$ can vary from a minimum of zero (when everyone is in the same income category) to a maximum of $\log K$ (when each of the $K$ categories contains exactly $1/K$ of the sample). Statistically, this maximum state is the “most probable state,” or most likely state. It is also the least predictable state, and thus the one that exhibits the most uncertainty or disorder.

That is, if one wishes to predict the social class of an individual, the prediction is certain for the case where $H$ equals zero, as all persons have the same income. Thus, when $H$ is
a minimum, there is no uncertainty, and social class is perfectly predictable. However, when $H$ is a maximum, then the probability of assignment to each category is basically random, resulting in an even distribution of individuals across class categories. This means that uncertainty is maximized, and it is difficult to predict a person’s social class. Since $H$ was developed within information theory, it is often referred to as a measure of information. The conundrum here is that minimum $H$ yields minimum uncertainty (high information) while maximum $H$ represents maximum uncertainty (low information). Thus, by calling $H$ information, scholars must face the uncomfortable anomaly or irony that minimum information yields maximum certainty, and that maximum information yields minimum certainty. If one is comfortable with an equation of information and uncertainty, then $H$ can be termed a measure of information. If one is more comfortable with $H$ as a direct measure of uncertainty or disorder (which it clearly is), then it is better to term $H$ a measure of entropy.

In contrasting $H$ with Clausius’ thermodynamic entropy and Boltzmann’s entropy, several differences are apparent. Thermodynamic entropy is defined solely in terms of heat and temperature, as shown in Equation (1). In contrast, Boltzmann’s entropy, as shown in Equation (4), is written in terms of the behavior of gas molecules. $H$ is a more open or generic measure, which is more content-free, and can be applied to any set of discrete categories for which data exists.

Shannon’s $H$ is strikingly similar to Boltzmann’s entropy in terms of its equation. Since Boltzmann’s entropy and Shannon’s $H$ are statistical formulations presented in terms of probabilities, and since the formulas are so similar, many statisticians may have little difficulty making the transition from Boltzmann’s entropy to Shannon’s $H$ (or vice versa). However, students of Shannon’s $H$ may have a more difficult time understanding thermodynamic entropy, and resolving differences between thermodynamic entropy and Shannon’s $H$.

Similarly, students whose first introduction to entropy was in thermodynamics, may not be sure that Boltzmann’s entropy is “real” entropy, but may accept it as a valid physics entity. However, they may consider this statistical computation to be less rigorous than the original thermodynamic entropy. These scholars may have real difficulty in accepting Shannon’s $H$ as an entropy measure. They may consider it a valid information-theory measure, or entropy analog, but may not consider it a valid entropy measure to the same degree that they would accept thermodynamic entropy or even Boltzmann’s entropy.

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Biography


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Biographical Sketch

Kenneth D. Bailey is Professor of Sociology at the University of California, Los Angeles. He was educated at the University of Texas at Austin, where he was awarded the B.Sc. degree in Mathematics, and the M.A. and Ph.D. in Sociology. Professor Bailey’s chief research areas are general systems theory, social entropy theory, research methods, and environmental sociology. He is the author of over 100 books and articles. Among his books are *Methods of Social Research*, Fourth Edition (The Free Press, 1994), *Social Entropy Theory* (State University of New York Press, 1990), *Sociology and the New Systems Theory: Toward a Theoretical Synthesis* (State University of New York Press, 1994), and *Typologies and Taxonomies: An Introduction to Classification Techniques* (Sage Publications, 1994). He also edited (with Yong Pil Rhee), *Systems Thinking, Globalization of Knowledge, and Communitarian Ethics*, Proceedings of the International Society for the Systems Sciences International Conference (Seoul, Korea, 1997). In addition to his published work, Professor Bailey has presented over 130 conference papers, and has presented invited lectures in Germany, Sweden, Italy, and Korea. His books have been translated into Italian, Korean, Chinese, and Malaysian. He has served as Associate Editor of *the American Sociological Review*, and is currently a member of the Editorial Board of *Systems Research and Behavioral Science*. He was Vice President for North America, Research Committee 33, Logic and Methodology, International Sociological Association; Secretary/Treasurer of the International Society for the Systems Sciences; President of Research Committee 51, Sociocybernetics, International Sociological Association; and a member of the Committee on Professional Ethics of the American Sociological Association. His current research is on new developments in social entropy theory, and on a systems approach to environmental sociology.