OVERLAPPING GENERATIONS MODELS

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Summary

The specific feature of the *Overlapping Generations Model* is that new cohorts are born in every period, live for a number of periods and die. Consequently, at any point in time, society is composed of individuals belonging to the different generations that are currently alive. Because of the demographic structure of the population, no trade involving individuals about to die is possible and hence markets *per se* cannot bring about an efficient allocation of the existing resources. This natural restriction on the working of the markets makes the OLG model to behave very differently from the General Equilibrium. Specifically, we have that: i) typically there exists a vast number of equilibria; ii) competitive equilibria may fail to be Pareto efficient; and iii) Pareto efficient competitive equilibria may not belong to the core of the economy. Further, the introduction of intrinsically worthless fiat money induces the appearance of Pareto superior equilibria, otherwise unreachable. Such intergenerational transfers can also be performed by means of taxes and pensions, public debt or other forms of public intervention. These features make clear that the parallelism between the OLG and the GE model is limited. In order to stress the properties specific to OLG economies we start by examining the simplest model possible. This is the canonical OLG model due to Samuelson. In the subsequent sections we present a detailed discussion of the most salient "anomalies" in a much more general model of a pure exchange economy. We examine first the main properties of competitive equilibria: existence, uniqueness and efficiency. Next we analyze the role of fiat money and the conditions for existence and efficiency of monetary equilibria. Finally, we address the issue of the trustworthiness of intergenerational (implicit) agreements. We conclude with some remarks on future research directions.

1. Introduction

David Hume argued that political power could not emanate from a *social contract*

because society was not made of butterflies -who are born and die at the same time-, but composed of overlapping cohorts of humans: the unborn generations could not feel obliged by prior social agreements. Indeed we live in a society in which many social or economic contracts, as well as rules and norms of behavior, involve obligations to future, still unborn generations. Accordingly to Plutarch (Licurgus XV), a young man refused to give his seat at the assembly to old Dercilides of Sparta (IV BC), thus violating existing social rules. He argued that, since Dercilides had remained a bachelor, he had not contributed any children who in their turn might one day would give their seat to him when in his old days.

Intergenerational agreements pervade our social life. Parents give education, values and bequests to their off-springs. But, at the same time, they also shift to their children the burden of public debt, fiat money, and pensions. Further, intergenerational interactions transcend the family nucleolus. At a societal level, current investment in physical capital or in R&D spreads its benefits into future generations. However, our present consumption of non-renewable resources (or partially so, including pollution and global warming) is at the expense of the wellbeing of our descendants.

At first sight, this problem does not look essentially different from the standard allocation of resources over competing needs –the core of modern Economics. Market prices reveal the intensity of the needs relative to the abundance of the necessary resources. Resources end up being allocated efficiently by the interdependence of actions taken by egotistic agents. However, in the case of intergenerational arrangements, a part of the concerned individuals are not present to express the intensity of their preferences. Further, even if we could anticipate their preferences and resources, the sequential nature of the population structure imposes restrictions on the feasibility of trades. We cannot conceive direct (implicit) agreements with generations one century ahead of us without the intermediation of the generations in between. A competitive market proper cannot exist by nature.

There are two main ways of modeling the intergenerational allocation of resources. They essentially differ in the way they treat consumers. In one case, consumers are represented by one single, infinitely lived individual. This highly simplified representation has to be understood as a short hand for a more complex model that can be formalized in a number of alternative ways. The competing way of modeling the inter-temporal allocation of resources is the *Overlapping Generations Model* (OLG hereafter). In the OLG model new cohorts are born in every period, live for a number of periods and die. Therefore, at any point in time, society is composed of individuals belonging to the different generations that are currently alive. As it turns out, the kind of interactions or trades that can occur are constrained by the fact that they concern individuals at different points of their life-span.

The first formal model of an economy with overlapping generations is due to Samuelson (1958). In his seminal paper, the highly stylized economy consists of one individual per generation who lives for two periods and only one (perishable) commodity. At each period there are two players only: the young and the old. All possible trades necessarily involve individuals of different, subsequent generations. Suppose that consumers own the entire stock of commodities when young and nothing

when old. Clearly, they would be better off by lending part of these commodities to another consumer to be repayed in the second period of his lifetime. Yet, the old consumer he has nothing to offer in exchange. Thus, because of the demographic structure of the population no trade is possible and markets *per se* cannot bring about an efficient allocation of the existing resources. Then, Samuelson goes on to show that if the government introduces valueless fiat money notes, they will attain a strictly positive market price and the economy will shift to a Pareto superior allocation.

In spite of its extreme simplicity this model contains most of the essential features specific to OLG models which make it substantially different from the standard, static General Equilibrium (GE thereafter) model. In the (Arrow-Debreu) GE model, the time horizon is finite and consumers are all contemporary. Consumers are assumed to behave competitively, that is, make plans and trade in markets using exogenous prices as the sole information. Under some fairly general conditions, competitive equilibria exist -in the sense that there are prices for which all individual plans are mutually compatible- and are locally unique. In addition, (First Fundamental Theorem of Welfare Economics) competitive equilibria are Pareto optima. Finally, competitive equilibrium allocations belong to the *core* of the economy.

These results reinforce each other and render the notion of competitive equilibrium logically coherent. The existence of equilibrium warrants that the definition of competitive equilibrium is not void and the local uniqueness makes the notion determinate: the description of the economy is neither unnecessarily stringent, nor too loose. That equilibira are Pareto optimal shows that competitive exchange through markets leads to efficient outcomes. Lastly, the result that the equilibrium allocations belong to the core of the economy shows in a formal way that it does not pay to individuals (or subsets of) to deviate from competitive behavior and seek for arrangements with other consumers, by-passing the market exchanges. Thus, individuals do rightly by behaving accordingly with the competitive conjecture.

In contrast to the results above, in OLG models we shall have that: i) typically there exists a vast number of equilibria; ii) competitive equilibria may fail to be Pareto efficient; and iii) Pareto efficient competitive equilibria may not belong to the core of the economy. Consequently, we have that, in an OLG environment, the description of the economy as an uncoordinated interaction that takes place through markets in which agents act competitively might be insufficient to be able to yield determinate outcomes. In addition, the working of the markets does not seem to be sufficient to warrant the efficient allocation of resources. Finally, the assumption that consumers take decisions conjecturing a competitive environment might be inconsistent with rational behavior.

The arguments we have just presented make it clear that the trade between coexisting generations will have distinct features. The parallelism with the GE model is limited. In order to stress the properties specific to OLG economies we shall start by examining the simplest model possible. This is the canonical OLG model due to Samuelson. In the subsequent sections we shall present a detailed discussion of the most salient "anomalies" in a much more general model of a pure exchange economy. We shall examine first the main properties of competitive equilibria: existence, uniqueness and efficiency. Next we analyze the role of fiat money and the conditions for existence and

efficiency of monetary equilibria. Finally, we address the issue of the trustworthiness of intergenerational (implicit) agreements.

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The OLG model initiates in Samuelson's (1958) seminal paper. Cass and Yaari (1966) and Gale (1973) provide a good and clear account of the OLG model. Cass-Okuno-Zilcha (1979) introduced the "reflected offer curve" as a useful device for the analysis of existence and stability of equilibria.

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2. Intertemporal efficiency

Our discussion of Pareto Efficiency closely follows Balasko and Shell (1980). Okuno and Zilcha (1980) proved virtually the same result on the characterization of efficiency by the divergence of the sum of the inverse of the supporting prices. Both are based on the Cass (1972) price characterization of efficient accumulation programs. The reading of Shell (1971) is useful, since it gives good insights into the problem of efficiency with a countable infinity of individuals. Burke (1987) shows that the Balasko-Shell's curvature assumption on indifference curves is inappropriate. Little is know on the properties of the set of efficient allocations. Balasko-Shell (1981) prove that it is arc-connected. Their proof is again amended by Burke (1989). Balasko (1997) explores the traditional characterization of efficient allocations as the outcome of the constrained maximization of a social welfare function for all possible weights on individual consumers.

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Existence of competitive equilibria with n-commodities was first formally proven by Balasko and Shell (1980), later generalized by Balasko-Cass-Shell (1981). The extent of multiplicity of equilibria has been examined by Kehoe and Levine (1985) and later generalized by Muller-Woodford (1988). Balasko-Shell (1981) have shown that with log-linear preferences one can obtain uniqueness. But, this result is not robust. Kehoe-Levine (1984) and Geanakoplos-Polemarchakis (1984) have shown that in a stationary model with intertemporal separability of preferences generically there is a countable infinity of equilibria. As for the "causes" of the inefficiency of competitive equilibria, Cass and Yaari (1966) argue that this is due to the fact that there are missing markets. Pingle and Tesfatsion (1991) make the point that in the innefficient equilibria there are potential gains from intermediation left. Finally, Aiyagari's (1992) associates inefficiency to the violation of Walras Law at infinity. We have not covered “sunspot” equilibria. The interested reader will find a good overview in Guesnerie and Woodford (1992).

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4. Monetary Equilibria: Existence and Efficiency.

Balasko-Shell (1981) examine properties of monetary equilibria for the case of many commodities. The first proof of existence of an efficient monetary equilibrium in a stationary OLG model with a constant stock of money is due to Benveniste and Cass (1986). The case of "active" monetary policies, in which the money supply may vary from period to period, has received little attention. Besides Balasko and Shell (1981) and (1986), there are the contributions of Burke (1987) and (1988) and Esteban, Mitra and Ray (1994). Burke proves the existence of efficient moneraty equilibria for monetary policies consisting in the introduction of money followed by a sequence of budget surpluses. Esteban, Mitra and Ray (1994) completely characterize the sequences of money supply for which an efficient monetary equilibrium exists.

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In the text we have not covered the OLG model with durable goods and/or production. Money can be seen as a particular technology to transfer "income" across periods. When goods can be stored we shall have two competing way of transferring purchasing power from present to future. This case is studied by Koda (1984) and Maeda (1992). Note that fiat money has the efficiency property of performing the same role without having to save real goods to transfer purchasing power across periods. The basic OLG model with productive capital and labor was first developed by Diamond (1965) and Cass and Yaari (1967), later generalized by Galor and Ryder (1989). The results are qualitatively similar to the case of the pure exchange OGL economy that we have examined. Since capital is productive, the role of money is now played by interest bearing public debt. Intergenerational transfers can also by carried out by a pay-as-you-go pensions system, as it was pointed out by Samuelson (1975). Jones and Manuelli (1992) and Boldrin (1992) entend the model to the case of increasing returns. Finally, Galor (1992) and Fisher (1992) study the properties of the growth models with two sectors: consumption and production goods. Contrarily to the results in standard growth models, in OLG economies the capital good sector has to be more capital intensive in order to have locally stable steady state equilibria.

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Biographical Sketch

Joan Maria Esteban is Professor at the Institut d'Analisi Economica (Campus Universitat Autonoma de Barcelona). He has made definitive contributions to the literature on welfare economics and on the overlapping-generations models of intertemporal equilibrium theory.