TURBULENCE

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Keywords: Turbulence, vortex dynamics, energy cascade, turbulence model, large eddy simulation

Contents

Summary

The scope of this chapter is to recall some of the bases of turbulence theory and modeling methods. The emphasis is put on turbulent flow features that are of primary interest for turbulent flow prediction and modeling: turbulent scales, energy cascade, vorticity, turbulence production and dissipation. Then, some basics notions about turbulence modeling are reviewed and the advantages and limitations of the more frequently used types of turbulence models are discussed. The reader interested in more in-depth discussions of the subject is referred to relevant textbooks.

1. What is Turbulence?

Most fluid flows occurring in nature as well as in engineering applications are turbulent. Even though many turbulent flows can be easily observed, it is very difficult to give an accurate and generally accepted definition of turbulence: actually, many definitions have been given along more than a century of turbulence studies, reflecting progresses in turbulence understanding. However, researchers and engineers generally agree on some characteristics of turbulent flows, i.e. on their general phenomenology.

1.1. Phenomenology

Turbulence or turbulent flow is a flow regime characterized by chaotic, stochastic fluctuations, in space and time, of the flow properties. These fluctuations cover a large range of space and time scales, ranging from the macroscopic scale, which essentially depends on the problem geometry, to the molecular scale, dominated by viscous effects, which contribute to dissipate the kinetic energy of turbulent fluctuations into heat. When present, turbulence usually dominates all other flow phenomena such as mixing, heat transfer, and drag. The lack of a satisfactory understanding of turbulence represents one of the great remaining fundamental challenges to scientists –and to engineers as well– since most technologically important flows are turbulent. Because of these difficulties, turbulence is often considered the last major unsolved problem in classical physics.

It is often claimed that there is no good definition of turbulence, and many researchers are inclined to forego a formal definition in favor of intuitive characterizations. One of the best known of these is due to Richardson, in 1922:

"*Big whorls have little whorls,*
which feed on their velocity;
And little whorls have lesser whorls,
And so on to viscosity."

This reflects the physical notion that mechanical energy injected into a fluid is generally on fairly large length and time scales, but this energy undergoes a "cascade" whereby it is transferred to successively smaller scales until it is finally dissipated (converted to thermal energy) on molecular scales. This description underscores a second physical phenomenon associated with turbulence: dissipation of kinetic energy.

In a 1937 lecture von Kármán defined turbulence by quoting G.I. Taylor as follows:

"*Turbulence is an irregular motion*
which in general makes its appearance in
fluids, gaseous or liquid, when they flow
past solid surfaces or even when
neighboring streams of the same fluid
flow past or over one another."

That definition is acceptable but is not completely satisfactory, since many irregular flows cannot be considered turbulent. To be turbulent, they must have certain stationary statistical properties analogous to those of fluids when considered on the molecular scale. Hinze, in one of the most widely-used texts on turbulence, offers a more accurate definition, which takes into account statistical aspects:

"*Turbulent fluid motion is an irregular condition of the flow in which the various quantities show a random variation with time and space coordinates, so that statistically distinct average values can be discerned.*"

Note that none of the preceding definitions offers any precise characterization of turbulent flow in the sense of predicting, *a priori*, on the basis of specific flow conditions, when turbulence will or will not occur, or what would be its extent and intensity.

The Navier-Stokes (N-S) equations, which are now almost universally believed to embody the physics of all fluid flows (within the confines of the continuum hypothesis), including turbulent ones, were introduced in the early to mid 19th Century by Navier and Stokes, and contain *a priori* all necessary information to describe the fluid motion accurately. For an incompressible flow of a constant density fluid, they may be written in vector notation as:

$$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} = -\frac{\mathbf{1}}{\rho} \nabla p + \nu \nabla^2 \mathbf{u} \qquad (1)$$

where $\mathbf{u}$ is the velocity vector, which must satisfy the incompressibility condition $\nabla \cdot \mathbf{u} = 0$, ρ is the density, p is the pressure, ν is the fluid cinematic viscosity, and ∇ is the classical nabla differential operator. In Eq. (1) the right-hand side represents the acceleration of a fluid particle, whereas terms at the right-hand side represent, respectively, pressure and viscous forces.

The N-S equations are nonlinear and difficult to solve. As is well known, just a few exact solutions are available, and all of these have been obtained at the expense of introducing simplifying, often physically unrealistic, assumptions. Thus, little progress in the understanding of turbulence can be obtained via analytical solutions to these equations, and as a consequence early descriptions of turbulence were based mainly on experimental observations.

In Cartesian components, Eq. (1) becomes:

$$\frac{\partial u_i}{\partial t} + u_j \frac{\partial u_i}{\partial x_j} = -\frac{1}{\rho} \frac{\partial p}{\partial x_i} + \nu \frac{\partial^2 u_i}{\partial x_j \partial x_j}, \qquad (2)$$

with the incompressibility condition

$$\frac{\partial u_i}{\partial x_i} = 0.$$

In the above, Einstein's summation convention has been used (According to which terms where an index appears twice, imply a summation over that index. For instance, given two vectors of the three-dimensional space **x**=(x_1, x_2, x_3) and **y**=(y_1, y_2, y_3), then their inner product $\mathbf{x}\cdot\mathbf{y}=\sum_{i=1}^{3}x_i y_i$ can be re-written using Einstein's shorthand notation as $\mathbf{x}\cdot\mathbf{y}=x_i y_i$ where the summation symbol has been omitted).
Introducing a suitable reference velocity scale U and length scale L, the Navier-Stokes equations can be recast in non dimensional form:

$$\frac{\partial u_i}{\partial t}+u_j\frac{\partial u_i}{\partial x_j}=-\frac{\partial p}{\partial x_i}+\frac{1}{Re}\frac{\partial^2 u_i}{\partial x_j \partial x_j}. \qquad (3)$$

From Eq. (3) it appears that, in the absence of body forces, the only free parameter in the N-S equations is the Reynolds number

$$Re=\frac{UL}{\nu},$$

which represents the ratio of inertial to viscous forces in the flow and is strongly related to the first appearance of turbulent flow. For "small" values of the flow Reynolds number (where the concept "small" is strongly problem-dependent) the flow is dominated by viscous diffusion, and the N-S equations admit stable "regular" solutions, such that flow properties vary in an "ordered" way in space and time. In this case, the flow is said to be *laminar*. For higher values of the Reynolds number, inertial forces dominate the flow and the flow becomes unstable. In these conditions, both velocity and pressure fluctuations (and, for compressible flows, density fluctuations) appear, and the flow field becomes essentially three-dimensional and unsteady. When this phenomenon occurs, the flow is said to be *turbulent*.

The fact that the deterministic Navier-Stokes equations, completed by deterministic initial and boundary conditions, may give rise to (at least apparently) random solutions may seem contradictory. Such a dilemma has been solved by Lorentz in 1963, when it was shown that some nonlinear systems of ordinary differential equations may be so extremely sensitive to initial conditions that an infinitesimal perturbation gives quickly rise to solutions that are completely different point wise, even if they display the same statistical properties (for instance, mean values). In the case of fluid motion, since it is impossible to reproduce a flow field with infinite experimental or numerical accuracy, two realizations of a given flow field will never be identical and, if the flow Reynolds number is sufficiently high, the two flows will be characterized, locally, by completely different values of the instantaneous flow properties.

1.2. Some Notions in Hydrodynamic Stability and Transition

Turbulence has its origins in the inherent instabilities of laminar flow. In flows which are originally laminar, turbulence arises from instabilities at large Reynolds numbers. A flow becomes unstable when it undergoes a suitable perturbation (where how "suitable" depends on the kind of flow and on its Reynolds number).

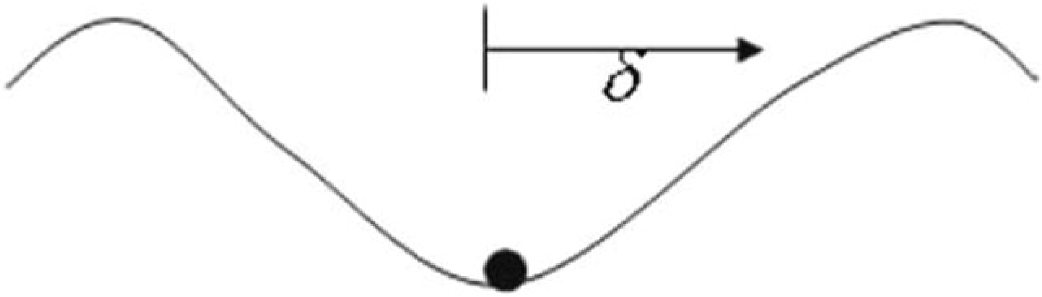

Figure 1. Simple example illustrating the notion of stability.

For instance, let us consider a point-like mass lying in a "valley" (potential energy minimum) as in Figure 1: if the mass is displaced by a small quantity δ from its equilibrium position, it tends to return to it as soon as the perturbation is removed. If however, the perturbation amplitude is large enough, the mass leaves the valley and returns back no more to its previous equilibrium position. Similarly, a realizable fluid flow has not only to be a valid solution of the Navier-Stokes equations, but also a stable one for the flow conditions considered. Using the words of Landau and Lifshits (1959):

"*Yet not every solution of the equation of motion,*
even if it is exact, can actually occur in Nature.
The flow that occurs in Nature must not only obey
the equations of fluid dynamics but also be stable."

For each given laminar flow, it is possible to find a finite value of the Reynolds number, referred-to as the *critical Reynolds number*, beyond which the flow may no longer exist in reality. Going further with the previous example, note that it is possible to establish some criterion to predict if the configuration will have a stable or unstable behavior under a given perturbation; however, nothing can be said about the new equilibrium state that will be reached due to an unstable perturbation. Similarly, for viscous fluid flows, it can be proved that a given laminar flow is unstable beyond a given critical Reynolds number; however, stability analysis does not predict turbulence. Rather, turbulence is something that is observed experimentally. It has never been demonstrated mathematically that turbulence is the stable state of a given flow at high Reynolds numbers. Also note that, in general, turbulence cannot maintain itself, but depends on its environment to obtain energy. A common source of energy for turbulent fluctuations is shear in the mean flow. If turbulence is generated in an environment where there is no shear or other maintenance mechanism, it decays, and the flow tends to become laminar again.

Experiments have shown that transition to turbulence is commonly initiated by a primary instability mechanism, which in simple cases is two-dimensional. The primary mechanism produces secondary motions, which are generally three-dimensional and become unstable themselves, generating further instabilities, and so on. The sequence of instabilities generates intense localized three-dimensional disturbances, usually called *turbulent spots*, which arise at random times and positions. These spots grow quickly and merge each other forming at last a developed turbulent flow. Figure 2 shows an example of this mechanism for a flat plate boundary layer.

A more general mathematical definition of turbulence within the context of deterministic instabilities of the Navier-Stokes equations may be formulated as follows:

"*Turbulence is any chaotic solution to the 3-D Navier-Stokes equations that is sensitive to initial data and which occurs as a result of successive instabilities of laminar flows as a bifurcation parameter is increased through a succession of values.*"

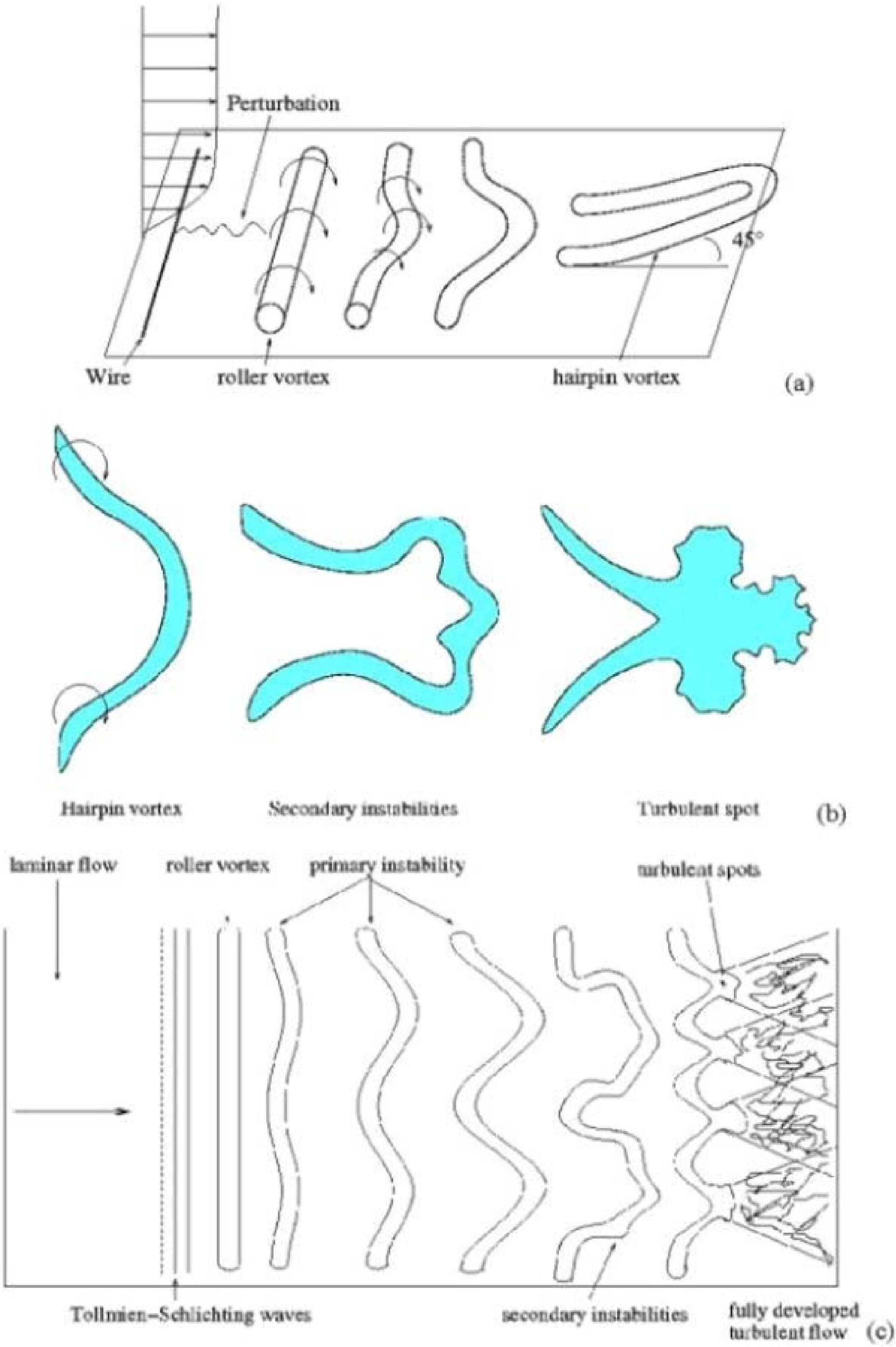

Figure 2: a) Growth of instabilities in a flat-plate boundary layer; b) turbulent spot formation; c) transition to turbulence of a boundary layer.

While this definition is still somewhat vague, it contains specific elements that permit detailed examination of flow situations relating to turbulence. First, it acknowledges that turbulent flow is a solution of the N-S equations. Second, it requires that the fluid behavior be chaotic, i.e., erratic, irregular, as required in earlier definitions, but deterministic and not random, coherently with the deterministic nature of the governing equations. Third, it states that turbulence is three dimensional. This is consistent with the common classical viewpoint where generation of turbulence is ascribed to vortex stretching and tilting which can only occur in 3D flow, as will be discussed in more detail below. Finally, the preceding definition also imposes a requirement of sensitivity to initial data which allows one to distinguish highly irregular laminar motion from actual turbulence.

The notion of loss of stability of the laminar flow regime has both classical and modern roots. Stability analyses in the context of, mainly, normal mode analysis has been a mainstay in studies of fluid motion for at least a century, and their connections to transition to turbulence were already made in boundary layer studies. The modern contribution is to embed such approaches within bifurcation theory, thus opening the way to use of many powerful mathematical tools of modern analysis of dynamical systems.

1.3. Properties of Turbulent Flow

Once single perturbations have grown and multiplied such that they cover a continuous spectrum of frequencies, a velocity sensor in some given point of the flow field will detect a signal that is no longer “ordered”, i.e. can no longer be described in a deterministic way, but looks like a random signal.

A list of characteristic features of turbulent flows is reported below. For more detail the reader is referred, e.g. to the classical textbook by Tennekes and Lumley (1972).

- **Irregularity.** The flow is irregular, chaotic. This explains why statistical methods are often considered. Nevertheless, it is deterministic, and not random, and it is described by the Navier-Stokes equations. The flow exhibits a large spectrum of different length and time scales (eddy sizes).
- **Unsteadiness.** A fluctuating signal necessarily implies flow *unsteadiness*.
- **Three-dimensionality.** A turbulent flow is necessarily *three-dimensional*, since fluctuations in all space dimensions are equally probable. Actually, the major mechanism for perturbation propagation from one component to another in the frequency spectrum is the vortex stretching mechanism, which vanishes in two-dimensional flow, as it will be discussed later. This implies that turbulence is not only three dimensional but also necessarily *rotational*, i.e. $\nabla \times \mathbf{u} \neq 0$.
- **Dissipation.** Turbulence is a highly dissipative phenomenon since energy injected in an “ordered” form at a given frequency, is then distributed to a myriad of vortices that becomes smaller and smaller up to the molecular scale. At this level, the injected energy only contributes to intensify intermolecular collisions, i.e. system temperature. In other words, energy is converted into heat.
- **Diffusivity.** The turbulence increases the exchange of momentum, also increasing the resistance (wall friction). More generally, turbulence leads to enhanced mixing

(heat and mass transfer), with respect to laminar flow.

1.4. Brief Historical Survey

The earliest recognition of turbulence as a distinguished physical phenomenon is to be ascribed to Leonardo da Vinci (circa 1500). Figure 3 shows some sketches by Leonardo illustrating turbulent flows with hierarchically organized vortices in such a way that there are vortices within vortices. In practice however, no substantial progress in understanding turbulence has been done until the late 19th Century. The first observations of turbulent flow in a scientific sense were described by Hagen in 1839. He was studying flow of water through round tubes and observed two distinct kinds of flow, which are now known as laminar (or Hagen-Poiseuille) and turbulent. If the flow was laminar as it left the tube, it looked clear like glass; if turbulent, it appeared opaque and frosty. In 1854, he published a second paper showing that viscosity as well as velocity influenced the boundary between the two flow regimes. Another important contribution was due to Boussinesq who, in the year 1877, suggested that turbulent stresses are linearly proportional to mean strain rates. The Boussinesq assumption is still the cornerstone of most turbulence models. In 1883, Osborne Reynolds introduced a dimensionless parameter –the above-mentioned Reynolds number- that gave a quantitative indication of the laminar to turbulent transition. In an experiment, Reynolds demonstrated that, under certain circumstances, the flow in a tube changes from laminar to turbulent over a given region of the tube. He used a large water tank that had a long tube outlet with a stopcock at the end of the tube to control the flow speed. A thin dye was injected into the flow at the tube inlet. When the speed of the water flowing through the tube was low, the filament of colored fluid maintained its identity for the entire length of the tube. However, when the flow speed was high, the filament broke up into the turbulent flow that existed through the cross section. Looking at the flow by means of a stroboscope, it appeared that the dye filament had broken up into many vortices of fluctuating direction and intensity. Reynolds showed that the transition from one flow regime to the other did not depend on the flow velocity alone, but on combination of velocity, the fluid cinematic viscosity, and a flow characteristic length, which has since been known as the Reynolds number. A sketch of Reynolds experiment is provided in Figure 4. These experimental results and analyses set the “way of seeing” turbulence for many years to come. In particular, Reynolds concluded that turbulence was far too complicated ever to permit a detailed understanding, and proposed to use a statistical approach: he introduced the decomposition of flow variables into mean and fluctuating parts, which has resulted in a century of study in an effort to arrive at usable predictive techniques based on this viewpoint. Beginning with this work the prevailing view has been that turbulence is a random phenomenon, and as a consequence there is little to be gained by studying its details, especially in the context of engineering analyses.

Approximately the same time as Reynolds was proposing a random description of turbulent flow, the French mathematician Jules Henri Poincaré was finding that relatively simple nonlinear dynamical systems were capable of exhibiting chaotic random-in-appearance behavior that was, in fact, completely deterministic.

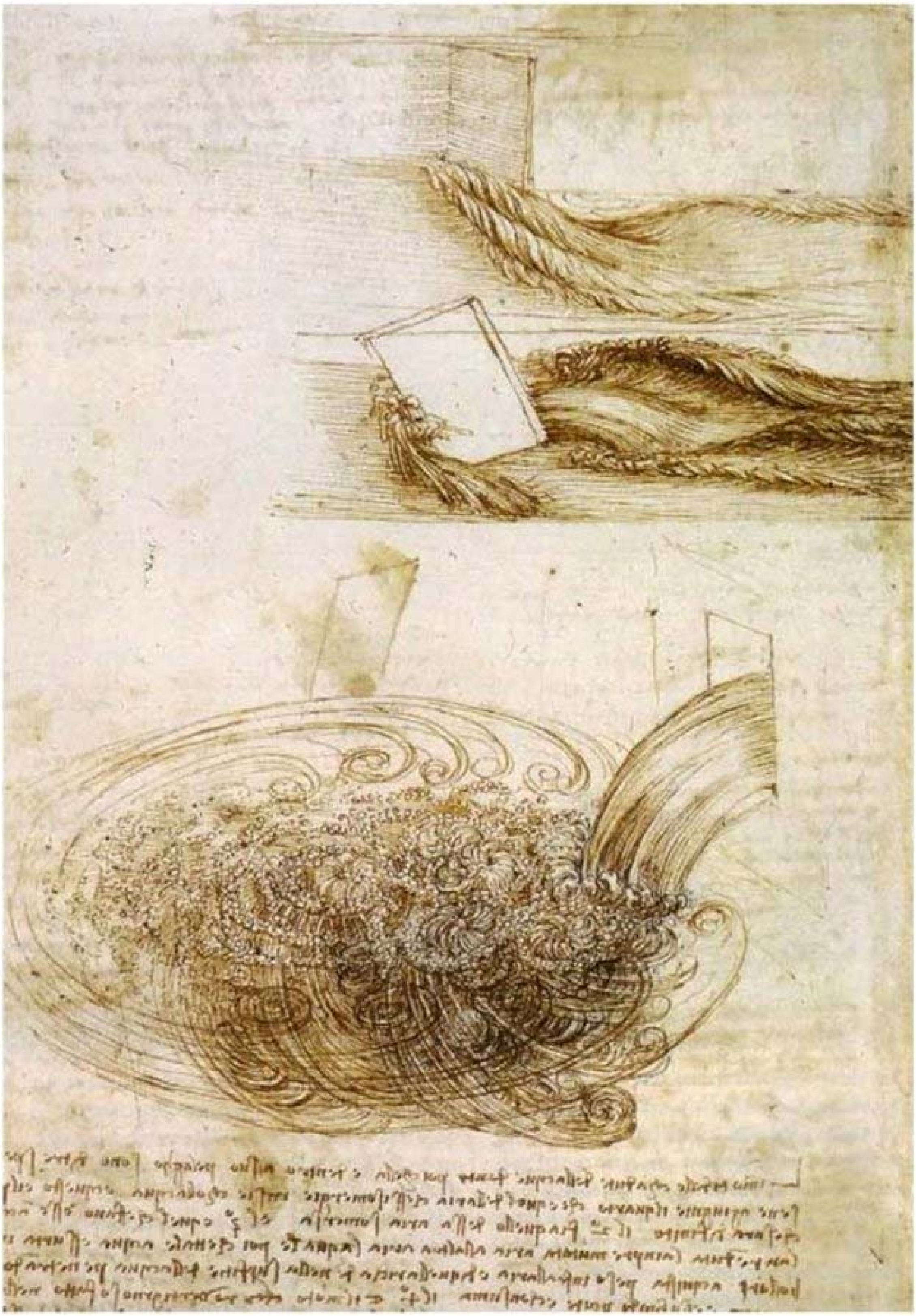

Figure 3: Leonardo Da Vinci: Studies of water passing obstacle and falling (c. 1508-1509).

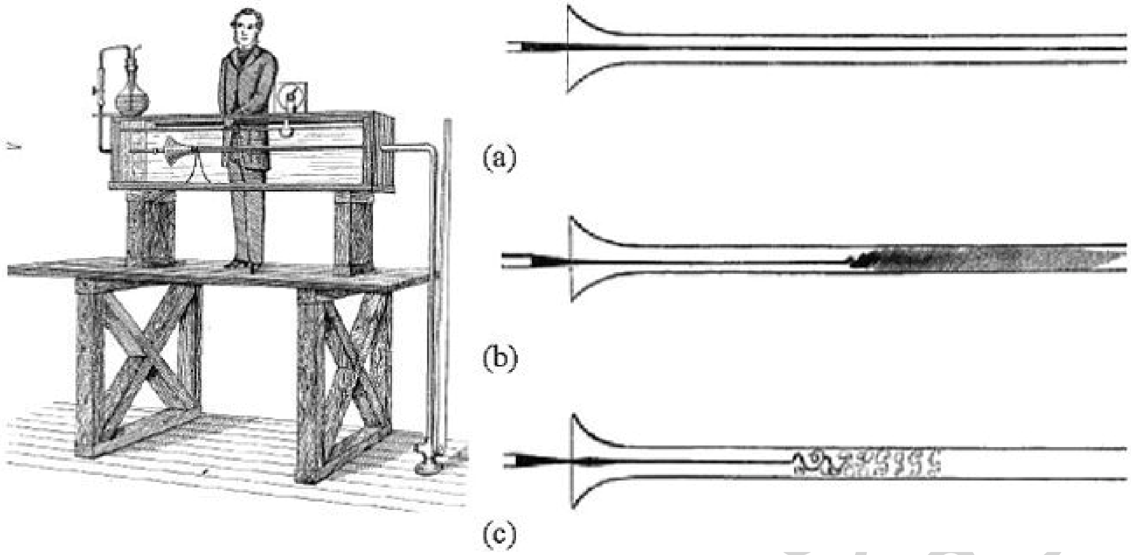

Figure 4: Reynolds apparatus for investigating the transition to turbulence in pipe flow, with sketches of laminar flow (a), turbulent flow (b) and turbulent flow in a stroboscopic light. From Reynolds (1883).

Following Reynolds' introduction of the random view of turbulence and proposed use of statistics to describe turbulent flows, essentially all analyses were along these lines. The first major result was obtained by Ludwig Prandtl in 1925 in the form of a prediction of the eddy viscosity, i.e. the proportionality coefficient in the linear stress-strain relation introduced by Boussinesq. Prandtl's "mixing-length theory", analyzed in more detail later, utilized an analogy between turbulent eddies and molecules or atoms of a gas to determine the length and velocity scales needed to construct an eddy viscosity. His reasoning followed the derivation of an analytical description of molecular viscosity used in the kinetic theory of gases. Despite the fact that this approach has essentially never been successful at making true predictions of turbulent flow, it does a fairly good job at making "post-dictions" of certain simple flows for which it has been calibrated.

The next major steps in the analysis of turbulence were taken by G. I. Taylor during the 1930s, who introduced formal statistical methods involving correlations, Fourier transforms and power spectra into the turbulence literature. In a 1935 paper he very explicitly presented the assumption that turbulence is a random phenomenon and he introduced statistical tools for the analysis of homogeneous, isotropic turbulence. The impact of this has lasted even to the present.

In 1941 the Russian statistician A. N. Kolmogorov published three papers (in Russian) that provide some of the most important and most-often quoted results of turbulence theory. These results, which will be discussed in some detail later, represent a distinct departure from the approach that had evolved from Reynolds' statistical approach (but are nevertheless still of a statistical nature). Kolmogorov's theory was derived purely from dimensional analysis and until recently, it has been used mainly as tests of other theories (or calculations). In recent years, theoretical turbulence studies have addressed breakdowns of Kolmogorov's theory.

During the 1940s Landau and Hopf (separately) proposed that as Re is increased a typical flow undergoes an infinity of transitions during each of which an additional incommensurate frequency (and/or wave number) arises due to flow instabilities, leading ultimately to very complicated, apparently random, flow behavior. This scenario was favored by many theoreticians even into the 1970s when it was shown to be untenable in essentially all situations. In fact, such transition sequences were never observed in experimental measurements, and they were not predicted by more standard approaches to stability analysis.

The first textbooks on turbulence theory began to appear in the 1950s. The best known of these are due to Batchelor, Townsend and Hinze. Again, as was true in the preceding decade, most of this work represented consolidation of earlier ideas. On the other hand, experimental work during this period and even somewhat earlier, was beginning to cast some doubt on the consistency, and even the overall validity, of the random view of turbulence. In particular, it become clear that a completely random viewpoint was not really tenable, and by the late 1950s measurement techniques were becoming sufficiently sophisticated to consistently indicate existence of so-called “coherent structures” contradicting the random view of turbulence.

By the beginning of the 1960s considerable advances were made possible by the introduction of the first digital computers. In 1963 the American meteorologist E. Lorenz published a paper, based mainly on machine computations that would eventually lead to a different way to view turbulence. In particular, this work presented a deterministic solution to a simple model of the N-S equations which was so temporally erratic that it could not (at the time) be distinguished from random. Moreover, this solution exhibited the feature of sensitivity to initial conditions, and thus essentially non repeatability. Furthermore, solutions to this model contained “structures” which might, at least loosely, be associated with the coherent structures being detected by experimentalists, although this was not recognized in 1963. The important point to take from this is that a deterministic solution to a simple model of the N-S equations had been obtained which possessed several notable features of physical turbulence.

In 1971 Ruelle and Takens published a seminal paper that contributed to delineating a new view of turbulence. In this work it was shown that the N-S equations, viewed as a dynamical system, are capable of producing chaotic solutions exhibiting sensitivity to initial conditions and associated with an abstract mathematical construct called a strange attractor. This paper also presents the sequence of transitions (bifurcations) that a flow will undergo as Re is increased to arrive at this chaotic state, namely: steady, periodic, quasi-periodic, and finally turbulent. This short sequence of bifurcations directly contradicts the then widely-held Landau-Hopf scenario mentioned earlier. Indeed, by the late 1970s and early 1980s many experimental results were showing this type of sequence.

Present-day turbulence investigations are deeply related to advances in computational techniques, starting from the early 1970s. In 1970 Dearoff proposed large-eddy simulation (LES). The first direct numerical simulation (DNS) by Orszag and Patterson dates from 1972. A wide range of Reynolds-averaged Navier-Stokes (RANS) approaches has been introduced also beginning around 1972. These initiated an enormous modeling effort that continues to this day, in large part because it has yet to be successful. At the same time, most other approaches are not yet computationally feasible, except for academic problems. In particular, DNS was not feasible for practical engineering problems (and probably will not be for at least another 10 to 20 years beyond the present), and in the 70s and 80s this was true as well for LES. Thus, great emphasis was placed on the RANS approaches despite their many shortcomings that we will note in the following. By the beginning of the 1990s computing power was reaching a level to allow consideration of using LES for some practical problems if they involved sufficiently simple geometry, and since then a tremendous amount of research has been devoted to this technique. It is fairly clear that for the near future this is the most promising method for turbulence modeling.

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Biographical Sketch

Paola Cinnella graduated *summa cum laude* in Mechanical Engineering at *Politecnico di Bari* (Italy) in 1995, with a dissertation on conservative numerical solutions of the incompressibile Navier-Stokes equations. Then she moved to France, where she got a master degree in Fluid Mechanics (*Diplôme d'Etudes Approfondies*) from *University Pierre and Marie Curie (*Paris VI*)* and *Ecole Nationale supérieure d'Arts et Métiers* (ENSAM-Paris). In 1999, she obtained a PhD in Fluid Mechanics from ENSAM-Paris, *summa cum laude*, with a dissertation about numerical schemes and turbulence models for complex compressible unsteady flows. After a post-doctoral research period at Politecnico di Bari, in 2001 she became Assistant Professor in Fluid Dynamics at the Engineering Department of *Università del Salento* (Italy). In 2006 she obtained the *Diplôme d'Habilitation à Diriger des Recherches* from University Paris VI. Since 2008 she is Professor at ENSAM-Paris. Her research interests are the numerical simulation of turbulent compressible flows in external and internal aerodynamics, aeroelasticity, dense gas dynamics and shape optimization. She has authored or co-authored more than 50 papers in the field of computational fluid dynamics, which appeared in international journals or refereed conference proceedings.