# **TYPES OF INTERACTIONS**

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# Summary

On the fundamental level there are four known distinct types of interactions in Nature which are, in ascending order of their strength at energies  $\ll 100 \text{GeV}$ : gravitation, weak interactions (involved in radioactivity), electromagnetism, and the strong interaction (responsible for binding nuclear matter). All four interactions are gauge interactions and are due to a local symmetry of the physical world that holds at each point of space-time. On the fundamental level they have to be described within quantum field theory which provides a general framework for gauge interactions consistent with quantum mechanics and Lorentz symmetry. When treated as per-quantities at the quantum level. In order to reach a physically meaningful description it is necessary to remove these infinites order by order of perturbation theory. In general this can be done if all interactions terms allowed by the symmetries of the system are included in the

Lagrangian .However, whereas for the three non-gravitational interactions this involves only readjustments of a finite number of masses and coupling constants, for gravity an infinite number of parameters is involved. Gravity is said to be non renormalizable , whereas the other interactions are renormalizable. This is also reflected by the fact that

the gravitational coupling constant (Newtons constant)  $G_N \simeq (1.22 \times 10^{19} \text{GeV})^{-2}$  has a

negative quadratic mass dimension, whereas the other coupling constants are dimensionless. On the classical (tree) level, non- gravitational interactions are therefore also invariant under rescaling of lengths and energies, whereas gravity is not. On the quantum gauge symmetries, renormalization leads to energy –dependent effective coupling constants. Under certain circumstances, the three relevant couplings converge at  $\simeq 2 \times 10^{16}$  GeV which suggests that these gauge symmetries unify to a larger internal

at  $\approx 2 \times 10^{10}$  GeV which suggests that these gauge symmetries unity to a larger internal symmetry. In contrast, the gauge group of gravity consists of the general covariant space-time coordinate transformations and it is presently unclear how they may unify with the non-gravitational symmetries within ordinary quantum field theory. A "Theory of Everything" may thus not even by a quantum field theory of particles and fields at all, but may rather reveal the currently known "fundamental "interactions as just effective field theories represent the low energy limits of a more fundamental description. Such a description could involve higher dimensional objects such as strings and branes in string theory.

# 1. Introduction

The fundamental interactions that we know of today are all described on the basis of quantum mechanics and relativity. We assume here that the reader is familiar with these subjects at least on an elementary level. For an introduction to these subjects as well as historical information the reader can consult the contribution in *Historical Review of Elementary Concepts In Physics*. The current article provides an exposition of the modern description of interactions on a more formal level.

The theory of quantum fields is the result of merging quantum mechanics with special relativity. Non-relativistic quantum mechanics describes the dynamics and interactions of a fixed number of particles. In contrast, special relativity requires a description allowing for processes which change the number of particles involved. This can already be seen from the fact that the relativistic relation.

$$\omega^2(k) = \mathbf{K}^2 + m^2 \tag{1}$$

for a particle of mass *m*, momentum **k**, and energy *E* allows positive as well as negative energies. If one now expands a free charged quantum field  $\psi(x)$  into its energy – momentum eigenfuctions  $u_{\omega(k),k}(X) \alpha \exp(ik X)$  and interprets the coefficients  $a_K$  of the positive energy solutions as annihilator of a particle in mode K, then the coefficients  $b_K^{\dagger}$  of the negative energy contributions have to be interpreted as creators of antiparticles of opposite charge,

$$\psi(x) = \sum_{\mathbf{k},\omega(k)>0} a_{\mathbf{k}} u_{\omega(k)}, \mathbf{k} e^{-i\omega(k)t} + \sum_{\mathbf{k},\omega(k)<0} b_{K}^{\dagger} u_{\omega(\kappa)}, \mathbf{k} e^{i\omega(k)t}.$$
(2)

Therefore, for each process involving factors of  $\psi(x)$  and conserving the number of particles, there are process producing pairs of particles and antiparticles, thus changing the total number of particles. Canonical quantization, discussed in Set.2.3 below shows, that the creators and annihilators indeed satisfy the expected relations,

$$\left[a_{i,\kappa},a_{i',\kappa'}^{\dagger}\right]_{\pm} = \left[b_{i,\kappa}b_{i'\kappa'}^{\dagger}\right]_{\pm} = \delta_{ii'}\delta(\mathbf{k}\cdot\mathbf{k}')$$
(3)

where *i*,*i*', now denote integral degrees of freedom such as spin, and  $[.,..]\pm$  denotes the commutator for bosons, and the anti-commutator for fermions, respectively.

All free non-interacting quantum fields are the form Eqs.(2) and (3).Interesting dynamics only occurs in the presence of interactions. Their quantum mechanical and relativistic description is the subject of the present contribution.

In the following, indices  $\mu, \nu$  etc. from the second half of the Greek alphabet label space-time indices, whereas indices  $\alpha, \beta$  etc. from the first half of the Greek alphabet represent gauge degrees of freedom in particular, and Latin indices a,b,etc. are related to general internal degrees of freedom or spinor indices. If not indicated otherwise, identical indices (both Latin and Greek) appearing as upper and lower indices are summed over. As usual, space –time indices are lowered and raised with the metric tensor  $g_{\mu\nu}$  and its inverse  $g^{\mu\nu} \equiv g_{\mu\nu}^{-1}$  respectively. We also use natural units, h = c = k = 1, in which all units are expressed in terms of energy units.

### 2. Description of Interactions in Quantum Mechanics and Quantum Field Theory

We know today that all interactions on the fundamental level should be described within the framework of quantum mechanics. We therefore start with a short review of basic concepts of quantum mechanics.

### 2.1 The Action Principle

On the fundamental level of a quantum mechanical system can be described within the *Hamiltonian* or *Lagrangian* formalism. Symmetries are more transparent in the Lagrangian formalism, so we will use the latter here. In general the *Lagrangian* is a functional  $L[\psi(t), \dot{\psi}(t)]$  of fields  $\psi_i(\mathbf{x}, t)$  and their time derivates  $\dot{\psi}_i(\mathbf{x}, t)$  at a given time *t*. Here , **x** is a space coordinate, and i is some discrete internal coordinate labeling particle type, indicates related to symmetry groups such as Lorentz –indices for non-trivial representations of the group of rotations and Lorentz-boosts, or indices related to internal symmetries such as color or electroweak charge (see further below). To avoid cluttering we will sometimes suppress **x** and *i*. the canonically conjugated momenta are defined as

$$\pi^{i}(\mathbf{x},t) \equiv \frac{\delta L[\psi(t),\dot{\psi}(t)]}{\delta \dot{\psi}_{i}(\mathbf{x},t)}$$
(4)

The action is then defined as the time integral of the Lagrangian,

$$S[\psi] \equiv \int_{-\infty}^{+\infty} dt L[\psi(t), \dot{\psi}(t)].$$
<sup>(5)</sup>

Demanding that the action is external under all infinitesimal variations  $\delta \psi(t)$  which vanishes for  $t \to \pm \infty$  yields the field equations, or equations of motion,

$$\dot{\pi}^{i}(\mathbf{x},t) = \frac{\delta L \left[\psi(t), \dot{\psi}(t)\right]}{\delta \psi_{i}(\mathbf{x},t)}$$

#### 2.2 Symmetries of the Action

Lorentz invariance suggests that the action should be the space-time integral of a scalar function of the fields  $\psi_i(\mathbf{x},t)$  and their space-time derivatives  $\partial_{\mu}\psi_i(\mathbf{x},t)$ , and thus the Lagrangian should be the space-integral of a scalar called the Lagrangian density L,

$$S[\psi] = \int d^4 x L[\psi_i(x), \partial_\mu \psi_i(x)], \qquad (7)$$

where  $x \equiv (x, t)$  from now on. In this case, the equations of motion Eq.(6) read

$$\partial_{\mu} \frac{\partial L}{\partial \left(\partial_{\mu} \psi_{i}\right)} = \frac{\partial L}{\partial \psi_{i}},\tag{8}$$

which are called *Euler-Lagrange* equations and are obviously Lorenz invariant if L is a scalar.

Symmetries can be treated in very transparent way in Lagrangian formalism. Assume that the action is invariant,  $\delta S = 0$  independent of whether  $\psi_i(x)$  satisfy the field equations or not, under a global symmetry transformation,

$$\delta\psi_i(x) = i \in \mathcal{F}_i\left[\psi_j(x), \partial_\mu \psi_j(x)\right],\tag{9}$$

for which  $\in$  is independent of x. here and in the following explicit factors of *i*, denote the imaginary unit, and not an index. Then, for a space-time dependent  $\in (x)$ , the variation must be of the form

$$\delta S = -\int d^4 x J^{\mu} \Big[ x, \psi_j (x), \partial_{\mu} \psi_j (x) \Big] \partial_{\mu} \in (x)$$
(10)

But if the fields satisfy their equations of motion,  $\delta S = 0$ , and thus

$$\partial_{\mu}J^{\mu}\left[x,\psi_{j}\left(x\right),\partial_{\mu}\psi_{j}\left(x\right)\right]=0,$$
(11)

which implies Noethers theorem, the existence of one conserved current  $J^{\mu}$  for each continuous global symmetry. If Eq. (9) leaves the Lagrangian density itself invariant, an explicit formula for  $J^{\mu}$  follows immediately,

$$J^{\mu} = -i \frac{\partial L}{\partial \left(\partial_{\mu} \psi_{i}\right)} \mathcal{F}_{i}$$

$$\tag{12}$$

where we drop the field arguments from now on.

An important example is invariance of the action *S* under space-time translations, for which  $\mathcal{F}_i = -i\partial_\mu \psi_i(x)$  in Eq. (9). In this case, the currents  $J^\mu$  for each  $\mu$  are given by

(13)

$$T^{\nu}_{\mu} = \delta^{\nu}_{\mu} \mathcal{L} - \frac{\partial \mathcal{L}}{\partial (\partial_{\nu} \psi_i)} \partial_{\mu} \psi_i,$$

which represents the energy momentum tensor.

# 2.3 Canonical and Path Integral Quantization

So far our discussion applies both to classical as well as quantum fields. In order to make the transition to quantum field theory, two main approaches are used in the literature. The first one is known as *canonical quantization* and elevates the fields  $\psi_i(x)$  and  $\pi_i(x)$  to operators  $\Psi_i(x)$  and  $\prod_i(x)$  acting on a Hilbert space of physical states, ad obeying the commutation relations

$$\left[\Psi_{i}(\mathbf{x},t),\prod^{j}(\mathbf{y},t)\right] = i\delta^{3}(\mathbf{x}\cdot\mathbf{y})\delta_{i}^{j}$$
(14)

The second approach is *path integral quantization* and is equivalent to canonical quantization. In this approach the expectation value of products of general operators  $\mathcal{O}[\Psi_i(x)]$  between the vacuum state at  $t = \pm \infty$ ,  $|0, \pm \infty\rangle$ , is given by a functional integral over c-number fields  $\psi_i(x)$  (which anti-commute in case of fermions),

$$\left\langle 0, +\infty \left| T \left\{ \mathcal{O}_{a} \left[ \Psi_{a} \left( t_{a} \right) \right] \mathcal{O}_{b} \left[ \Psi_{b} \left( t_{b} \right) \right] ... \right\} \right| 0, \infty \right\rangle$$

$$\propto \int \prod_{x,i} d\psi_{i} \left( x \right) \mathcal{O}_{a} \left[ \psi_{a} \left( t_{a} \right) \right] \mathcal{O}_{b} \left[ \psi_{b} \left( t_{b} \right) \right] ...$$

$$\exp \left[ i \int d^{4} y L \left[ \psi_{j} \left( y \right), \partial_{\mu} \psi_{j} \left( y \right) \right] \right]$$

$$(15)$$

where  $\Psi_a(t_a) \equiv \Psi_{ia}(x_a)$  etc., and  $T\{...\}$  signals time ordering with  $t_a > t_b > ...$ , Here we have assumed that *L* is at most quadratic in the derivates  $\partial_{\mu}\psi_i$ .

In general these path integrals cannot be evaluated exactly. However, one can perform a *perturbation expansion* in the following way: let us split the action into a free part  $S_0[\psi]$  quadratic in  $\psi_i(x)$ , and an interacting part  $S_1[\psi]$ , and write

$$S_0[\psi] = -\frac{1}{2} \int d^4x d^4y \sum_{i,i'} \mathcal{D}_{ix,i'y} \psi_i(x) \psi_{i'}(y) .$$
 (16)

The path integral for this quadratic part can then be reduced to Gaussian integrals for an arbitrary number of fields under the expectation value,

$$\frac{\int \left(\prod_{x,i} d\psi_i(x)\right) \psi_{i1}(x_1) \psi_{i,2}(x_2) \dots \exp\left(S_0[\psi]\right)}{\int \left(\prod_{x,i} d\psi_i(x)\right) \exp\left(S_0[\psi]\right)} = \sum_{\substack{x \in \mathcal{I} \\ \text{field} \\ \text{pairings}}} \prod_{\substack{\text{pairs} \\ ix, i'x'}} \left(-i\mathcal{D}_{ix, i'x'}^{-1}\right) \tag{17}$$

where  $-i\mathcal{D}^{-1}$  is called the *propagator*. The exponential of the full action can then be expanded into powers of  $S_1[\psi]$ , which contain *vertices* of at least three free fields, and the resulting path integrals can be evaluated using Eq.(17). Terms in that expansion that do not contain any space-time (or energy momentum in the Fourier transformed picture) integrations are said to be on the *tree or classical level*. All other terms contain such integrations or *loops* and are due to quantum fluctuations around the classical approximation. If one keeps Planck's constant  $\hbar$ , the umber of loops contributing to a given term in the perturbative expansion of Eq.(15) depends linearly on the power of  $\hbar$  in this term: The action S appears as divided by  $\hbar$  because it has the units of energy times time, thus each factor  $S_1$  contributes  $\hbar^{-1}$ , whereas according to Eqs (16) and (17) each propagator contributes  $\hbar$ , but the number of propagators minus the number of vertices, at which the space time coordinates of all fields coincide, is linear in the number of independent loops. Planck's constant therefore characterizes the size of quantum effects.

#### 2.4. Space-Time Symmetries and Their Representations

The Poincare' group is the symmetry group of special relativity and consists of all transformations leaving invariant the metric

$$ds^{2} = -\left(dx^{0}\right)^{2} + \left(dx^{1}\right)^{2} + \left(dx^{2}\right)^{2} + \left(dx^{3}\right)^{2},$$
(18)

where  $x^0$  is a time coordinate and  $x^1, x^2$  and  $x^3$  are Carstesian space coordinates. These transformations are the of the form

$$x'^{\mu} = \Lambda^{\mu}_{\nu} x^{\nu} + a^{\mu} \tag{19}$$

where  $a^{\mu}$  defined arbitrary space-time translations, and the constant matrix  $\Lambda^{\mu}_{v}$  satisfies

$$\eta_{\mu\nu}\Lambda^{\mu}_{p}\Lambda^{\nu}_{\sigma} = \eta_{p\sigma} \tag{20}$$

where  $\eta_{\mu\nu} = \text{diag}(-1,1,1,1)$ . The unitary transformations on fields and physical states  $\psi$  induced by Eq.(19) satisfy the composition rule

(21)

$$U(\Lambda_2 a_2)U(\Lambda_1 a_1) = U(\Lambda_2 \Lambda_1, \Lambda_2 a_1 + a_2).$$

Important subgroups are defined by all elements with  $\Lambda = 1$  (the commutative group of translations) and by all elements with  $a^{\mu} = 0$  [the homogenous Lorentz group SO(3,1) of matrices  $\Lambda^{\mu}_{\nu}$  satisfying Eq.(20).]. the latter contains the subgroup SO(3) of all rotations for which  $\Lambda^{0}_{0} = 1, \Lambda^{\mu}_{0} = \Lambda^{0}_{\mu} = 0$  for  $\mu = 1, 2, 3$ .

The general infinitesimal transformations of this type are characterized by an antisymmetric tensor  $\omega_v^{\mu}$  and a vector  $\in^{\mu}$ .

$$\Lambda^{\mu}_{\nu} = \delta^{\mu}_{\nu} + \omega^{\mu}_{\nu} a^{\mu} = \epsilon^{\mu}$$
(22)

any element  $U(1+\omega, \epsilon)$  of the Poincare group which is infinitesimally close to the unit operator can then be expanded into the corresponding Hermitian generators  $J^{\mu\nu}$  and  $P^{\mu}$ ,

$$U(1+\omega,\epsilon) = 1 + \frac{1}{2}i\omega_{\mu\nu}J^{\mu\nu} - i\epsilon_{\mu}P^{\mu}$$
<sup>(23)</sup>

It can be shown that these generators satisfy the commutation relations

$$i \begin{bmatrix} J^{\mu\nu}, J^{p\sigma} \end{bmatrix} = \eta^{\nu} J^{\mu\sigma} - \eta^{\mu p} J^{\nu\sigma} - \eta^{\sigma\mu} J^{p\nu} + \eta^{\sigma\nu} J^{p\mu}$$
$$i \begin{bmatrix} P^{\mu}, J^{p\sigma} \end{bmatrix} = \eta^{\mu p} P^{\sigma} - \eta^{\mu\sigma} P^{p}$$
$$\begin{bmatrix} P^{\mu}, P^{\nu} \end{bmatrix} = 0.$$
(24)

The  $P^{\mu}$  represent the energy-momentum vector, and since the Hamiltonian  $H \equiv P^0$  commutes with the spatial pseudo-three-vector  $J \equiv (J^{23}J^{31}J^{12})$ , the latter represents the angular –momentum which generates the group of rotations SO(3).

Fields and physical states can thus be characterized by their energy-momentum and spin, which characterize their transformations properties under the group of translations and under the rotation group, respectively. Let us first focus on fields and stats with non-vanishing mass. In this case one can perform a Lorentz boost into the rest frame where  $P^{\mu} = (M, 0, 0, 0)$  with M the mass of the state  $P^{\mu}$  is then invariant under the rotation group SO(3). The irreducible unitary representations of this group are characterized by a integer r half –integer valued spin j such that the 2j+1 states are characterized by the eigenvlaues of  $J_i$  which run over -j, -j+1, ..., j+1, j note that an eigenstate with eigenvalue  $\sigma$  of  $J_i$  is multiplied by a phase factor  $e^{2\pi i \sigma}$  under a rotation around the *i*-axis by  $2\pi$ , and a half –integer spin state thus changes sign. Given the fact that a rotation by  $2\pi$  is the identity this may at first seem surprising. Note, however that normalize states in quantum mechanics are only defined up to phase factors and thus a general unitary projective representation of a symmetry group on the Hilbert space of states can in general include phase factors in the composition rules such as Eq.(21). This is indeed the case for the rotation group SO(3) which is isomorphic to  $S_3/Z_2$  the threedimensional sphere in Euclidean four-dimensional space with opposite points identified, and thus doubly connected. This means that closed curves winding n times over a closed path are continuously contractible to a point of n is even, but are not otherwise. Half integer spins then correspond to representations for which  $U(\Lambda_1)U(\Lambda_2) = (-)^n U(\Lambda_1\Lambda_2)$ , where *n* is the winding number along the path from 1 to  $\Lambda_1$ , to  $\Lambda_1, \Lambda_2$  and back to 1, whereas integer spins do not produce a phase factor.

With respect to homogenous Lorentz transformations, there are then two groups of representations. the first one is formed by the tensor representations which transform just as products of vectors,

$$W_{\nu\dots}^{\prime\,\mu\dots} = \Lambda_p^{\mu} \Lambda_{\nu}^{\sigma} \dots W_{\sigma\dots}^{p\dots}$$
(25)

These represent bosonic degrees of freedom with maximal integer spin j given by the number of indices. The simplest case is complex spin-zero scalar  $\phi$  of mass *m* whose standard free Lagrangian

$$L_{\phi} = -\frac{1}{2} \Big( \partial_{\mu} \phi^{\dagger} \partial^{\mu} \phi - m^2 \phi^{\dagger} \phi \Big)$$
<sup>(26)</sup>

leads to an equation of motion Eq.(8) known as Klein -Gordon equation.,

$$\left(\partial_{\mu}\partial^{\mu} - m^2\right)\phi = 0. \tag{27}$$

Its free solutions are of the general form Eq.(2). By comparing Eqs.(16) and (26) one finds that the propagator of a scalar of mass *m* is  $-i/(p^2 - m^2)$  in four momentum space. In the static case  $p^0 = 0$  this leads to an interaction potential.

$$V(r) = g_1 g_2 \frac{e^{-mr}}{r} \tag{28}$$

between two "charges"  $g_1$ , and  $g_2$  at which the propagator ends at which correspond to the vertices from the pertubation part  $S_1$  of the action. The potential for the exchange of bosons of non-zero spin involve some additional factors for the tensor structure. Note that the *range* of the potential is given by  $\approx m^{-1}$ . Constraints on a potential *fifth force* beyond the known four fundamental interactions are usually expressed in terms of g and m.

The second type of representation of the homogeneous Lorentz group can be constructed from any set of *Dirac matrices*  $\Gamma^{\mu}$  satisfying the anti-commutation relations.

$$\left\{\Gamma^{\mu},\Gamma^{\nu}\right\} = 2\eta^{\mu\nu} \tag{29}$$

also known as Clifford algebra. One can then show that the matrices

$$J^{\mu\nu} \equiv -\frac{i}{4} \Big[ \Gamma^{\mu}, \Gamma^{\nu} \Big] \tag{30}$$

indeed obey the commutation relations in Eq.(24). The objects on which these matrices act are called *Dirac spinors* and have spin  $\frac{1}{2}$ . In 3+1 dimensions, the smallest representation has four complex components, and thus  $\Gamma^{\mu}$  are 4×4 matrices. The standard free Lagrangian for a spin -  $\frac{1}{2}$  Dirac spinor  $\psi$  of mass *m*,

$$L_{\psi} = \overline{\psi} \Big( \Gamma^{\mu} \partial_{\mu} + m \Big) \psi \,, \tag{31}$$

where  $\overline{\psi} \equiv \psi^{\dagger} i \Gamma^0$ , leads to an equation of motion Eq.(8) known as *Dirac equation*,

$$\left(\Gamma^{\mu}\partial_{\mu} + m\right)\psi = 0 \tag{32}$$

Its free solutions are again of the form Eq.(2) and represent fermionic degrees of freedom.

It is easy to see the matrix

$$\Gamma_5 \equiv -i\Gamma^0 \Gamma^1 \Gamma^2 \Gamma^3 \tag{33}$$

is a pseudo –scalar because the spatial  $\Gamma^i$  change sign under parity transformation, and satisfies

$$\Gamma_{5}^{2} = 1 \quad \left\{ \Gamma^{\mu}, \Gamma_{5} \right\} = 0 \left[ J^{\mu\nu}, \Gamma_{5} \right] = 0.$$
(34)

A four –component Dirac spinor  $\psi$  can then be split into two inequivalent Weyl representations  $\psi_L$  and  $\psi_R$  which are called left-chiral and right –chiral,

$$\psi = \psi_L + \psi_R \equiv \frac{1 + \Gamma_5}{2} \psi + \frac{1 - \Gamma_5}{2} \psi.$$
(35)

Note that according to Eqs. (34) and (35) the mass term in the Lagrangian Eq. (31) flips chirality, whereas the kinetic term conserves chirality.

The general irreducible representations of the homogenous Lorentz group are then given by arbitrary direct products of spinors and tensors. We note that massless from representations of the group SO(2) leaving invariant  $P^{\mu}$ , instead of SO(3). The group SO(2) has only one generator which can be identified with *helicity*, the projection of spin onto three-momentum. For fermions this is the chirality defined by  $\Gamma_5$  above.

Most of what was discussed in this section carries over directly to an arbitrary number D of space-time dimensions, where the homogeneous Lorentz group is SO(1, D-1). The spatial rotation group for massive and massless particles are SO(D-1) and SO(D-2), with D-1 and D-2 components for each vector representation, respectively. As can be seen by pairing dimensions, a D-dimensional Dirac spinor has  $2^{[D/2]}$  complex components where [...] denotes the integer part. If D is even, the chirality  $\Gamma \equiv i^{-[D/2]+1}\Gamma^0\Gamma^1...\Gamma^{D-1}$ , corresponding to  $\Gamma_5$  for D=4, allows again to split Dirac spinors  $\psi$  into two Weyl spinors  $\psi_L$  and  $\psi_R$ . If D=2 or D=10 one can simultaneously define real (Majorana) spinors. These two cases play an important role in string theory.

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