STRUCTURAL STABILITY

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Keywords: Structural stability, bifurcation instability, limit point instability, finite disturbance instability, snap-through instability, columns, beams, plates, shells, trusses, boundary condition effects, eigenvalue problem, differential equation approach, energy approach, Galerkin's approach, finite element method, numerical methods, notional load, effect of geometrical imperfections, effect of material inelasticity, design for stability.

Contents

Summary

Fundamental concepts of structural stability as applied to columns, beams, plates, shells and trusses are discussed. Analytical and numerical methods to obtain critical or buckling load are presented. The effects of boundary conditions, geometrical imperfections and material inelasticity are briefly described. Application of stability concepts to structural design is addressed.

1. Introduction

Structural stability is a field of mechanics that studies the behavior of structures under compression. When a structure is subjected to a sufficiently high compressive force (or stress), it has a tendency to lose its stiffness, experience a noticeably change in geometry, and become unstable. When instability occurs, the structure loses its capacity to carry the applied loads and is incapable of maintaining a stable equilibrium configuration. Examples of structural instability include: buckling of a column under a compressive axial force, lateral torsional buckling (LTB) of a beam under a transverse load, sideways buckling of an unbraced frame under a set of concentric column forces, buckling of a plate under a set of in-plane forces, and buckling of a shell under longitudinal or axial stress, etc.

2. Types of Instability

Instability can generally be classified into: Bifurcation instability, limit point instability, finite disturbance instability, and snap-through instability. A short description of each is given below.

2.1. Bifurcation Instability

Bifurcation instability refers to the scenario when deformation that occurs in one direction suddenly changes to another direction. An example is that of a perfectly straight column subject to a concentric compressive load. When the load is first applied, the column shortens or experiences axial deformation in the direction of the applied force. When the applied load gradually increases, there comes a point when the mode of deformation suddenly switches from one of axial to one of lateral in which the column buckles in a direction perpendicular to the direction of the applied force. The load at which this occurs is referred to as the bifurcation, or critical, load.

Bifurcation instability can be symmetric or asymmetric. As shown in Figure 1, for symmetric bifurcation the secondary equilibrium path (i.e., the equilibrium path that corresponds to the buckled configuration of the structure) is symmetric about the primary equilibrium path (i.e., the equilibrium path that corresponds to the pre-buckled configuration of the structure). The symmetric bifurcation is *stable* if the secondary equilibrium path rises above the critical load (Curve a), and it is *unstable* if the secondary equilibrium path drops below the critical load (Curve b). Examples of structures that exhibit stable symmetric bifurcation are elastic buckling of a perfectly straight slender column subjected to a concentric compressive force and a geometrically perfect thin plate subjected to an in-plane compressive force. An example of unstable symmetric bifurcation is the elastic buckling of a guyed tower.

In asymmetric bifurcation, the secondary equilibrium path is not symmetric about the primary equilibrium path (Curve c). An example of asymmetric bifurcation is a geometrically perfect L-shaped frame subject to a concentric column axial force. The secondary equilibrium path drops below or rises above the critical load depending on whether the frame buckles in a direction that results in the beam shear acting down or up on the column, respectively.

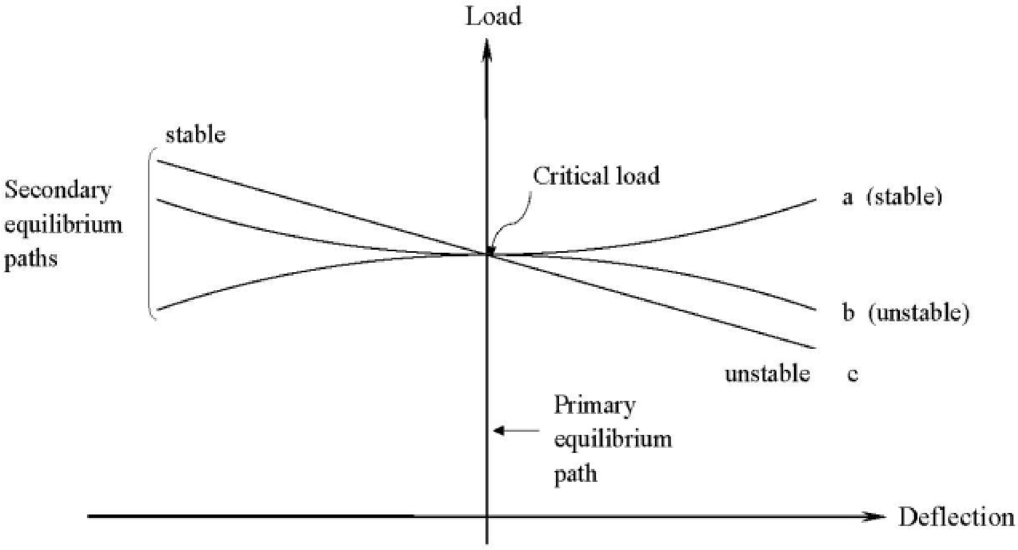

Figure 1. Bifurcation instability

2.2. Limit Point Instability

Limit point instability refers to the scenario when a single deformation mode exists throughout the load history. The deformation increases when the load increases from start of loading to final failure. The load-deflection behavior of a structure that experiences limit point instability is shown in Figure 2. The maximum load that the structure can carry before failure is referred to as the limit load. Examples of structures that exhibit limit point instability are geometrically imperfect (crooked) columns subject to concentric compressive forces, and frames subject to gravity loadings that are eccentric to the longitudinal axes of the columns.

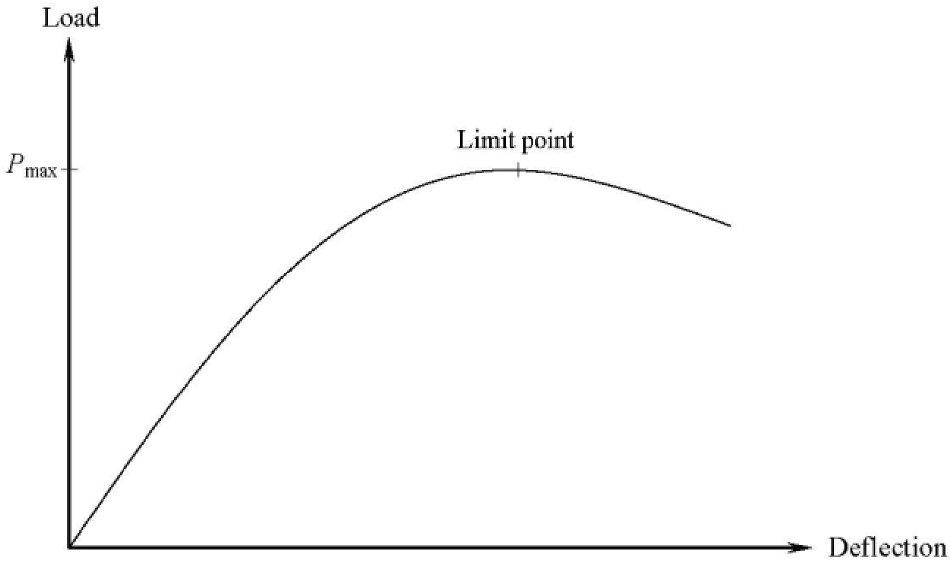

Figure 2. Limit point instability

2.3. Finite Disturbance Instability

Finite disturbance instability occurs when a compressive force is applied along the longitudinal or axial direction of a thin-walled cylinder shell. As depicted schematically in Figure 3, the load deflection curve rises to the (theoretical) critical load N_{cr}, then drops suddenly to a lower value in order for the structure to maintain equilibrium. The value of N_{cr} has been shown by Donnell and Wan to be very sensitive to the initial geometrical imperfections present in the shell. The slightest imperfections drastically reduce N_{cr}.

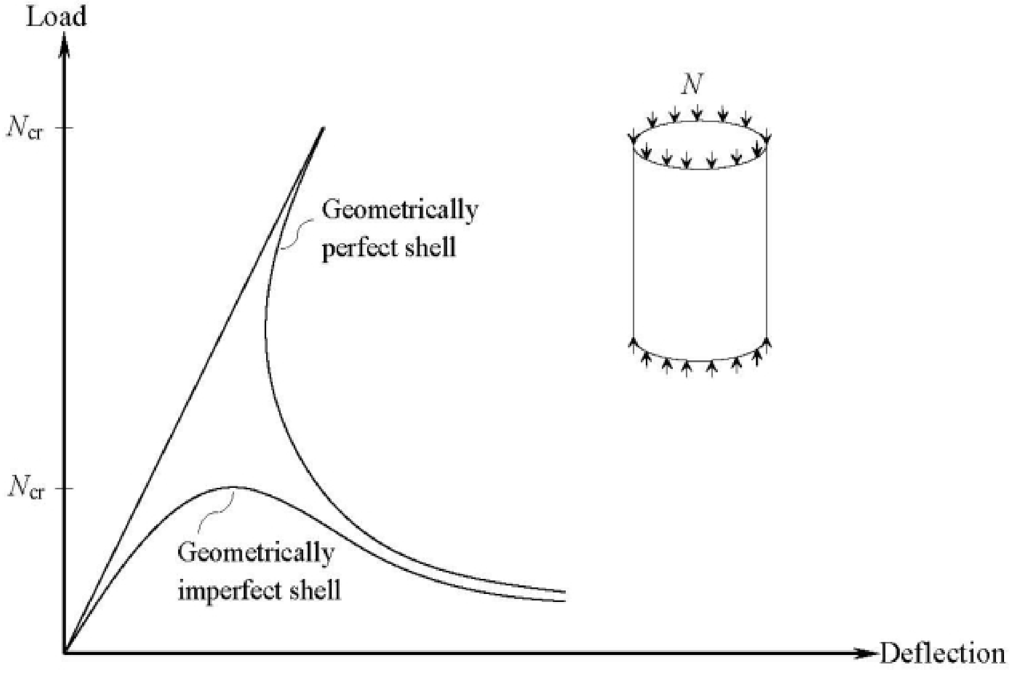

Figure 3. Finite disturbance instability

2.4. Snap-Through Instability

Snap-through instability is characterized by a sudden increase in deformation under a constant load.

For some systems, such as in-plane buckling of a shallow truss or arch subject to a transverse load, and buckling of a shallow spherical cap subject to a radial load, once a load reaches a certain value as denoted by point A in Figure 4, the system can maintain equilibrium only if the displacement snaps suddenly from point A to point B as shown by the solid horizontal line.

The dotted curved line represents an unstable equilibrium state and can be observed only if the system is subjected to an ideal displacement controlled condition.

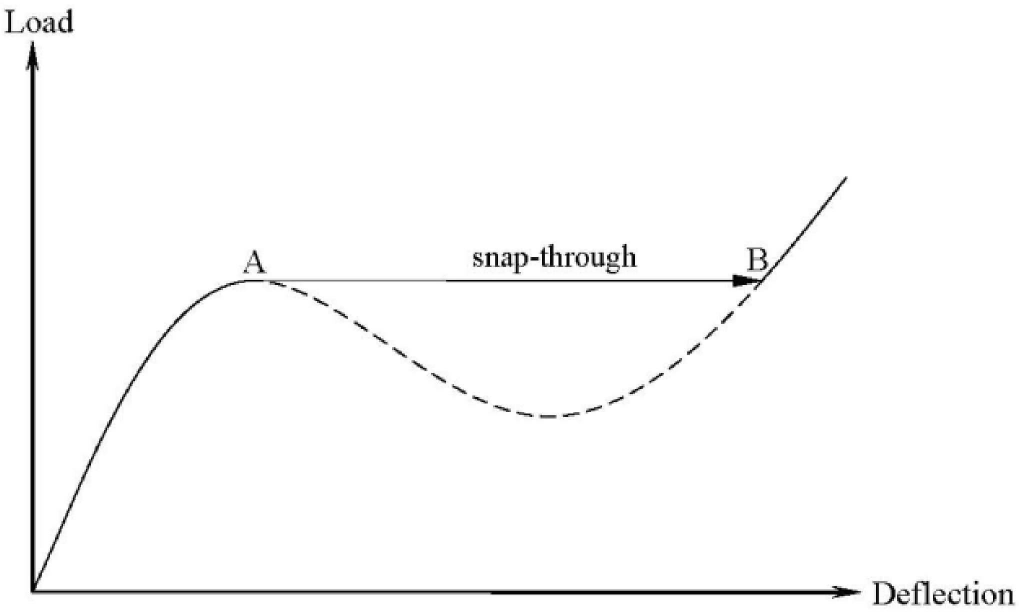

Figure 4. Snap-through instability

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Biographical Sketch

Dr. Eric M Lui is Laura J. and L. Douglas Meredith Professor in the Department of Civil & Environmental Engineering at Syracuse University. He received his BSCE with high honors from the University of Wisconsin-Madison, and his MSCE and Ph.D. from Purdue University. Professor Lui's research and teaching interests are in the area of Structural Engineering with an emphasis on structural stability, structural dynamics, structural materials, numerical methods, computer-aided analysis and design, and earthquake engineering.