# INTRODUCTORY TOPICS IN THE MATHEMATICAL THEORY OF CONTINUUM MECHANICS

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### Contents

- I. GENERAL PRINCIPLES
  - 1. Introduction
  - 2. The well-posed problem
- **II. BASIC CONTINUUM MECHANICS** 
  - 3. Introduction
  - 4. Material description
  - 5. Spatial description
  - 6. Constitutive theories
- **III. EXAMPLE: HEAT CONDUCTION** 
  - 7. Introduction. Existence and uniqueness
  - 8. Continuous dependence. Stability
  - 9. Backward heat equation. Ill-posed problems
  - 10. Non-linear heat conduction
  - 11. Infinite thermal wave speeds
- IV. CLASSICAL THEORIES
  - 12. Introduction
  - 13. Thermoviscous flow. Navier-Stokes Fluid
  - 14. Linearized and linear elastostatics
  - 15. Linearized Elastodynamics
  - 16. Linearized and linear thermoelastostatics
  - 17. Linearized and linear thermoelastodynamics
  - 18. Viscoelasticity
  - 19. Non-linear Elasticity
  - 20. Non-linear thermoelasticity and thermoviscoelasticity
- V. NON-CLASSICAL THEORIES
  - 21. Introduction
  - 22. Isothermal models
  - 23. Thermal Models

Glossary

Bibliography

Biographical sketches

## Summary

Qualitative properties of well-posedness and ill-posedness are examined for problems in the equilibrium and dynamic classical non-linear theories of Navier-Stokes fluid flow and elasticity. These serve as prototypes of more general theories, some of which are also discussed. The article is reasonably self-contained.

# I. GENERAL PRINCIPLES

## 1. Introduction

The mathematical treatment of equations descriptive of phenomena in continuum mechanics in particular requires techniques from non-linear partial differential equations. Such techniques rely mostly on analysis and geometry (including topology) and to a lesser extent (for certain wave motions) on algebra. The resulting qualitative properties not only explain how solutions behave but also provide firm foundation for numerical procedures.

The governing equations of continuum mechanics themselves are derived from postulated integral balance laws of mass, momentum and energy. For smooth processes these axioms imply differential equations but on surfaces of discontinuity they generate jump conditions. Constitutive assumptions are introduced to account for different types of materials and interrelate, for example, motion, stress, heat transfer, electro-magneto-mechanical effects, and transport phenomena. Invariance requirements and thermodynamical restrictions further define the mathematical problem whose specification is completed by suitable initial and boundary conditions. This enables qualitative and quantitative properties to be determined, and special methods developed for particular problems. These notes describe the types of qualitative results possible and the mathematical arguments used to achieve them, as well as explaining potential difficulties and open problems. Not discussed are special solutions, particular properties (e.g., universal deformations, pattern formation), or methods of solutions (e.g., matched asymptotic expansions, Weiner-Hopf techniques). Inclusion of such topics, although of obvious intrinsic interest, would expand the article considerably beyond its intended scope.

Hadamard's notion of a well-posed problem provides a convenient ordering of our treatment. Hadamard (1923) defined a problem to be well-posed when its solution exists, is unique, and depends continuously upon the data. Indeed, it was claimed that a mathematical model lacking these properties cannot be relevant to any creditable phenomenological process, including those of continuum mechanics. Nevertheless, while in very broad terms the criticism is obviously justified, caution should be exercised in its precise interpretation. The notion of well-posedness remains formal until the individual elements and associated function spaces are adequately and precisely defined. Moreover, a problem not well-posed according to one set of function spaces may become well-posed with respect to another.

Length restrictions limit this account to a description of selected principle themes and developments with reference confined to relevant main monograph and research

literature. Primary sources are included where these are either reasonably accessible or acknowledged classics. Consequently, the publications mentioned are necessarily only part of the vast total available and inevitably reflect the authors' interests. It is definitely not intended to imply that those omitted are secondary.

The remainder of Part I amplifies and illustrates the notion of a well-posed problem by means of general remarks on existence, uniqueness, and continuous data dependence. Spatial and dynamic stability are discussed along with the concept of an ill-posed problem. Part II reviews basic principles of continuum mechanics and, in particular, their application to the classical non-linear theories of thermoviscous flow and thermoelasticity, both of which contain the important special case of heat conduction. This comparatively simple theory is used in Part III to discuss various mathematical techniques required in Part IV for the discussion of existence, uniqueness, continuous data dependence, stability, spatial stability, and ill-posed problems concerned with equilibrium and non-equilibrium processes of thermoviscous flows, thermoelasticity, and their isothermal counterparts. Part V briefly reports progress in corresponding studies of several non-classical theories. Notation is either direct or indicial, when the summation and comma conventions are adopted. Further explanation is provided in Section 4.

A knowledge is assumed of basic kinematical and mechanical concepts, which may be found in standard introductory texts, e.g., Chadwick (1976), Green and Zerna (1968), Jauzemis (1967), and Ogden (1984). A limited understanding of analysis and partial differential equations will also be useful.

These notes contain hardly any new material. They obviously rely heavily upon previously published leading accounts, full acknowledgement to which is given at appropriate places in the text. Nevertheless, it is a pleasure to repeat here our indebtedness to these authors.

# 2. The Well-Posed Problem

# 2.1. Basic Notion

To achieve desirable generality, we formulate the definition of well-posedness in an abstract context. Further discussion and elementary examples of the concept may be found in standard textbooks on differential equations. We consider topological vector spaces X, Y, Z with  $Y \subset Z$ . It is supposed that the data is contained in the space X, and that the solution u is a mapping  $u: X \to Y$ . Notice that data includes the initial and boundary data, source terms, material parameters, and the geometry of the space (-time) region over which the governing system of partial differential equations is defined, while the space Y represents the set of values of all solutions. The problem is *well-posed* when:

- 1. The mapping *u* exists.
- 2. The mapping u is uniquely determined by the data.
- 3. The mapping  $u: X \to Z$  is continuous at a given element of X.

A problem that is not well-posed is *ill-posed* or *improperly posed*.

Whether or not a problem is well-posed depends upon the choice of the function spaces X, Y, Z, selected usually to be a Hilbert or Sobolev space. Another fundamental question concerns the conditions under which well-posedness of a non-linear problem is inherited by the corresponding linearized one. Simple counter-examples demonstrate that the statement is not universally true, although in fluid mechanics and similar dissipative systems, continuous dependence in the linear system under suitable conditions, implies that in the non-linear system.

We now discuss in greater detail the constituent elements in the definition of well-posedness.

### 2.2. Existence

The basic axioms of conservation of mass, momentum, and energy express the mathematical modeling of many physical systems, including those exhibiting chaotic behavior, in terms of (stochastic) integral equations involving both volume and surface integrals. These are the so-called balance laws in integral form, which reduce to conservation laws in the absence of supply terms. Constitutive relations, subject to appropriate invariance and thermodynamics restrictions, specify particular continuum theories such as elasticity, thermoelasticity, viscoelasticity, the Navier-Stokes fluid, magnetohydrodynamics, and multi-polar and Cosserat materials. Prescribed initial and boundary conditions complete the specification of the problem. Existence of a solution, however defined, cannot immediately be inferred. For example, without sufficient smoothness of the boundary, the surface integrals may be meaningless, invalidating applications of the divergence theorem. Attempts to model microstructure, granular materials, and fractal boundaries encounter such difficulty and have contributed to increasing interest in the application of geometric integration theory (Silhavý (1997)), unfortunately beyond the scope of these notes. An account of these and related issues is provided by Capriz and Podio-Guidugli (2004). Non-smoothness of constitutive parameters and other data likewise may prevent volume integrals from becoming properly defined. Consequently, an important element in studying existence of solutions is to establish minimal smoothness conditions on the data in order that the integral equations composing the model are well-defined and possess what is termed a *weak* solution. Weak solutions have limited smoothness, and their discontinuities may correspond quite naturally to certain static and dynamical physical phenomena, for instance, phase boundaries, rupture, cracks, cavitation, and shock waves. Further conditions must be imposed in order to reduce the integral equations to a system of partial differential equations, whose solution in a relevant smoothness class must be separately established. Such solutions are termed strong when they are continuous together with their spatial and temporal derivatives to sufficient order. Weak solutions must be discussed in the context of Sobolev and other abstract functional spaces, or in a distributional sense.

Especially in dynamics, well-known one-dimensional examples, many cited in the books by Straughan (1998), Dafermos (2006) and Tartar (2006), demonstrate that globally (in time) smooth solutions are not to be expected. We select one example from

elasticity due to F. John (1974), (see also John (1979, 1981)), in which the spatial and temporal scalar variables are x and t respectively and u(x,t) is the scalar displacement. The equation of motion in the absence of body force becomes

$$\frac{\partial \sigma(u_{,x})}{\partial x} = \frac{\partial^2 u}{\partial t^2},\tag{1}$$

where without loss the uniform density is supposed equal to 1, and a subscript comma denotes partial spatial differentiation. The second order equation (1) may be rewritten as the first order system

$$\frac{\partial U}{\partial t} + A \frac{\partial U}{\partial x} = 0,$$

where the vector U and matrix A are given by

$$U = \begin{bmatrix} w \\ v \end{bmatrix}, A = \begin{bmatrix} 0 & -1 \\ -\sigma'(w) & 0 \end{bmatrix},$$

and  $w = \partial u / \partial x$ ,  $v = \partial u / \partial t$ , while a superposed prime indicates differentiation with respect to the argument of a function.

Suppose that system (2) is *strictly hyperbolic*, that is, for each u, the matrix A has distinct real eigenvalues and associated real eigenvectors. Consider Lipschitz continuous deformations which depend upon the variables x, t only through the single function  $\phi(x,t)$ , and set

$$U(x,t) = H(\phi), \tag{4}$$

where  $H'(\phi)$  is an eigenvector of A with eigenvalue  $a(\phi)$ . Consequently, on substitution in (2), we obtain

$$\frac{\partial \phi}{\partial t} + a(\phi) \frac{\partial \phi}{\partial x} = 0, \tag{5}$$

and along characteristic curves, defined by

$$\frac{dx}{dt} = a(\phi),\tag{6}$$

we have that  $\phi(x,t) = \text{constant}$ . Let  $\phi(x,0) = \tilde{\phi}(x)$ . Then, by (6), the characteristic curve through the point  $(x_0,0)$  is the straight line  $x = x_0 + a(\tilde{\phi}(x_0))t$ . Next, assume that  $\sigma$  is such that  $a(\tilde{\phi}(x_0))$  decreases with  $x_0$  and consider the characteristic lines through the initial points  $(y_1,0)$  and  $(y_2,0)$  where  $y_1 < y_2$ . Then  $a(\tilde{\phi}(y_1)) > a(\tilde{\phi}(y_2))$ , and the

two characteristic lines intersect at critical time  $t_{crit}$  given by

$$t_{\rm crit} = \frac{y_2 - y_1}{a(\tilde{\phi}(y_1)) - a(\tilde{\phi}(y_2))}.$$
(7)

The vector function U is constant along a characteristic curve and therefore at the intersection has conflicting values. We conclude that at  $t_{crit}$  the solution is neither continuous nor differentiable, and consequently smooth solutions cannot exist globally with respect to time. For  $t > t_{crit}$ , a solution, if it exists, must be a weak solution here defined as satisfying

$$\int_{\mathcal{Q}} \left( B \frac{\partial \Phi}{\partial x} + C \frac{\partial \Phi}{\partial t} \right) dx = 0,$$

for all vector test functions  $\Phi \in C_0^{\infty}(Q, \mathbb{R}^2)$ , where Q is the *space-time* region over which (1) is defined, and the matrices B, C are given by

(8)

(9)

$$B = \begin{bmatrix} \sigma & 0 \\ 0 & v \end{bmatrix}, C = \begin{bmatrix} -v & 0 \\ 0 & -w \end{bmatrix}.$$

Alternative definitions of a weak solution are possible. (See, for example, Marsden and Hughes (1983), Ball (2002), Dafermos (2006), and Tartar (2006)).

The broad array of methods deployed to investigate existence include spectral analysis, direct methods of the calculus of variations, the Lax-Milgram lemma, the implicit function, fixed point and inverse function theorems for equilibrium problems; and energy arguments, the Galerkin method, and contractive semi-group theory for problems in dynamics. Some of these techniques are described in later Sections.

# 2.3. Uniqueness

The importance of knowing whether or not a solution is unique for given data is almost self-evident. For example, such information is vital for numerical evaluation, and for ensuring completeness of solutions constructed by semi-inverse and similar methods.

But uniqueness is not necessarily a universally desirable property. Bifurcation and buckling would be impossible without loss of uniqueness in the associated (linear) problem. Turbulence and cavitation would not occur without failure of uniqueness in the non-linear problem, and indeed in non-linear elastostatics there are well-known counter-examples demonstrating that unqualified uniqueness is physically untenable. In other systems, there may be uniqueness of smooth solutions but non-uniqueness of weak solutions. To illustrate the last remark, consider the one-dimensional Burgers equation (cp., Dafermos (1975, 2006))

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} = 0, \qquad t \ge 0.$$
(10)

Suppose (10) is defined on the whole real line  $\mathbb{R}$  with initial data v(x) = +1 for x < 0, and v(x) = -1 for x > 0. The characteristic curves are respectively  $x \pm t$ , and the piecewise smooth solution is u(x,t) = +1 in the quarter plane x < 0, t > 0 and u(x,t) = -1 in the quarter plane x > 0, t > 0, with the axis x = 0, t > 0 being a line of shock discontinuity. The piecewise smooth solution satisfying different initial data v(x) = -1, x < 0, and v(x) = +1, x > 0, has characteristic curves  $x \mp t$ , and is given by  $u(x,t) = \pm 1$  in the quarter planes x > 0, t > 0 and x < 0, t > 0, respectively. However, a second piecewise smooth solution with the same initial data is given by



with shocks occurring on the lines  $x \pm t = 0$ . It is easy to check that the *Rankine-Hugoniot condition* is satisfied by both solutions, so that clearly there is non-uniqueness. Non-uniqueness similarly may be shown for the first example.

Uniqueness may be recovered when the solution is subject to a suitable selection criterion satisfied by at most one solution. Various criteria have been proposed each motivated by a different physical argument. For hyperbolic conservation laws in  $\mathbb{R}^m$ , admissible solutions are assumed to satisfy the inequality

$$\frac{\partial S}{\partial t} + \sum_{i=1}^{m} q_{i,i} \le h, \tag{14}$$

where the *S* is a scalar entropy function,  $q \in \mathbb{R}^n$  the entropy flux, and  $h \in \mathbb{R}$  the entropy production.

Inequality (14) must be interpreted in the sense of distributions when weak solutions are considered. In continuum mechanics, selection criteria often correspond to entropy production inequalities. A full account is provided by Dafermos (2006).

Uniqueness in linear systems is equivalent to proving that at most only the trivial solution exists for homogeneous data, whereas in non-linear systems it must be shown that specified data admit at most one solution within a given function class consistent with that for existence. When discontinuity surfaces develop, an appropriate function space for both equilibrium and dynamic solutions is the class of functions of bounded variation.

Energy arguments are amongst the most frequently employed to establish uniqueness in

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linear and non-linear systems, especially those of continuum mechanics. There are, however, many other approaches, including analytic functional methods used for continuous data dependence, and differential inequalities and convexity techniques for spatial stability and ill-posed problems respectively. A selection is illustrated later.

# 2.4. Continuous Data Dependence

Continuous data dependence is of practical and numerical importance. Actual physical measurements are seldom possible to the accuracy required by mathematical prescription and data in this respect contain unavoidable error. Furthermore, a measurement cannot be taken at a precise point in space or time, but is either in some neighborhood of the given point, or represents an average over a space-time interval about the point. The continuous distribution of data usually assumed in mathematical treatments can be obtained only by theoretical interpolation from data measured "pointwise" in the sense just described. Again, it is seldom absolutely certain that initial data is simultaneously measured over a spatial region at the same instant of time. Errors also are introduced by imprecise constitutive parameters, or geometry of the region. Numerical data can be prescribed to only limited accuracy in numerical computations. In all these situations, it is vital to know whether or not small errors in data generate correspondingly small errors in the solution. The conclusion has been proved in standard problems of elliptic, parabolic, and hyperbolic type, but fails, by definition, for ill-posed problems. Nevertheless, it is later explained how continuous data dependence in certain ill-posed problems may be recovered in a weakened sense for classes of constrained solutions.

Continuous data dependence is closely related to the concept of continuity, and in discussing this relationship it is preferable to introduce the same abstract (topological) function spaces used to treat existence and uniqueness. The next Section explains how continuous dependence upon initial data in dynamic problems is related to, and is refined by, the notion of stability and its associated theorems. Meanwhile, as a basis for subsequent discussion, we elaborate upon the mathematical definition of continuous data dependence.

A neighborhood is defined in terms of a positive-definite function  $\rho: X \times X \to \mathbb{R}$  with the properties

1.  $\rho(x, y) \ge 0, \quad \forall x, y \in X.$ 2.  $\rho(x, y) = 0 \Leftrightarrow x = y.$ 

These functions define a norm on the respective topological spaces provided the following additional conditions are satisfied:

1. 1.  $\rho(x, y) = \rho(y, x)$ . 2. 2.  $\rho(x, y) \le \rho(x, z) + \rho(y, z)$ ,

where  $x, y, z \in X$ .

The precise definition of continuous dependence (or continuity of maps) is given by:

**Definition 1** (Continuous Dependence) Let  $\rho_1$  and  $\rho$  be positive definite functions defined on the spaces  $X \times X$  and  $Y \times Y$  respectively, and let  $\phi \in X$  and  $\psi \in X$ correspond to data for solutions  $u, v : X \to Y$  respectively. The solution u is continuous at  $\phi$  if and only if for each  $\varepsilon > 0$  there exists  $\delta(\varepsilon) > 0$  such that

$$\rho_1(\psi,\phi) < \delta \implies \rho(v,u) < \varepsilon. \tag{15}$$

The definition applies equally to static and dynamic problems, but in dynamic problems the concept of dependence upon initial data corresponds to that of stability. Indeed, we regard stability as a property of dynamical perturbations of a system whether in equilibrium or in motion. We avoid the convention, especially in elasticity, of adopting the minimum energy criterion as a definition of stability. The criterion has encountered justified criticism, and at best is a test for stability whose mathematical proof awaits a complete existence theory for elastodynamics.

Because of its practical importance, we devote the next subsection to a brief discussion of the fundamental elements of stability theory based primarily upon the treatments by Movchan (1960a,b), Gilbert and Knops (1967), and Knops and Wilkes (1973).

# 2.5. Stability

A solution either in equilibrium or in motion is stable when perturbed initial data produce small disturbances as the system evolves with time. When the disturbances vanish as time increases indefinitely, the solution is said to be asymptotically stable.

It is obvious from these rough ideas that the time variable t is a preferred variable. Consequently, let us consider a time interval of existence, [0,T], possibly semi-infinite in length, and the evolutionary maps  $\phi:[0,T] \rightarrow Y$ , where Y is the function space in which the solution u(x,t) is represented by a sequence of elements as time evolves. Let  $\mathcal{B}([0,T],Y)$  designate the set of functions defined on [0,T] taking values in Y. Let initial data belong to the set X equipped with the positive-definite function  $\rho_1$ , and let the space Y be equipped with the positive-definite function  $\rho_2$ .

**Definition 2** (Liapunov stability) The solution  $u \in \mathcal{B}([0,T],Y)$  is Liapunov stable if and only if the mapping  $\phi$  from X to  $\mathcal{B}([0,T],Y)$  is continuous at u. That is, for  $v \in \mathcal{B}([0,T],Y)$  and for each  $\varepsilon > 0$  there exists  $\delta(\varepsilon) > 0$  such that  $\rho_1(u(0),v(0)) < \delta$ implies  $\rho(u,v) < \varepsilon$ , where

$$\rho(u,v) = \sup_{t \in [0,T]} \rho_2(u(t), v(t)).$$
(16)

**Definition 3** (Asymptotic stability) The solution  $u \in \mathcal{B}([0,T],Y)$  is asymptotically stable if and only if (a) u is stable; and (b) for  $v \in \mathcal{B}([0,T],Y)$  there exists  $\delta > 0$  such

that  $\rho_1(u(0), v(0)) < \delta$  implies  $\rho_2(u(t), v(t))$  tends asymptotically to zero as  $t \to \infty$ .

**Definition 4** (Instability) A solution that is not stable is unstable. Equivalently, a solution  $u \in B([0,T],Y)$  is Liapunov unstable if and only if the mapping from X to  $\mathcal{B}([0,T],Y)$  is discontinuous at u. That is,  $u \in \mathcal{B}([0,T],Y)$  is unstable if and only if for  $v \in \mathcal{B}([0,T],Y)$  there exists  $\varepsilon > 0$  such that for all  $\delta > 0$  there holds

 $\rho_1(u(0), v(0)) < \delta \Longrightarrow \rho(u, v) \ge \varepsilon.$ (17)

These are natural and precise definitions in terms of dynamics which generalize corresponding Lagrange-Dirichlet definitions for discrete systems. Obviously, the choice of positive-definite functions  $\rho_1, \rho_2$  crucially affects whether or not a solution is stable. A given solution to the same initial boundary value problem may be stable or unstable according to the choice of positive-definite functions and the underlying spaces X, Y. Examples illustrating this point are described by Knops and Wilkes (1973) and extend those familiar in the calculus of variations.

It immediately follows from these definitions and from well-known properties of continuity that a stable solution u is unique, and that the corresponding mapping  $\phi$  is bounded at u.

There are two general methods for establishing stability, namely, (a) maximum principles; and (b) the direct, or second, method of Liapunov. We dispose immediately of maximum principles since the method simply states that the solution u(t) is stable if there exists a bounded real function M(t) on [0,T] such that for  $v \in \mathcal{B}([0,T],Y)$  we have

$$\rho(u,v) \le M(t)\rho_1(u(0),v(0)).$$
(18)

The solution u is *uniformly stable* when M(t) is independent of t, whereas when  $M(t) \rightarrow 0$  as  $t \rightarrow \infty$  it is asymptotically stable. Inequalities of type (18) frequently occur in stability analyzes for non-linear fluid dynamics using the so-called *energy method* when the kinetic energy is used as a positive-definite measure. They likewise appear in discussions of spatial stability and stabilization of ill-posed problems.

The other main method for stability, commonly referred to as Liapunov's second method, generalizes the Lagrange-Dirichlet theorem for discrete systems. It finds formal application to non-linear elastodynamics and assists in clarifying concepts associated with the energy criterion for stability. Liapunov's theorem, originally developed for ordinary differential equations, was extended to continuous systems by Movchan (1960a, 1960b).

**Theorem 1** (Liapunov stability) The solution  $u \in \mathcal{B}([0,T],Y)$  is stable if and only if there exist positive-definite functions  $V_t$ , where  $t \in [0,T]$ , defined on  $Y \times Y$  with the properties that (a) for given real  $\varepsilon > 0$  there exists  $\delta(\varepsilon) > 0$  such that for all  $v \in \mathcal{B}([0,T],Y)$ 

$$\rho_1(u(0), v(0)) < \delta \implies V(u, v) < \varepsilon, \tag{19}$$

(b) for given real  $\eta > 0$  there exists real  $\zeta > 0$  such that for  $v \in B$  ([0,T],Y)

$$V(u,v) < \zeta \quad \Rightarrow \quad \rho(u,v) < \eta, \tag{20}$$

where

$$V(u,v) = \sup_{t \in [0,T]} V_t(u(t),v(t)).$$

We remark that the solution u is asymptotically stable when Condition (a) is supplemented by

(21)

(22)

 $\lim V_t(u(t), v(t)) = 0.$ 

Movchan (1960a) replaces Condition (a) by the following two subordinate conditions, which, however, are only sufficient for stability:

(c) given real  $\varepsilon > 0$  there exists real  $\delta(\varepsilon) > 0$  such that

$$\rho_1(u(0), v(0)) < \delta \implies V_0(u(0), v(0)) < \varepsilon.$$
(23)

(d)  $V_t(u(t), v(t))$  is non-increasing with respect to t; that is

$$V(u,v) \le V_0(u(0), v(0)). \tag{24}$$

The proof of these statements depends upon the composition law for continuous maps, and is given in the references previously cited.

Liapunov's theorem states necessary and sufficient conditions for stability and consequently yields necessary and sufficient conditions for instability. Nevertheless, it is convenient to state explicit conditions which, of course, should automatically exclude the trivial instability due to non-unique and unbounded solutions.

**Theorem 2** (Liapunov instability) The solution  $u \in \mathcal{B}([0,T],Y)$  is unstable if and only if there exist positive-definite functions  $V_t$  that satisfy (a) there is  $\varepsilon > 0$  such that for all  $\delta > 0$  there holds

$$\rho_1(u(0), v(0)) < \delta \implies V(u, v) \ge \varepsilon, \tag{25}$$

(b) for given  $\eta > 0$  there exists  $\zeta(\eta) > 0$  such that

$$\rho(u,v) < \zeta \implies V(u,v) < \eta. \tag{26}$$

A principal objective is to establish necessary and sufficient conditions for a solution to satisfy one or other of the above stability definitions. A major obstacle for many non-linear conservative systems is the lack of a complete global existence theory so that many of the known results remain formal.

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