CONFIGURATIONAL FORCES

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Contents

Summary

Configurational forces are those thermodynamic (co-vectorial) forces that are associated by duality with any local manifestation of a material inhomogeneity, whether this is a real material inhomogeneity (foreign inclusion, rapid but smooth or abrupt change of property) or a more or less localized defect (dislocation, disclination, phase-transition front, shock wave). They are calculated from the usual field solution by means of the so-called Eshelby material stress and their arena is the material manifold itself. They have, therefore, no Newtonian nature, and are often associated with a local structural rearrangement of matter. When inserted in a criterion of evolution, they allow for the future determination of the evolution of the material inhomogeneity or defect. Examples of such "forces" are the Peach-Koehler force acting on a dislocation in elasticity, the *J*-integral of fracture theory, and the driving force acting on an evolving phase-transition front. The present contribution presents the "thermomechanics" of this fruitful concept in the absence or presence of intrinsic dissipative effects while it itself often is the patent mark of a dissipation of topological origin. Configurational forces also find useful applications in the implementation of various numerical schemes.

1. Introduction

Continuum mechanics in its simplest form has been the paragon of field theory and developed in parallel with the mathematical field of partial differential equations since the inception of this concept by d'Alembert in his studies of wave motion in a string and his elements of hydrodynamics in the mid 1700s. Thereafter progress was relatively slow due to the mathematical difficulties in obtaining appropriate solutions to problems of complex geometries and finding the most appropriate functional classes to allow for the existence of the looked for solutions (see Chapter on "Mathematical issues" in this encyclopedia volume). Now often considered, with some scorn - or the least, condescension - as an « old » and almost closed science by some physicists, it is true that further progress at the conceptual level was also slow and perhaps not as spectacular as in other branches of "natural philosophy". Had to be grasped and mathematically formulated the difficult notion of dissipation, whether in fluids in the form of viscosity, and then in solids in the form of plasticity and damage.

Until recently all these advances were made in the framework of three tenets of 19th century physics: linearity, isotropy, homogeneity. Of course there are exceptions to these such as the early introduction of finite strains in elasticity by Cauchy in the 1820s and the inherent nonlinearity of some problems of fluid mechanics. Anisotropy was conquered next due to the consideration of crystals. This even reached fluids in the form of liquid crystals (see a foregoing chapter in this volume). Considerations of material heterogeneities were to come last as we shall briefly see. Apart from mathematical advances with the introduction of new functional spaces (Sobolev spaces, distribution theory), the main advance that emerged after the rejuvenation (in fact a true "rebirth") of the field by authors such as C.A.Truesdell (e.g., Truesdell and Toupin, 1960;

Truedsell and Noll, 1965), was the firm grounding of continuum mechanics in a thermo-mechanical framework, to the posthumous satisfaction of Pierre Duhem (see Maugin, 1999a). That very much helped to classify and logically arrange the field, however sometimes to a useless extreme "bourbakism", as also to incorporate some multi-physical effects (e.g, electromagnetism, see Eringen and Maugin, 1990), and to prepare the way for enlarging the categories of modeling, including multiscale continuum mechanics and the introduction of scale effects (characteristic internal lengths, nonsymmetric Cauchy stress, micropolar and micromorphic continua; gradient theories). All these advances of the second half of the 20^{th} century are more generalizations than new conceptual thinking.

It is only at this point that more attention was paid to material heterogeneities, whether in the case of composite materials or that of polycrystals, and the necessary accompanying notion of defects. This, in our opinion, is the last great conceptual advance in continuum mechanics, in particular due to the recognition of the conceptual unity of the sub-field of continuum mechanics related to the notion of **configurational force**, the subject matter of this chapter. Indeed, the first example of such "forces" is the Peach-Koehler (1951) force that drives a dislocation line, while the second is the force on a material elastic inhomogeneity (e.g., inclusion) and a field singularity in the pioneering work of J.D.Eshelby (1951), whom we consider the "founding father" of our field. The remarkable feature of these developments in a half century, but accelerated in the years 1990s-2000s, has been the new interrelation of continuum mechanics with recent fields of mathematical physics, in particular in so far as invariances are concerned. This is shown in the forthcoming sections.

The basic thinking here is a typical reflex of a good "mechanician". Whatever apparently moves or progresses in the matter in an observable manner is thought as being acted by a "force" dual to the observed displacement of that "object". But this is not a force of the Newtonian type, for the object can be a material defect of mathematically vanishing support, a dislocation line, a mathematical surface of discontinuity (e.g., a phase-transition front, a shock wave), a material inclusion, a hole, a field singularity such as a crack tip, a strongly localized mathematical field solution (e.g., structured shock waves, solitons), etc. In the framework of continuum mechanics all these take place on the material manifold M^3, i.e., the set of material points constituting the body in a more or less smooth manner. This is directly related to the notion of material heterogeneity since that property describes the dependency of the material properties on the material point (not the point occupied in physical space), hence on the local configuration. The problem with such "configurational" forces is that they are not directly accessible, but what is shown in their theory, is that they may be computed once more classical entities are obtained, and then further progress of their point of application can be envisaged depending on the implementation of a criterion of progress. One easily imagines the practical, engineering, interest for such a procedure in problems of fracture (progress of a crack tip) or phase transformations because of its predictive nature.

The exposition that follows is a rational ordered reconstruction of the field rather than a linear history of it. First is recalled the important notion of Piola transformation, and then follows that of quasi-static configurational (Eshelby) stress (Section 2). Configurational forces are introduced in Section 3 along with material and so-called inhomogeneity forces. Effects analogous to material inhomogeneity and plasticity in so far as configurational forces are concerned are considered in Section 4. Section 5 is devoted to the paradigmatic case of inhomogeneous pure elasticity (hyperelasticity). This gives an opportunity to use a variational formulation, apply Noether's theorem, and introduce the notion of canonical conservation laws, next to that of the basic balance laws. The dynamical material Eshelby stress and material momentum come up in this nondissipative approach. Section 6 presents the setting for balance and canonical balance laws in the general case when dissipation is present and is subjected to the second law of thermodynamics. Section 7 deals with configurational forces acting on field singularities. This shows the intimate relationship of the subject matter with the theory of fracture and that of the propagation of singular surfaces. Section 8 deals in some discursive manner with the interaction between configurational forces and numerical schemes of various types. Finally, Section 9 gives a far from complete overview of the field. This is complemented by a bibliography, too short to render justice to all contributors and the wealth of recent publications.

2. Concepts of Piola Stress and Configurational Stress

The classical transformation between Cauchy's stress $\boldsymbol{\sigma}$ and the *first Piola stress* $\mathbf{T}$ is given by

$$\mathbf{T} = J_{\mathrm{F}}\mathbf{F}^{-1}\sigma \quad , \quad \sigma = J_{\mathrm{F}}^{-1}\mathbf{F}\,\mathbf{T}\,, \tag{2.1}$$

where $\mathbf{F}$ is the deformation gradient between the reference configuration K_{R} (local coordinates $\mathbf{X}$) and the actual configuration at time t, K_t (local coordinates = placement $\mathbf{x}$), and $J_{\mathrm{F}} = \det\mathbf{F}$. As a matter of fact, the transformation (2.1) that goes back to Piola (1836, 1848) accounts for the basic fact that the stress is a quantity defined per unit surface, so that (2.1) in fact relates to the form invariance of the applied stress vector in the actual configuration, a vector field, i.e.,

$$\mathbf{N}.\mathbf{T}dS = \mathbf{n}.\sigma ds \tag{2.2}$$

and

$$\mathbf{N}\,dS = J_{\mathrm{F}}^{-1}\mathbf{n}.\mathbf{F}ds \;\;,\;\; \mathbf{n}ds = J_{\mathrm{F}}\mathbf{N}.\mathbf{F}^{-1}dS \tag{2.3}$$

where $\mathbf{n}$ and $\mathbf{N}$ are unit normals in correspondence between K_t and K_{R}.. That is why (2.1) is essentially a specific **vectorial** transformation and not a second-order tensorial one. Only the *second Piola stress* defined by

$$\mathbf{S} = \mathbf{T}\mathbf{F}^{-\mathrm{T}} = J_{\mathrm{F}}\mathbf{F}^{-1}\sigma\,\mathbf{F}^{-\mathrm{T}} \quad , \quad \sigma = J_{\mathrm{F}}^{-1}\mathbf{F}\,\mathbf{S}\,\mathbf{F}^{\mathrm{T}}\,, \tag{2.4}$$

is a nice material stress tensor also referred to as the *energy stress* (see below).

All above equations refer to one reference configuration only K (no subscript R to simplify the notation) and this is sufficient in most continuum mechanics. Accordingly, that reference configuration is chosen as the most convenient one for computations depending on the geometry of the deformable body under study. With the consideration of the physics of the problem this may also be chosen as a stable solution providing a minimum of energy (cf. Lardner, 1974)). We should have been more careful in noting $\mathbf{F}_K$, $\mathbf{T}_K$ and $\mathbf{S}_K$ the various objects where the relation to the selected reference configuration K is understood. The question naturally arises of a *possible change of reference configuration*, e.g., between configurations K and K'. Let $P_{KK'}$ the transformation between K and K' at a material point $\mathbf{X}$. Given the tensorial nature of $\mathbf{F}$ and $\mathbf{T}$ - these are in fact two-point tensor fields, i.e., geometric objects having their two feet on different manifolds -, we have the following transformations

$$\mathbf{F}_{K'} = \mathbf{F}_K \, P_{KK'} \quad , \quad \mathbf{F}_K = \mathbf{F}_{K'} \, P_{K'K} \, , \tag{2.5}$$

$$\mathbf{T}_{K'} = J^{-1}_{K'K} \, P_{K'K} \, \mathbf{T}_K \quad , \quad \mathbf{T}_K = J^{-1}_{KK'} \, P_{KK'} \, \mathbf{T}_{K'} \tag{2.6}$$

where

$$P_{KK'} \, P_{K'K} = \mathbf{I} \quad , \quad P_{K'K} \, P_{KK'} = \mathbf{I} \, , \tag{2.7}$$

$\mathbf{I}$ being the identity transformation. Of course (2.6) are *Piola transformations*.

Now consider the case of energy-based elasticity for which there exists a potential energy per unit volume of the considered reference configuration, e.g., $W_K\left(\mathbf{F}_K\right)$, such that

$$\mathbf{T}_K = \partial W_K / \partial \mathbf{F}_K \, . \tag{2.8}$$

Accordingly, for another reference configuration *K'* we would have

$$\mathbf{T}_{K'} = \partial W_{K'} / \partial \mathbf{F}_{K'} \, . \tag{2.8'}$$

Since *W* is per unit volume, we have

$$W_{K'} = J^{-1}_{K'K} \, W_K \quad , \quad W_K = J^{-1}_{KK'} \, W_{K'} \, . \tag{2.9}$$

By direct computation of (2.8)' and use of (2.5) and (2.9), we check that (2.6) hold identically.

Now let us do something more original by computing the quantity

$$\mathbf{b}_{KK'} = \frac{\partial W_K}{\partial P_{KK'}} = \frac{\partial}{\partial P_{KK'}} \left(J^{-1}_{KK'} W_{K'} \left(\mathbf{F}_K \, P_{KK'} \right) \right) . \tag{2.10}$$

The result is

$$\mathbf{b}_{KK'} = -J_{KK'}^{-1}\left(P_{K'K}\, W_{K'} + \mathbf{T}_{K'}\,\mathbf{F}_{K}\right). \qquad (2.11)$$

We call **configurational stress** the geometric object defined in the *K* configuration by

$$\mathbf{b} = \mathbf{b}_K := -\mathbf{b}_{KK'}\,P_{KK'}, \qquad (2.12)$$

i.e., as shown by a simple calculation

$$\mathbf{b} = \mathbf{b}_K = -\frac{\partial W_K}{\partial P_{KK'}}P_{KK'} = W_K\,\mathbf{I}_K - \mathbf{T}_K\mathbf{F}_K\,. \qquad (2.13)$$

This will also be called the **quasi-static Eshelby material stress**.

Let $\mathbf{P}$ the two-point tensor field representing the transformation $P_{K'K}$. Accordingly, (2.5) and (2.13) read (T = transpose)

$$\bar{\mathbf{F}} = \mathbf{FP} \quad , \quad \mathbf{b} = -\frac{\partial \bar{W}}{\partial \mathbf{P}}\mathbf{P}^{\mathrm{T}} = W\,\mathbf{1}_R - \mathbf{TF}, \qquad (2.14)$$

where $\mathbf{1}_R$ is the identity in $K_R = K$, and

$$\bar{W} = J_{\mathbf{P}}^{-1}W\left(\bar{\mathbf{F}}\right) = \tilde{W}\left(\mathbf{F},\mathbf{P}\right). \qquad (2.15)$$

This follows Epstein and Maugin (1989,1990) so that

$$\mathbf{T} = \frac{\partial \tilde{W}\left(\mathbf{F},\mathbf{P}\right)}{\partial \mathbf{F}} = \frac{\partial W\left(\mathbf{F}\right)}{\partial \mathbf{F}}, \; \mathbf{b} = -\frac{\partial \tilde{W}\left(\mathbf{F},\mathbf{P}\right)}{\partial \mathbf{P}}\mathbf{P}^{\mathrm{T}} = W\,\mathbf{1}_R - \mathbf{TF}\,. \qquad (2.16)$$

We can also note, on account of the reciprocal of $(2.4)_1$ that

$$\mathbf{TF} = \mathbf{SF}^{\mathrm{T}}\mathbf{F} = \mathbf{S.C} =: \mathbf{M}\,, \qquad (2.17)$$

where $\mathbf{C} = \mathbf{F}^{\mathrm{T}}\mathbf{F}$ is the Cauchy-Green finite-strain on the configuration K_R, and $\mathbf{M}$ is the so-called **Mandel stress tensor** in K_R (cf. Lubliner, 1990; Maugin, 1992). Therefore, configurational stresses and Mandel stresses are intimately related since they differ only by the presence of an energy isotropic term, i.e.,

$$\mathbf{b} = W\mathbf{1}_R - \mathbf{M} \quad \text{or} \quad \mathbf{b} + \mathbf{M} = W\mathbf{1}_R\,. \qquad (2.18)$$

This difference reduces to a pure change of sign for an isochoric deformation associated with $\mathbf{b}$ or $\mathbf{M}$.

If the Cauchy stress is symmetric (as happens in many cases), then we let the reader check with the help of (2.1) and (2.14) that this results in the symmetry of $\mathbf{b}$ with respect to $\mathbf{C}$, considered as the deformed metric on the material manifold M^3, i.e.,

$$\mathbf{Cb} = \left(\mathbf{Cb}\right)^{\mathrm{T}} = \mathbf{b}^{\mathrm{T}}\mathbf{C}, \tag{2.19}$$

as first noticed by Epstein and Maugin (1989). If, furthermore, the material considered is *isotropic*, then classical symmetry (i.e., with respect to a neutral unit covariant metric) applies because $\mathbf{S}$ becomes a function of the basic invariants of $\mathbf{C}$.

3. Configurational Force

If K_R is a global reference configuration over the material body B, and $P_{K'K}$ is smooth and integrable over the material manifold, then $\mathbf{P}$ will be a gradient of a deformation in a classical sense, so that (2.6) is not distinguishable from a standard Piola transformation. The situation may be altogether different in the case when the body is not materially homogeneous. Indeed, the case when $\mathbf{T}$ is function of $\mathbf{F}$ and $\mathbf{F}$ only, where $\mathbf{F}$ is true gradient, represents the essence of **pure homogeneous elasticity -** a paradigmatic case as we shall see herein after **-** with

$$\mathbf{T} = \bar{\mathbf{T}}\left(\mathbf{F}\right) = \frac{\partial W\left(\mathbf{F}\right)}{\partial \mathbf{F}}. \tag{3.1}$$

As soon as W becomes an explicit function of additional arguments, we are no longer in this ideal framework. This happens whether the additional argument is another field variable such as temperature in thermoelasticity, or electric polarization or magnetization in electro-magneto-elasticity (cf. Maugin, 1988), or else any variables such as so-called internal variables of state supposed to account for the hidden complexity of microscopic processes which have a macroscopic manifestation in the form of thermodynamic irreversibility (i.e., dissipation; cf. Maugin, 1999a). These cases will be examined later on. Another frequent possibility is that the energy W depends explicitly on the material particle $\mathbf{X}$, in which case $W = \bar{W}\left(\mathbf{F};\mathbf{X}\right)$ and the elastic material is said to be *materially inhomogeneous*. We call **material force of inhomogeneity** the material co-vector defined by

$$\mathbf{f}^{\,\mathrm{inh}} := -\left.\frac{\partial \bar{W}}{\partial \mathbf{X}}\right|_{\mathrm{expl}}, \tag{3.2}$$

if $\bar{W}$ is a sufficiently smooth function of $\mathbf{X}$, and where the subscript *expl* means that the material gradient is taken at fixed field (here $\mathbf{F}$). In composite materials where inhomogeneities manifest abruptly by jumps in material properties, (3.2) must be replaced by a distributional (generalized functions) definition. The *force* $\mathbf{f}^{\,\mathrm{inh}}$ belongs in the world of **material forces** (cf. Maugin, 1992, 1995) since it is a co-vector on the material manifold. It is a directional indicator of the changes of elastic properties as it is oriented opposite to the direct explicit gradient of W.

Now we can exploit the thought experiment of Epstein and Maugin (1990a,b). To that purpose, imagine that at each material point $\mathbf{X}$ we can give to the material deformation energy the *appearance* of that of a pure homogeneous elastic body (dependence on one deformation only and nothing else) by applying the appropriate local (at $\mathbf{X}$) change of reference configuration. We consider this along with the concomitant change of volume (compare to (2.9))

$$W = \bar{W}\left(\mathbf{F};\mathbf{X}\right) = J_{\mathbf{K}}^{-1} W\left(\mathbf{F}\mathbf{K}\left(\mathbf{X}\right)\right) = \tilde{W}\left(\mathbf{F},\mathbf{K}\right). \tag{3.3}$$

Performing the same operation as in (2.16), we clearly have

$$\mathbf{T} = \frac{\partial \bar{W}\left(\mathbf{F};\mathbf{X}\right)}{\partial \mathbf{F}}\ ,\quad \mathbf{b} = -\frac{\partial \tilde{W}\left(\mathbf{F},\mathbf{K}\right)}{\partial \mathbf{K}}\mathbf{K}^{\mathrm{T}} = W\mathbf{1}_R - \mathbf{T}\mathbf{F}\,. \tag{3.4}$$

Thus there exits a relationship between the notion of material inhomogeneity and that of configurational (or Eshelby) stress. This is made more visible by applying the definition (3.2):

$$\begin{aligned} \mathbf{f}^{\,\mathrm{inh}} = -\left.\frac{\partial \bar{W}\left(\mathbf{F};\mathbf{X}\right)}{\partial \mathbf{X}}\right|_{\mathrm{expl}} &= -\frac{\partial \tilde{W}\left(\mathbf{F},\mathbf{K}\right)}{\partial \mathbf{K}}.\frac{\partial\,\mathbf{K}}{\partial \mathbf{X}} \\ &= \mathbf{b}.\mathbf{K}^{-\mathrm{T}}.\frac{\partial\,\mathbf{K}}{\partial \mathbf{X}} = \mathbf{b}.\left(\mathbf{K}^{-\mathrm{T}}.\left(\nabla_{\mathrm{R}}\mathbf{K}\right)^{\mathrm{T}}\right) \end{aligned}. \tag{3.5}$$

On the other hand, if we compute the material divergence of $\mathbf{b}$ in the case of quasi-statics in the absence of body force, for which the equilibrium at $\mathbf{X}$ is simply given by $\mathrm{div}_R\mathbf{T} = \mathbf{0}$, we have

$$\begin{aligned} \mathrm{div}_R\mathbf{b} &= \nabla_R W - \left(\mathrm{div}_R\mathbf{T}\right).\mathbf{F}\text{-}\mathbf{T}.\left(\nabla_R\mathbf{F}\right)^{\mathrm{T}} \\ &= \left(\frac{\partial W}{\partial \mathbf{F}} - \mathbf{T}\right).\left(\nabla_R\mathbf{F}\right)^{\mathrm{T}} + \left.\frac{\partial\,W}{\partial \mathbf{X}}\right|_{\mathrm{expl}} \end{aligned}, \tag{3.6}$$

or, on account of $(3.4)_1$ and (3.5),

$$\mathrm{div}_R\mathbf{b} = -\mathbf{f}^{\,\mathrm{inh}}\,. \tag{3.7}$$

Here the material force of inhomogeneity is deduced from (or balanced by) the material divergence of the configurational stress. It is justified to call **configurational forces** these forces that are deduced through an operation acting on the configurational stress, whether by differentiation or integration (e.g., over a material surface, along a material contour in 2D). If we combine the results of (3.5) and (3.7), we also obtain an equation for $\mathbf{b}$ which involves the local transformation $\mathbf{K}$ in a source term, that is,

$$\mathrm{div}_R \mathbf{b} + \mathbf{b}.\Gamma = \mathbf{0}\,, \tag{3.8}$$

where we have defined a *material connection* $\Gamma(\mathbf{K})$ by

$$\Gamma(\mathbf{K}) = (\nabla_R \mathbf{K}).\mathbf{K}^{-1} = -\mathbf{K}.\left(\nabla_R \mathbf{K}^{-1}\right)^{\mathrm{T}}. \tag{3.9}$$

The result (3.8) is due to Epstein and Maugin (1990a,b). If $\mathbf{K}$ is the same for all points $\mathbf{X}$, then $\nabla_R \mathbf{K} = \mathbf{0}$, and (3.8) reduces to the strict conservation law

$$\mathrm{div}_R \mathbf{b} = \mathbf{0}\,, \tag{3.10}$$

in the case (we remind the reader) of the absence of body force and neglect of inertia (quasi-statics). Otherwise, the above-reported intellectual construct means that the operation carried out brings the neighborhood of each material point $\mathbf{X}$ into a prototypical situation of the pure elastic type which allows one to compare the response of different points. Since this is point-like, the operation will not result in an overall smooth manifold, but in a collection of non-fitting neighborhoods or infinitesimal chunks of materials, and $\mathbf{K}$ will not, accordingly, be itself a gradient. It may at most be a Pfaffian form. Of course, if $\mathbf{K}$ is not integrable, so is the case of $\bar{\mathbf{F}} = \mathbf{FK}$. With Eqs.(3.8)-(3.9) we enter the geometrization of continuum mechanics that we shall not pursue here although this was started in the mid 1950s by scientists such as Kondo, Kröner, Noll, Wang, etc (cf. Maugin, 1993, 2003a).

Remark: All material forces are not translated into useful configurational forces. First there are material forces that are but the convection back to the material manifold of usual physical forces, such as mass body force $\mathbf{f}_0$ of Newtonian or Lorentzian origin which may also be represented by material forces of the type

$$\mathbf{f}^{\mathrm{ext}} = -\rho_0\, \mathbf{f}_0.\mathbf{F}\,. \tag{3.11}$$

Here we cannot help but $\mathbf{f}_0$ is always a function of the actual placement $\mathbf{x}$ in physical space. True material forces are those material forces that the full material formulation (projection onto the material manifold) makes apparent while they did not manifest themselves in physical space This is the case of the inhomogeneity force (3.2) as also of some material forces due to the nonuniformity of some physical fields on the material manifold (e.g., temperature; see below). The case of inertial forces not treated for the moment is more subtle because if one can define a material (co-vectorial) momentum ***P*** by

$$P = -\rho_0 \mathbf{v}.\mathbf{F} = \rho_0 \mathbf{C}.\mathbf{V} \quad , \quad \mathbf{V} = -\mathbf{F}^{-1}.\mathbf{v}\,, \tag{3.12}$$

the inertial force in the physical frame work does not translate directly in an analogous inertial force on the material manifold. As a matter of fact, the material inertial force is naturally defined as

$$\mathbf{f}^{\text{inertia}} = -\frac{\partial P}{\partial t} = \left(\frac{\partial}{\partial t}(\rho_0 \mathbf{v}) \right).\mathbf{F} - \rho_0 \mathbf{v}.(\nabla_R \mathbf{v})^{\mathrm{T}}, \qquad (3.13)$$

as is easily checked. With ρ_0 depending on $\mathbf{X}$ (case of material inertial inhomogeneities), the last term in (3.13) will contribute to both the dynamical configurational stress and the dynamical force of inhomogeneity since

$$\rho_0 \mathbf{v}.(\nabla_R \mathbf{v})^{\mathrm{T}} = \mathrm{div}_R \left(\left(\frac{1}{2} \rho_0 \mathbf{v}^2 \right) \mathbf{1}_R \right) - \left(\frac{1}{2} \mathbf{v}^2 \right) (\nabla_R \rho_0). \qquad (3.14).$$

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Biographical sketch

Gérard A. Maugin, Born 1944 in France. Received his Mechanical (1966) and Aeronautics (1968) Engineering Diplomas in Paris, a Ph.D. (1971) at Princeton University, USA, and a D.Sc. in Mathematics from Paris University in 1975. He is the head of the institute "Jean Le Rond d'Alembert" (Mechanics, Acoustics, Energetics) at University Pierre et Marie Curie-Paris 6, France. Recipient of several dr hc, member of various academies, Max Planck Award 2001, Eringen Medal of the SES 2003. Author of numerous research papers and books in mathematical physics, theoretical continuum mechanics, and applied mathematics. His research interests include the electrodynamics of deformable continua, surface waves, the thermomechanics of continua, the theory of material defects, nonlinear waves and, more generally, nonlinear phenomena in crystal physics and mechanobiology.