THE PHYSICS OF STARS

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Summary

In this chapter we describe the physics required to understand the structure and evolution of stars. It begins with a description of how we measure a star's luminosity and temperature from its brightness and color. The four basic equations of stellar structure that describe mass, hydrostatic equilibrium, heat transport and energy generation are then derived in one dimension. We consider the various contributions of gas, electron degeneracy and radiation, along with minor corrections to the equation of state. Radiation transport and convective transport of heat are examined in some detail and we explain the various sources of energy generated in a star, paying particular attention to nuclear burning. The stellar atmosphere as a boundary condition to our interior models is briefly discussed and then we go on to describe how stars evolve. Finally we discuss white dwarfs, neutron stars and black holes, along with supernovae as the end points of stellar evolution.

1. Introduction

The physics of stars is in one sense simple because the high interior temperatures preclude the complexities of chemistry. On the other hand every aspect of physics from high energy nuclear to solid state is involved as a star lives out its complex and varied life. Stars have fascinated mankind since ancient times but it is only in the last century that we have begun to fully understand them and only in the last fifty years that we have been able to model their evolution in quantitative detail. Indeed there are many aspects of their evolution that still await elucidation. Here we give a comprehensive overview of the basic physics needed to make stellar models and describe the structure and evolution of typical stars.

2. Observable Quantities

When we look at stars in the night sky they have two apparent properties: they vary in brightness and color. The brightness is assessed in terms of magnitudes. Historically, and we are going back to the ancient Greeks here, stars fall into six magnitude classes. The brightest stars are of first magnitude and the faintest stars visible to the naked eye are sixth magnitude, though these are rarely visible with today's city lights. The eye measures brightness logarithmically so that a star of magnitude 6.0 turns out to be one hundred times fainter than a star of magnitude 1.0. Modern photometry can measure the magnitude of stars extremely accurately and in different wavelength ranges.

But these magnitudes are only apparent. A star can vary in brightness for two reasons. First it may be brighter because it is intrinsically more luminous. Alternatively it might just be brighter because it is close to us. Indeed, in the eighteenth century, the astronomer Sir William Herschel, like Sir Isaac Newton before him, hoped that all stars were of similar intrinsic luminosity so that he might map the Galaxy by taking variations in brightness to indicate variations in distance. By the turn of the century he had recognized that some double stars of very different brightness are gravitationally bound and therefore near neighbors. These he called binary systems. Indeed the Revd. John Michel had already noted that this must be the case simply because there are more close pairs of stars on the sky than would be expected in a random distribution.

Today we know that binary stars are very common. Our nearest neighbor, α-Centauri, is a binary star and has even a third fainter companion known as Proxima Centauri that currently lies on the nearside of the system. In fact the Sun is relatively unusual in not having a companion. About nine out of ten stars have stellar companions. Half of these are close enough to affect one another by tides at some point in their lives. Some, closer still, end up transferring matter between them but this is all another fascinating story. First we must understand how stars evolve in isolation. Today the distances to nearby stars can be determined by accurate trigonometric parallaxes. The motion of the star is measured against the background of distant, apparently immovable, stars and galaxies as the Earth moves around its orbit. Once the distance is known an absolute magnitude can be calculated from the observed apparent magnitude and from this we get an estimate of the luminosity of the star.

The second observable quantity is a star's color. Some stars, such as Betelgeuse, appear red while others, like Sirius, are distinctly blue. The color of a star is related to its surface temperature. The apparent surface or photosphere of a star is the locus of points at which the majority of photons were last emitted or scattered before they began their journey through space to the Earth. Typically the spectrum of radiation emitted by a star is close to that of a black body. The hotter the black body the bluer is the peak in its spectrum.

Thus blue stars are hot while red stars are relatively cool. Another way to determine the surface temperature of a star is to look at the dark lines in its spectrum. These generally occur at wavelengths where an atomic transition of an electron makes the absorption of a photon particularly favorable. The light traveling from the photosphere directly towards us is absorbed and re-emitted in all directions so that the line appears dark against the uninhibited background. Historically spectra were classified by the strength of their hydrogen lines. Those with the strongest hydrogen lines are of type A while those with the weakest are of type M. Hydrogen ionizes at about 10,000K and it is stars of this temperature that have the most prominent hydrogen lines. As the temperature rises fewer and fewer atoms have bound electrons and the lines disappear from the spectra. As the temperature falls the electrons around the hydrogen nuclei become more and more energetically confined to the ground state orbits. This in turn leads to fewer weaker hydrogen lines in the spectra. However lines from the more weakly bound electrons of other atoms and molecular rotation and vibration bands become more prominent. So it is easy to distinguish the very hot O stars from the relatively very cool M stars. The sequence of spectral types from the hottest to the coolest normal stars follows

O B A F G K M.

Once we know the temperature and nature of a star's atmosphere we can relate its absolute magnitude to a bolometric luminosity. This bolometric luminosity L is the total energy radiated by the star per unit time.

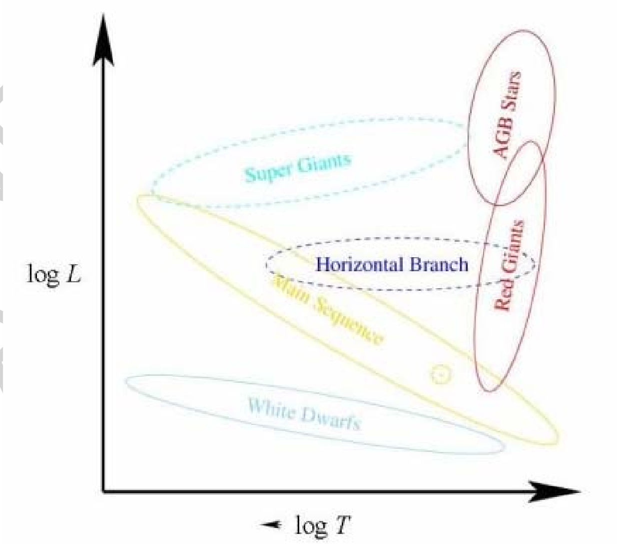

Figure 1. A schematic Hertzsprung-Russell diagram showing the position of stars in a surface temperature-luminosity or color magnitude diagram. Temperature increases from right to left along the horizontal axis. Color changes from blue to red from left to right. Most stars, like the Sun, lie along the main sequence but other distinct groups of stars are visible, particularly when the diagrams are plotted for clusters of stars.

In the early years of the twentieth century Henry Norris Russell, an American who worked partly in Cambridge at the time, and the Danish astronomer and chemist Ejnar Hertzsprung examined the correlations of these two quantities with each other. The resulting Hertzsprung-Russell diagram has become the major tool for describing the evolution of stars over their lifetimes. Rather than populating the whole of such a diagram we find that most of the stars lie on a band running from hot bright stars to cool faint stars (Fig. 1). The Sun takes its place in one of the most populated parts of this band which is called the main sequence. Because the radiation from stars is very close to a black body the temperature of the photosphere is close to the effective temperature T_{eff} .

The total luminosity L is the energy flux, that depends only on the effective temperature for a blackbody, multiplied by the area of the star.

$$L = 4\pi\sigma R^2 T_{\mathrm{eff}}^4 , \quad (1)$$

where σ is the Stefan-Boltzmann constant and R is the radius of the photosphere. So the loci of stars of constant radius are straight lines of slope -4 in the Hertzsprung-Russell diagram. This means that stars at the top left of the main sequence are blue giants or supergiants while those at the bottom right are red dwarfs. In a diagram of the brightest stars another region to the right and nearly vertically upwards from the main sequence is prominent. These are the red giants. In diagrams of globular clusters this giant branch splits into two distinct parts, the normal red giants and the asymptotic giant branch. We shall see later how these are populated by stars in quite distinct evolutionary phases. In Hertzsprung-Russell diagrams of nearby stars the fainter but relatively common white dwarfs appear in a band below the main sequence. Also discernible as separate, though not so distinct regions, are the supergiants from blue to red across the very top of the diagram and the subgiants between the main sequence and the true red giants.

Many stars are actually members of clusters. Often all the stars in the cluster formed together and so today they have the same age. There are open clusters, such as the Pleiades and Hyades, that are easily found in the northern sky. These tend to contain thousands of stars relatively loosely bound and tend to be rather young. Their age can be determined by the extent of the main sequence because the more massive stars, which are bluer and brighter, use up their energy first and then evolve away (Section 9). Globular clusters, such as 47 Tucanae and the more unusual ω Centauri easily found in the Southern hemisphere, or M13 in the constellation of Hercules in the northern, are much denser and older. They have the advantage that the stars, numbering millions, all lie at approximately the same distance so that relatively, though not absolutely, the errors associated with distance measurements are significantly reduced. Today some very beautiful Hertzsprung-Russell diagrams of globular clusters have been plotted with data from large telescopes and these reveal all sorts of interesting details. Of particular note is the horizontal branch at relatively constant luminosity extending from red to blue across from the red giants. The structure and population of this feature varies considerably from cluster to cluster and contains clues to the age and initial chemical composition of the constituent stars. Because the Sun lies right in the middle of the most populated part of the main sequence we can deduce that it is typical of the majority of stars. In the next sections we shall investigate the physics and the mathematical models that have allowed us to unravel the life of a star as it moves about the Hertzsprung-Russell diagram from the main sequence to the red giant branch, perhaps to the horizontal branch or back to the subgiant area, then on to the asymptotic giant branch and, in the case of the Sun, finally to a white dwarf (Section 11). A white dwarf remnant is left whenever the star has lost enough mass to avoid a supernova explosion (Section 12). More massive stars end their lives as the even more exotic neutron stars or black holes.

3. Structural Equations

The structure of a star can be described remarkably well by a one dimensional model with spherical symmetry. At a radius r from the centre the properties of the star are identical over the surface of the sphere described by r = const. This structure can be described in essence with four differential equations. Two of these, that describe the variation of mass and pressure with radius, can be called the structural equations. They are the subject of this section. With any equation of state that can relate pressure to density they form the building blocks of a stellar model.

The first equation is easily derived by considering a thin shell of mass δm and thickness δr at radius r in the star (Fig. 2). The mass in the shell is just its volume multiplied by the local density $\rho(r)$ and when we take the limit as δr tends to zero we obtain

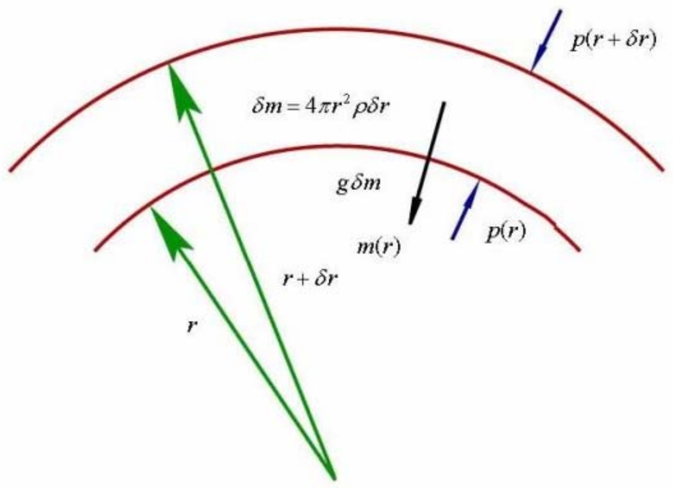

Figure 2. The structure of a fluid sphere. The mass enclosed by a spherical surface of radius r is $m(r)$. A shell of mass δm and thickness δr at this radius is supported against gravity by a pressure gradient.

$$\frac{dm}{dr} = 4\pi r^2 \rho, \tag{2}$$

the mass equation in which the mass $m = m(r)$ is the mass interior to the sphere of radius r.

The mass inside this shell exerts on it an attractive radial force of magnitude $g\delta m = 4\pi r^2 \rho g \delta r$, where $g(r) = Gm/r^2$ is the local gravitational acceleration. This must be balanced by the difference in the force of the pressure P on either side of the shell $4\pi r^2 \left(P(r+\delta r) - P(r)\right)$. Again in the limit as δr tends to zero we obtain

$$\frac{dP}{dr} = -\frac{Gm\rho}{r^2}. \tag{3}$$

This is the equation of hydrostatic equilibrium. Equations (2) and (3) are special spherically symmetric cases of the more general equations of mass conservation and the Euler momentum equation of fluid dynamics when the velocity in the fluid is everywhere zero and remains zero.

If we can write P explicitly as a function of ρ only we can obtain a full solution to the structure of the star. The simplest boundary conditions to apply are, at $r = 0$,

$$m(0) = 0 \tag{4}$$

and, because the gravity vanishes at the origin by symmetry,

$$\frac{dP}{dr} = 0 \tag{5}$$

and, at $r = R$,

$$m(R) = M \qquad \rho(R) = 0, \tag{6}$$

where M is the total mass of the star. It turns out that the equation of state of very degenerate matter takes just such a form and white dwarfs (Section 11) can be modeled with just these equations.

4. Equation of State

In practice pressure does not depend only on density. Fig. 3 illustrates the various contributions to the pressure as temperature and density vary. Typically the state of stellar material depends on its composition and any two state variables. The relation between all the state variables is what we call the equation of state. In general there are many contributions to the equation of state but for most normal stars the fluid behaves very much like an ideal gas, for which the pressure may be written as a function of density, temperature T and mean molecular weight μ,

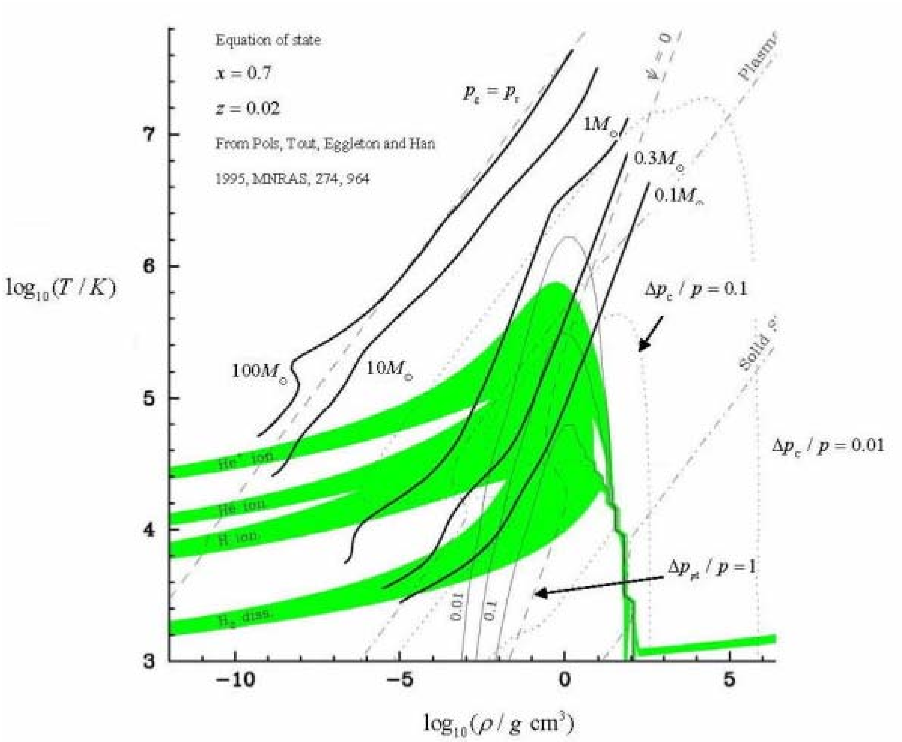

Figure 3. Contributions to the equation of state as a function of temperature and density. The thick solid lines are the run of temperature and density through zero-age main-sequence stars of masses 0.1, 0.3, 1, 10 and 100 $M_\odot$. Their centers are towards the top right of the figure. A dashed line marks where gas and radiation pressure are equal with increasing P_r/P_g to the left. A second dashed line indicates where the electron chemical potential $\psi = 0$. To the right of this line material becomes progressively more electron degenerate. The shaded regions represent the range over which ionization of H, He and He^+ and dissociation of molecular hydrogen take place. Thin solid lines indicate the effects of pressure ionization and dotted lines other corrections. These lines are labeled in the sense that, when $\Delta P_I/P = 1$ then the process of ionization doubles the total pressure, while $\Delta P_C/P = 0.1$ means that plasma effects change the pressure by one tenth etc. Dot-dashed lines indicate when the fluid can be considered a plasma and when it begins to crystallize into the solid state.

$$P = \rho \frac{\Re T}{\mu} \tag{7}$$

where $\Re$ is the gas constant per unit mass (*Note that this is not the usual gas constant per mole and care must be taken with the units used*). The mean molecular weight is the reciprocal of the number of particles, each of which contributes to the pressure equally at a given temperature, per atomic mass unit. Thus neutral hydrogen contributes one particle for each mass unit and has $\mu = 1$ while fully ionized hydrogen contributes two particles, an electron and a proton, for each mass unit and so has $\mu = 1/2$. Fully ionized helium contributes two electrons and a helium nucleus made up of two protons and two neutrons for its four mass units and so has $\mu = 4/3$. Anything heavier than hydrogen and helium is designated a metal and when fully ionized contributes approximately half as many particles as its atomic mass because the nucleus typically consists of equal numbers of protons and neutrons, especially for the most abundant ^{12}C, ^{14}N and ^{16}O, and each positively charged proton is balanced by an electron. Thus metals have $\mu \approx 2$. For a fully ionized mixture, adding the numbers and masses, we find

$$\frac{1}{\mu} = 2X + \frac{3}{4}Y + \frac{1}{2}Z, \tag{8}$$

where X is the mass fraction of hydrogen, Y is that of helium and Z that of all metals and $X+Y+Z=1$. In the deep interiors of stars temperatures are such that nearly all atoms are ionized but as the temperature falls so electrons recombine with their nuclei to form atoms in various ionization stages. The most strongly bound electrons recombine at the highest temperatures. Thus in the Sun hydrogen recombines between about 10,000 and 20,000K while iron is still not completely ionized at 1,000,000 K.

An important consequence of Eq. (8) is that the equation of state changes as nuclear reactions convert one element to another. This is the major driving force behind stellar evolution and is responsible for the Sun gradually expanding and brightening with time. The Sun's radius is currently growing at just under one inch per year.

At high temperatures the pressure exerted by energetic photons becomes comparable with that exerted by the particles and we must include a radiation pressure term

$$P_{\mathrm{r}} = \frac{1}{3}aT^4 \tag{9}$$

where a is the radiation constant. As stars become hotter so radiation pressure becomes more important. Its effect is quite negligible in the Sun but for a $100\,M_{\odot}$ star it contributes about half of the total pressure throughout the star on the main sequence.

At high densities electrons contribute a degeneracy pressure. This arises because free electrons must occupy a discrete set of momentum states and, as the volume to which an electron is confined is reduced, so the energies of its available states increase. Thus squeezing an electron gas increases the momenta of the electrons and this requires energy. So work must be done and the gas exerts a force against compression. The contribution to this degeneracy pressure P_{e} can be evaluated according to the Fermi-Dirac distribution in statistical physics. It becomes important when the electron chemical potential ψ becomes positive. There is a small contribution in the core of the Sun, and lower mass main-sequence stars, and electron degeneracy dominates the pressure in white dwarfs where it provides sufficient support against gravity even when the gas is cold. Although we might expect a cold gas to consist of neutral atoms this is not the case at very high densities because the nuclei are so close to one another, much nearer than the radius of an atom, that the electrons are not bound to a particular nucleus but behave as a free gas similar to those in metallic elements at room temperature. This effect, known as pressure ionization, is also important to some extent in the Sun.

There are various other corrections to the pressure that must be included such as plasma effects at high densities and eventually liquefaction and crystallization to the solid state as density increases and temperature falls. The coolest white dwarfs have a crystalline structure that can be seen in the seismic oscillations which we are beginning to observe in great detail. For most of stellar evolution we can construct a very good model with only a meager equation of state that incorporates the physics appropriate to the state of the material. However if we are to use the detail of oscillation frequencies that can now be obtained with asteroseismology we must use a very accurate equation of state. Indeed it is often found that new additions must be made to fit the observations sufficiently well.

5. Radiation Transport

When temperature is important for the equation of state we require two further equations to describe the star. The first is for the temperature gradient. This depends on the rate at which energy can be transported from where it is generated, usually at the hot centre, through the star. One of three processes dominates energy transport under different conditions. Radiation, or the diffusion of photons, dominates in the central parts of the Sun. Conduction, or the diffusion of particles, is prevalent in degenerate material. Convection, or energy transport by bulk fluid motion, operates when the temperature gradient becomes too large for stable radiative transfer. This is the case in the outer layers of the Sun.

In radiative regions we can estimate the temperature gradient by considering two surfaces of different temperatures separated by the small distance λ that a photon typically moves between interactions with the matter and over which it maintains memory of the conditions when it last interacted (Fig. 4). Deep in the star everything is in local thermodynamic equilibrium so that a surface at temperature T emits energy as a blackbody with a flux of energy per unit area of $F = \sigma T^4$, where the Stefan Boltzmann constant $\sigma = ac/4$, a is the radiation constant and c is the speed of light. Consider two such surfaces, one at temperature T and one at $T + \delta T$. In our spherically symmetric star the surfaces are spheres of area $4\pi r^2$ and T usually decreases as r increases. We call the net energy flow through a sphere of radius r the local luminosity L_r and we have

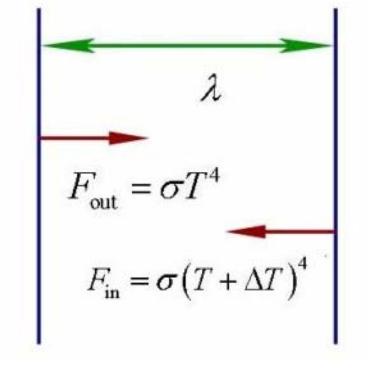

Figure 4. Radiation diffuses through the star. The interior of the star is locally in thermodynamic equilibrium so that the radiation flux emitted by any surface depends on the temperature of that surface. Photons travel until they are absorbed or scattered,

typically a mean free path length from where they were emitted or last scattered. In this way heat diffuses from hotter to cooler regions. To approximate the process we consider two surfaces at temperatures T and $T+\delta T$ separated by a distance λ. Both surfaces emit as black bodies and the difference $F_{\text{out}}-F_{\text{in}}$ between their fluxes gives rise to the luminosity.

$$L_r = 4\pi r^2 \left(F_{\text{out}} - F_{\text{in}}\right), \tag{10}$$

where

$$F_{\text{out}} - F_{\text{in}} \approx -4\sigma T^3 \delta T \tag{11}$$

is the difference between the inward flux from the surface at temperature $T+\delta T$ and the outward from the surface at T. The difference in temperature is just the temperature gradient multiplied by the distance between the surfaces

$$\delta T = \lambda \left(\frac{dT}{dr}\right). \tag{12}$$

So we have

$$L_r \approx -16\pi\sigma r^2 \lambda T^3 \left(\frac{dT}{dr}\right). \tag{13}$$

Note that L_r is usually positive because dT/dr is usually negative.

The typical distance traveled by a photon between interactions, its mean free path, depends on the opacity of the material κ. Opacity is defined to be the effective cross-section per unit mass seen by a photon, that is the effective target area of the absorber or scatterer seen by photons divided by its mass and summed over all appropriate absorbers and scatterers. The probability of interaction of a photon passing along a cylinder (Fig. 5) of cross-section equal to κ times the mass in the cylinder and length λ is unity. Thus for material of density ρ we have

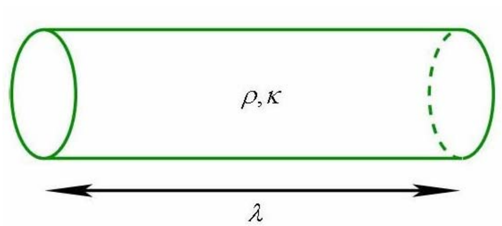

Figure 5. The relation between mean free path and opacity. A photon is likely to be absorbed or scattered once within a cylinder of length λ and cross-sectional area κm, aligned with its motion, that contains one target of mass m.

$$\rho\kappa\lambda = 1. \tag{14}$$

Combining this with Eq. (13) we find

$$\frac{dT}{dr} = \frac{-\kappa\rho L_r}{4\pi acr^2T^3}\,. \qquad (15)$$

We have been rather vague about how we averaged the paths of all photons but we have in fact done it correctly! We should have considered surfaces separated by the distance l traveled by each photon and then weighted each contribution to the luminosity by the number of photons traveling a distance in a small range $l+\delta l$ but this average just gives us the mean free path λ. However our formula (15) is not quite correct because we have not taken proper account of the fact that the radiation field from a point on a surface is isotropic and not directed towards the other surface. Taking this into account, and with somewhat more effort, we can obtain

$$\frac{dT}{dr} = \frac{-3\kappa\rho L_r}{16\pi acr^2T^3} \qquad (16)$$

which is the equation of radiative transfer.

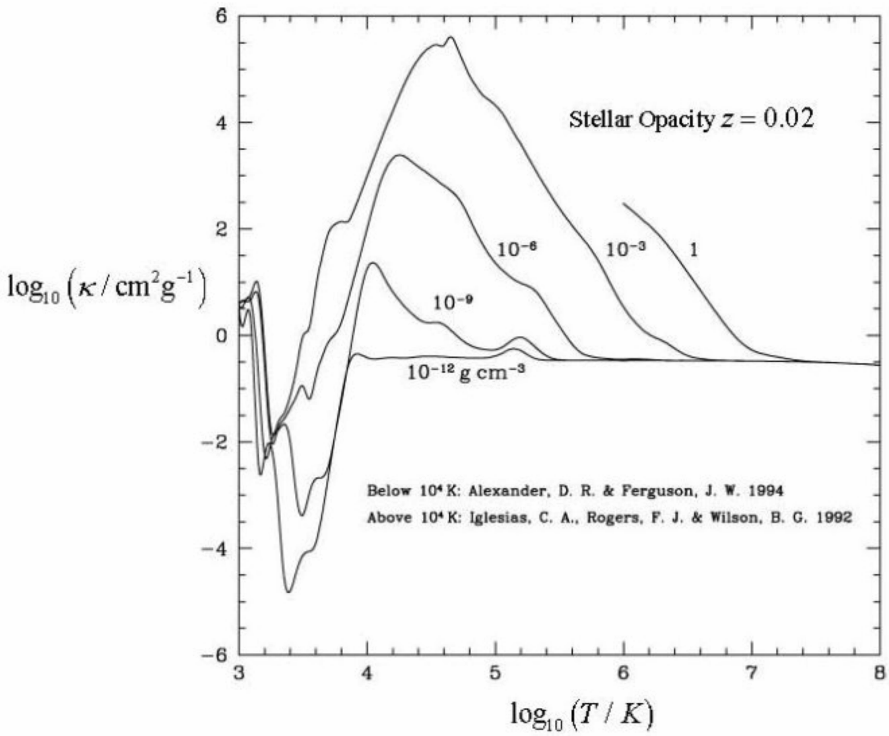

Figure 6. Opacity as a function of temperature for various stellar densities.

The detailed calculation of opacity is a long and complex procedure. Fig. 6 illustrates how it varies with temperature and density in stellar material. At high temperatures all material is ionized and the only source of opacity is scattering by electrons which is independent of temperature and density until at very high temperatures when the relativistic effects of Compton scattering become important. At intermediate temperatures atomic processes, where electrons are moved from one state to another by absorption of a photon, dominate. The states may be either bound or free and a dependence

$$\kappa \propto \rho T^{-3.5} \tag{17}$$

emerges. Just above 10,000K the opacity drops rapidly with decreasing temperature as hydrogen recombines and fewer and fewer photons have sufficient energy to change the electronic states. At lower temperatures it begins to rise again as H^- ions and various molecules become important sources. The calculation becomes even more complex when we must include molecular vibration and rotation.

Conductivity can be described in a similar way with electrons replacing the photons as the energy carriers. Usually the mean free path of electrons is much shorter than that of photons. This means that their effective opacity is much larger so that radiation transport dominates. However in degenerate material electrons are not easily scattered because they must end up in an empty momentum state but all neighboring momentum states are already occupied. The mean free path becomes very large and the fluid is effectively superconducting. In practice this means that degenerate regions of stars are close to isothermal.

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Biographical Sketch

Christopher Tout was born in Billericay, England on 25th April 1964. He was educated at Brentwood School and Emmanuel College, Cambridge. He has a BA (1985), MA (1989) and PhD (1989) in astronomy from the University of Cambridge. His major field of study is stellar evolution.

He is the John Couch Adams Reader in Astronomy at the University of Cambridge and a Fellow of Churchill College. He was previously a NATO Fellow at Lick Observatory, a Research Fellow at Sidney Sussex College, a Postdoctoral Fellow at the Space Telescope Science Institute, a Royal Society Overseas Fellow at Konkoly Observatory and a SERC/PPARC Advanced Fellow. He is a regular visitor to the Department of Mathematics at Monash University and at the Australian National University. He has published over one hundred articles in learned journals. His research interests include interacting binary stars, accretion discs and flows, magnetic fields in fluids and the origin of the elements.

Dr Tout is a Fellow of the Royal Astronomical Society. He is a keen cyclist with a general love of the outdoors and the natural world.