THE QUANTIFICATION OF SYSTEM DOMAINS

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Contents

Summary

The methodology of quantification is applied to system domains which can be considered "hard" or "soft", depending on whether the variables which describe them, pertain to domains in the physical/natural sciences or to domains in the biological/behavioral sciences. In this article, emphasis is definitely placed on the latter.

First, the article discusses what is the essence of the quantification problem in Science.

Then, with examples drawn from the real-world, it shows what are considered appropriate versus inappropriate forms of quantification. It continues by discussing traditional as well as novel approaches to quantification and the difference between qualitative and quantitative forms of measurement.

Implicit quantification and quantifiers are also introduced with more examples. A

hierarchy of conditional imperatives is used to illustrate implicit quantification which resides in the meaning of terms rather than in their explicit description.

1. Introduction

In the last few decades, most social sciences disciplines have attempted to develop problem solving methodologies which are based on mathematical modeling and quantitative approaches. These developments have flourished without asking whether the type of quantification used or chosen is adequate, in relation to the nature of the problem domains at hand.

In this article, various approaches to quantification are reviewed to assess the extent to which quantification is permissible.

The quantification problem--taken in its widest sense-- is not particular to the social sciences, but represents an important epistemological issue which concerns all of Science.

2. Quantification, Mathematization and Measurement

This article encompasses *mathematization, measurement and quantification.*

- *Quantification* has several meanings. In its most general sense it includes *mathematization, measurement* as well as the use of *quantifiers.*
  In its strictest sense, the term "*quantification*" originates in modal logic and predicate calculus and refers only to the use of *quantifiers.*
- *Mathematization* refers to the application of mathematical symbolism and mathematical methods to a problem, in order to solve it. A scientific discipline "*improves its image as a science*" to the extent that its problems can be *formalized*, preferably through the use of mathematics--a process which is called "*mathematization*".

Unfortunately, some disciplines have taken this motivation much too seriously and are driving *mathematization* to absurd extremes.

- *Measurement* refers to the assignment of numerals and numbers to represent attributes and properties in order to make choices and practical decisions.

The distinction between *quantitative* and *qualitative* measurement depends on the "*strength*" of the measurement scale which is used to evaluate the attribute or the property of a domain. See *Formal Approaches to Systems.*

3. The Scientific Imperative and the Quantification Problem

3.1. How Does A Scientific Discipline become more Rigorous?

The scientific disciplines of the Western world are still, to this day, placing too much emphasis on the traditions of Cartesianism and of the failed Positivist movement.

There is an over-reliance on the so-called "scientific method", as "the sole" method to reach scientific truths, and, on quantification and quantitative methods. Indeed, in certain scientific circles, *quantification* has become the unqualified litmus test that "*guarantees*" *scientific rigor*. (See *The Formalization and Quantification of Complexity*)

These tendencies neglect the fact that scientific disciplines are not "homogeneous" with respect to the nature of their domain.

- At the "*exact/hard*" end of the spectrum, are sciences such as physics and mathematics which are *rigorous and formalized.*
- At the "*inexact/soft*" end of the spectrum, are sciences such as the behavioral sciences, environmental science, conservation science (a discipline which deals with the historical and cultural aspects of our heritage), management science, cognitive science, social sciences, education etc. which admit *less rigorous and less formalized* methodologies.

The Quantification Problem refers then to an assessment of the extent to which mathematical and other quantitative approaches are used to solve a particular problem, in light of the hard vs. soft nature of its domain.

4. Quantification means Representation and Evaluation

Quantification can be considered a form of representation involving evaluation. (See Traditional Approaches to the Evaluation Problem).

The "representation" portion of the question will be omitted here. It is the subject matter of other specialized disciplines, such as cognitive science and artificial intelligence.

The "evaluation" portion entails:

- A determination of *the appropriate form of quantification*, given the domain of the properties in question
- The choice of *the suitable scale of measurement*
- The *choice between qualitative-type and quantitative-type* evaluations
- The *selection of the correct methodology* by which alternatives for decision making can be evaluated and weighed
- *The assignment of value* to attributes whose properties may change, depending on the logic level (of the inquiring system) where the problem is being considered.

Our main concerns are *epistemological* and *semantic:*

- *Epistemological*: To show the relative adequacy of different forms of quantification depending on their domain of application.
- *Semantic*: To show that certain concepts imply quantification by *the meaning* they acquire in certain contexts and domains.

5. Quantification. Formal Definition.

Quantification is a form of *representation* where quantifiers express the notion of *plurality*.

A distinction is drawn between *singular* terms and *general (plural)* terms. A *predication* is a formal combination of singular and general terms.

Quine provides the following example and definition:

"*In 'Mama is a woman' ('a is an F'):*

'a' represents a singular term and 'F' a general term."

"The general term is *what is predicated*."

"*'F' is the term which expresses that*

there is more than one 'a' in 'F'."

In the sentence, "*All men are mortal*", "*mortal*" is what is *predicated* or what is being affirmed or denied of the subject [here, the subject is "*all men*"].

6. Adequacy in the form of Quantification

It is difficult to establish milestones to determine the "*degree of quantification*". (See *Glossary & Quantification. Formal Definition*)

In certain cases, *mathematical formalisms* represent the purest form of quantification. In other cases, a natural language text may include a great number of quantifiers (*implicit or explicit*).

We prefer to qualify quantification as either, *adequate/suitable or, inadequate/unsuitable*.

Quantification is deemed *Adequate or Suitable* if:

- *The strength of quantification is appropriate for the problem domain, and if,*
- *The degree or extent of quantification used adds information to the problem situation.*

Quantification is *Inadequate or Unsuitable* if*:*

- *The strength of quantification is inappropriate for the problem domain, and if,*
- *The degree or extent of quantification fails to add information to the result.*

The selection of *the right strength* and *the right degree* or *extent of quantification* poses serious methodological difficulties.

Examples below illustrate the point.

7. Quantification of Attributes in Soft System Domains

7.1. An Unfinished Business

Quantification/measurement is a pervasive function which is not exclusive to the exact-hard domains. Because quantification is an integral and pervasive part of natural language, it cannot be dismissed as a technique "solely understood by engineers or technicians". As will be described below, applying the art of measurement in a knowledgeable way does not necessarily require "formal quantification" (i.e. mathematization).

The attributes of soft system domains listed below illustrate areas of discourse where *adequate/suitable* forms of quantification are still lacking:

- *The value of life, the value of euthanasia*
- *The value of health*
- *The value of avoiding drugs*
- *The value of eradicating poverty, delinquency*
- *The value of recreation*
- *The value of education*
- *The value of democracy*
- *The value of freedom(s) such as freedom of speech, freedom from want, etc.*
- *The value of exercising social responsibility*
- *The value of the quality of life, of ethical values, the evaluation of rights (be they, legal rights--written and/or unwritten, constitutional rights, human rights, etc.)*
- *The value of enjoyment of music and art*
- *The value of participation in community and cultural events,*
- *Historical value, cultural heritage value*
- *The value of clean air and potable water*
- *The value of contemplating a beautiful sunset, the value of a tree in an old-growth forest*
- *The value of biodiversity, the value of preserving the spotted owl and other endangered species, etc.*

The above list illustrates the kind of attributes of soft systems that can be found in the context of traditional domains which have defied quantification and measurement. (See *Implicit Quantification and Implicit Quantifiers*)

In the rest of this article, examples to illustrate various applications of quantitative formalization are given. In turn, the validity of each application is judged and a rating is given. See *Soft Systems Methodology.*

7.2. Examples of Inadequate/Unsuitable Quantification of a Soft-System Domain

EXAMPLE. # 1. THE QUANTIFICATION IN POLITICAL SCIENCE DOMAIN: A COMPARISON OF US PRESIDENTS.

One of the favorite pastimes of US historians has been to rate the performance of past presidents. After the 1996 election, thirty-two scholars were surveyed to ask them to judge President Clinton.

They were allowed six possible ratings: *Great, Near Great, Average-High, Average-Low, Below Average and Failure.*

President Clinton received 17 "*average*", *five* "*below average*" *and two* "*failure*" *ratings, which placed him among the overall Average (Low) in this President's* "*race*".

Can this attempt to quantify a president's performance be considered legitimate? Ratings should not be confused with opinion polls which are based on proven statistical sampling methods.

These ratings are probably biased by partisan preferences and trends of the time. Opinions about past presidents change with time. Circumstances during which these individuals had to perform their duties vary from undeclared war to peacetime, from depression to periods of hostage crises and more. So what do they mean?

The exercise provides an example *of pointless quantification*. The complicated life of a US president does not lend itself easily to a simple rating, let alone a rating which must be compared with other ratings of past presidents whose performance is foreign to the people making the comparison.

The performance of different presidents, over the entire length of their mandate, are not *commensurate i.e. no common scale of measurement or variable to compare them can be found*. Each individual's character is distinct, and the performance of his mandate is subject to the vagaries of a million events, not always under his control.

In short, apart from grabbing the newspaper headlines, ratings of this nature are not very useful, and do not jive with opinion polls, either past or present. This example of quantification is rated *inadequate/unsuitable*.

EXAMPLE. # 2. THE QUANTIFICATION OF THE URBAN HAPPINESS QUOTIENT.

This is a another example of *inadequate/unsuitable* quantification. A recent newspaper article described the attempts of a city research team to create a formula that would reflect the city's mental health and social well-being of its inhabitants. The proposed *formula included all sorts of statistics on poverty, employment, wages, murders, education spending, the mental health budget, homelessness, etc.*

The final score was touted as "*an instrument for gauging the city's health*". While the research team *was* "*looking at the quality of life for the mentally ill*", a decision was made to use the same approach to figure out "*urban contentment*".

The formula used to develop mental health and social and mental well-being indices was calculated by manipulating variables such as:

x (the murder rate), y (the size of the mental health budget) and z (Economic factors, such as the average income of full-time workers). Then, the product was divided by w (the number of individuals with serious mental illness).

It is questionable whether a complex matter such as individuals' well-beings or their mental health, can be encompassed in a single formula aggregating all the scores in a single calculation. The proponents of such a formula thought that the formula was "*an objective measure*".

In the opinion of this author, searching for a single number that can represent the inherent complexity of human beings and their mental state or well-being is a waste of taxpayers money.

This example of quantification is rated *inadequate/unsuitable*.

7.3. Three Cases Illustrating Adequate/Suitable Quantification Through Mathematization

EXAMPLE # 3. THREE CASES (A, B & C)

The following examples of mathematical modeling illustrate the application of legitimate forms of quantification.

(a) The Formalization and Modelisation of A Heart Disease Risk Model.

Diseases of the heart are a major medical problem in the United States and in most industrial nations. A heart risk index was proposed which can be used by a health insurance company to quantify the factors associated with heart disease and calculate the premium it charges individuals in different risk categories.

The index is based on such common variables as blood pressure, amount of smoking, serum cholesterol level in blood, whether the candidate is male or female and age etc.

(b) Formalization and Modelisation of the Worth of a Stock Option

In 1973, two US professors of economics and business finance devised a formula by which the value of a stock option contract can be calculated. An option is an investment instrument that allows an individual to buy or sell an asset, a security or a commodity, at a set price during a set period of time. The formula calculates the value of the option, on the basis of the expected future price, the expected cost of exercising the contract and the volatility of the asset.

Today, the formula which has been programmed for pocket calculators, is used by farmers, grain dealers, purchasers of commodity futures and investment dealers to hedge future risks.

What seemed an insurmountable problem, was solved by formalization and quantification.

(c) Catastrophe Bonds

Investment bankers and insurance companies have found a way to devise a security to quantify the risk of catastrophes such as hurricanes and earthquakes. Bondholders and insurers are faced with gambles based on the destructive power of catastrophes brought about by natural forces such as hurricanes.

The risks of such catastrophes can be estimated to sell bonds for protection of such huge risks. The calculation is less exact than that of option contracts, because, the inherent uncertainty in large catastrophes is greater. However, the new bonds are rated not riskier than junk bonds. They attract investors who feel that "the odds of the event happening, [are] significantly less than the amount they are paying".

The quantification of the risk of catastrophe bonds uses computer modeling which is still deemed an "imprecise science".

The rating agencies have found a way to make the new investment instruments "look rational". They have quantified the statistical probability of large cataclysms in such a way to help investors weigh the chances of large losses when Nature creates havoc, against the benefits of earning a good return, when it does not.

The above examples are given to illustrate *adequate/suitable* use of quantitative analysis-- a methodology which is at the heart of the disciplines of Operations Research, Management Science, Systems Engineering, Quantitative Economics, Finance and other fields which rely on quantitative modeling and quantitative formalization.

Features of this type of quantitative modeling:

- *Formalization assumes that the causal relationship among the variables is well understood*
- *Formalization assumes that the mathematical function chosen to reflect the so-called the "essence" truly reflects the causal relationships among the variables.*
- *Modeling implies a closed system where all boundary conditions are known or assumed. The problem space is hypothetically constrained.*
- *Of necessity, modeling is a simplification of the real-world and conclusions drawn from a model have clear limitations.*

If properly used and applied, the kind of formalization and of quantitative modeling illustrated above, can be exceptionally useful. This paradigm is at the basis of all discoveries in mathematical physics and has contributed in no small measure to the progress of technology in our industrialized society.

Its main shortcomings are not inherent in the design of the methodology *per se* but, rather, in its misuse and the limited vision of those who apply it to real-world complex problems.

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