# STRATEGIC BEHAVIOR

**Prajit K. Dutta**
*Department of Economics, Columbia University, New York, NY 10027*

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**Contents:**



**Summary**

Strategic behavior is most widely studied within the social sciences by using the framework of Game Theory. This article is an introduction to game theory. It is organized in four sections. Section 1 will verbally discuss some examples of strategic behavior - or games - and give a more precise analysis of two selected examples. Section 2 will introduce the strategic form which is a formal construct within which strategic behavior can be studied and Section 3 will discuss the most popular solution concept, Nash equilibrium. Finally, in Section 4, I will discuss strategic behavior in the commons problem.

## 1. Examples of Games

The focus of Game Theory is *interdependence*, situations in which an entire group of people is affected by the choices made by every individual within that group. It is precisely this interdependence that gives rise to strategic behavior. In such an interlinked situation, the interesting **questions** include:

- What will each individual guess about the others' choices?
- What action will each person take? (This question is especially intriguing when the best action depends on what the others do.)
- What is the outcome of these actions? Is this outcome good for the group as a whole?
- Does it make any difference if the group interacts more than once?
- How do the answers change if each individual is unsure about the characteristics of others in the group?

The content of Game Theory is a study of these - and related - questions. I will give a more formal definition in a minute but consider first some **examples** of interdependence. These examples are drawn from economics, politics, resource utilization, law - and even our daily lives.

- *Art Auctions* (such as the ones at Christie's or Sotheby's where art works from Braque to Veronese are sold) and *Treasury Auctions* (at which the United States Treasury Department sells US government bonds to finance the federal budget deficit).
- *Voting at the United Nations* (for instance, to select a new Secretary General for the organization).
- *Animal Conflicts* (over a prized breeding ground, or scarce fertile females of the species, etc.).
- *Sustainable Use of Natural Resources* (the pattern of extraction of an exhaustible resource such as oil or a renewable resource such as forestry).
- *Random Drug Testing at Sports Meets and the Workplace* (the practice of selecting a few athletes or workers to take a test that identifies the use of banned substances).
- *Bankruptcy Law* (which specifies when and how much creditors can collect from a company that has gone bankrupt).
- *Trench Warfare in World War I* (when armies faced each other for months on end, dug into rival trench-lines on the borders between Germany and France).
- *OPEC* (the oil cartel that controls half of the world's oil production and hence has an important say in determining the price that you pay at the oil pump).

**Definition Game Theory is a formal way to analyze** *interaction* **among a** *group* **of** *rational* **agents who behave** *strategically.*

Consider each of the italicized items in the definition above:

1. *group* in any game there is more than one decision-maker; each decision-maker is referred to as a "player"
2. *interaction* what any one individual player does directly affects at least one other player in the group
3. *strategic* an individual player accounts for this interdependence in deciding what action to take
4. *rational* while accounting for this interdependence, each player chooses her best action

Let me illustrate these four concepts by discussing in detail some of the **examples** given above:

**Example from everyday life** - *Random drug testing* (at the Olympics): the *group* is made up of competitive athletes and the International Olympic Committee (IOC); the *interaction* is both between the athletes - who make decisions on training regimens as well as on whether or not to use drugs - and with the IOC, which needs to preserve the reputation of the sport; *rational strategic* play requires the athletes to make decisions based on their chances of winning and, if they dope, their chances of getting caught. Similarly, it requires the IOC to determine drug testing procedures and punishments on the basis of testing costs and the value of a clean-whistle reputation.

**Example from economics and finance** - *Treasury auctions*: On a regular basis, the United States Treasury auctions off US government securities. (*These securities are Bonds and Treasury Bills - financial instruments that are held by the pubic (or its* *representatives such as a mutual funds or pension funds). These securities promise to pay a sum of money after a fixed period of time, say three months, or a year or five years. Additionally, they may also promise to periodically pay a fixed sum of money over the lifetime of the security*.)

The principal bidders are investment banks such as Salomon Brothers or Merrill Lynch (who in turn sell the securities off to their clients) and so that is the *group* (the bidders in fact rarely change from auction to auction); they *interact* because the other bids determine whether a bidder is allocated any securities and possible also the price that he pays. Bidding is *rational* and *strategic* if bids are based on the likely competition and achieve the right balance between paying too much and the risk of not getting any securities.

**Example from biology** - *Animal behavior*: On the more fascinating applications of game theory in the last twenty-five years has been to biology and, in particular, to the analyses of animal conflicts and competition. Animals in the wild typically have to compete for scarce resources (such as fertile females or the carcasses of dead animals); it pays therefore to discover such a resource - or snatch it away from the discoverer - but the problem is that doing so can lead to a costly fight. Here the *group* of "players" are all the animals that have an eye on the same prize(s) and they *interact* because resources are limited; their choices are *strategic* if they account for the behavior of competitors; they are *rational* if they satisfy short-term goals such as satisfying hunger of long-term goals such as the perpetuation of the species.

**Example from law** - *Bankruptcy law*: In the United States once a company declares bankruptcy its assets can no longer be attached by individual creditors, but instead are held in safe-keeping till such time as the company and its creditors reach some understanding. However creditors can move the courts to collect payments before the bankruptcy declaration (although by doing so a creditor may force the company into bankruptcy). Here the *interaction* among the *group* of creditors arises from the fact that any money that an individual creditor can successfully seize is money that becomes unavailable to everyone else. *Strategic* play requires an estimation of how patient other creditors are going to be and a *rational* choice involves a trade-off between collecting early and forcing an unnecessary bankruptcy.

At this point, you may well ask what, pray, is not a game? A situation can fail to be a game in either of two cases - the *one* or the *infinity* case. By the *one* case, I mean contexts where your decisions affect no one but yourself. Examples include your choice about whether or not to go jogging, how many movies to see this week and where to eat dinner. By the *infinity* case, I mean situations where your decisions do affect others but there are so many people involved that it is neither feasible nor sensible to keep track of what each one does. For example, if you were to buy some stocks in AT&T then it is best to imagine that your purchase has left the large body of shareholders in AT&T entirely unaffected. Of course these two situations are precisely those in which there is no need for strategic behavior.

To fix ideas, let us now work though two games in some detail.

- *Nim and Marienbad*: These are two parlor games that works as follows:

There are two piles of matches and two players. The game starts with player 1 and thereafter the players take turns. When it is a player's turn to play, she can remove any number of matches from one pile. A player is required to remove some number of matches if either pile has matches remaining and she can only remove out of one or the other pile.

In Nim, whichever player remove the last match, wins the game. In Marienbad the player who removes the last match loses the game. The interesting question for either of these games is whether or not there is a winning strategy, i.e., is there a strategy such that if you used it whenever it is your turn to move, you can guarantee that you will win regardless of how play unfolds from that point on?

**Analysis of Nim**: Call the two piles *balanced* if there is an equal number of matches in each pile; and call them *unbalanced* otherwise. It turns out that if the piles are balanced, player 2 has a winning strategy. Conversely, if the piles are unbalanced, player 1 has a winning strategy.

Let us consider the case where there is exactly one match in each pile; denote this (1,1). It is easy to see that player 2 wins the game. It is not difficult either to see that player 2 also wins if we start with (2,2). For example, if player 1 removes two matches from the first pile, i.e., moves the game to (0,2), then all player 2 has to do is remove the remaining two matches. On the other hand, if player 1 removes only one match, i.e., moves the game say to (1,2), then player 2 can counter that by removing a match from the other pile. At that point the game will be at (1,1) and now we know player 2 is going to win.

More generally, suppose that we start with $n$ matches in each pile, $n > 2$. Notice that player 1 will never want to remove the last match from either pile, i.e., she would want to make sure that both piles have matches in them.( *Else, player 2 can force a win by removing all the matches from the pile which has matches remaining.*) However, in that case, Player 2 can ensure that after everyone of his plays, there is an equal number of matches in each pile (How?)( *Think of what happens if player 2 simply mimics everything that player 1 does, excpet with the other pile.* ) This means that sooner or later we will have arrived at the game with one match in each pile.

If we start with unbalanced piles, player 1 can balance the piles on her first play. Hence, by the above logic, she has a winning strategy. The reason for that is the following: once the piles are balanced, it is as if we starting afresh with balanced piles but with player 2 as the first mover. However, we know that the first mover loses when the piles are balanced.

Similar logic can be applied to the analysis of Nim's cousin, Marienbad.

**Analysis of Marienbad**: I claim that: - if the two piles are balanced with one match in each pile, player 1 has a winning strategy; - on the other hand; if the two piles are balanced with at least two matches in each pile, player 2 has a winning strategy. Finally,

- if the two piles are unbalanced, player 1 has a winning strategy.

Note, incidentally, that in both of these games the first player to move - in my discussion that player is also labeled Player 1 - has an advantage if the piles are unbalanced but not otherwise.

- **Prisoners' Dilemma.** This is the granddaddy of simple games. It was first analyzed in 1953 at the Rand Corporation, a fertile ground for much of the early work in game theory, by Melvin Dresher and Al Tucker.

The story underlying the Prisoners Dilemma goes as follows: Two prisoners, Calvin and Klein, are hauled in for a suspected crime. The DA speaks to each prisoner separately, and tells them that she more or less has the evidence to convict them but they could make her work a little easier (and help themselves) if they confess to the crime. She offers each of them the following deal: "*confess* to the crime, turn a witness for the State and implicate the other guy - you will do no time. Of course, your confession will be worth a lot less if the other guy confesses as well - in that case - you both go in for five years. You could *not confess*; however, be aware that we will nail you with the other guy's confession - and then you will do fifteen years. In the event that I cannot get a confession from either of you, I have enough evidence to put you guys away for a year."

Here is the representation of this situation:

| Calvin/Klein | Confess | Not Confess |
|---|---|---|
| Confess | 5,5 | 0,15 |
| Not Confess | 15,0 | 1,1 |

Notice that the entries in the above table are the prison terms: so the entry that corresponds to (Confess, Not Confess), i.e., the entry in the first row, second column, is the length of sentence to, respectively, Calvin and Klein when Calvin confesses but Klein does not. Note that since these are prison terms, a smaller number (of years) is preferred to a bigger number.

**Analysis**: From the pairs' point of view, the best outcome is (not confess, not confess). The problem is that if Calvin thinks that Klein is not going to confess, he can walk free by ratting on Klein. Indeed, even if he thinks that Klein is going to confess - the rat - Calvin had better confess to save his skin. Surely the same logic runs through Klein's mind - consequently, the both end up confessing.

Two remarks on the Prisoners Dilemma are worth making. There are outcomes in which both players can gain, such as (*Not Confess, Not Confess*). Second, this game has been used in many applications. Here are two - two countries are in an arms race, they would both rather spend little money on arms buildup (and more on education) but realize that if they outspend the other country they will have a tactical superiority. If they spend the same (large) amount though they will be deadlocked much the same way that they would be deadlocked if they both spent the same, but smaller amounts; - two parties to a dispute (a divorce, labor settlement, etc.) each have the option of either bringing in a lawyer or not. If they settle (50-50) without lawyers, none of their money goes to lawyers. If, however, only one party hires a lawyer they get better counsel and get more than 50% of the joint property; indeed, sufficiently more to also compensate for the lawyer's fees. If the both hire lawyers, they are back to equal shares, but now equal shares of a smaller estate.

I now turn to game theory's toolkit; the formal structure within which we can study strategic behavior. I will discuss one of the two principal ways in which a game can be written, the *Strategic Form* of a game.

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**Bibliography**

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**Biographical Sketch**

Prajit K. Dutta is a Professor of Economics and Columbia University. His research interests are in game theory with applications to resource economics, finance, and the Internet. He is the author of *Strategies and Games: Theory and Practice* (MIT Press, 1999).