MOIRÉ METHODS FOR SHAPE, DISPLACEMENT AND STRAIN ANALYSIS

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Summary

Moiré method is useful to measure shape and whole-field distributions of displacement and strain of structures. The theory of the fringe formation by superposing of two gratings or sampling of a grating is introduced firstly. Many kinds of moiré methods such as geometric moiré method, sampling moiré method, Fourier transform moiré method, moiré interferometry, shadow moiré method and moiré topography are explained. Grating method analyzing directly deformation of a grating without any moiré fringe pattern is also detailed. The relationship between the moiré fringe pattern or the grating pattern and the shape or the deformation is explained secondary. The analysis methods of the fringe pattern to obtain the shape or displacement and strain distributions are mentioned. Especially, the phase analysis methods of fringe patterns or grating patterns to obtain accurate results are detailed. The applications of these moiré methods and grating methods to shape measurement, displacement distribution measurement and strain distribution measurement are shown.

1. Introduction

To support the safety and security of human life, it is important to check the shape, deformation, strain and stress of the structures such as buildings, bridges, machines and human bodies. It is also expected to prolong the life of structures. The point measurement methods using a displacement gage, a strain gage etc. are popular in fields. Recently, in order to save resources and cost, optimal design of artificial structures is requested. The values of the stresses of a structure, therefore, become almost a constant, and it is difficult to detect the maximum stress value in a structure by using point measurement methods. In order to obtain distribution of these values, whole-field optical methods such as moiré method, photoelasticity, holography, and speckle interferometry are useful. These methods also provide easily recognizable results. Though the data amount of these optical methods is very huge, the development of computers and image processing techniques lead to high-speed analysis. In this chapter, moiré methods including grating methods which are the most popular whole-field methods are explained on the principle, analysis using image processing and some applications.

Moiré fringe patterns are observed by superposing two grating patterns such as net windows, and by taking a striped shirt with a TV camera. The moiré fringe patterns are different from the original grating patterns and the moving speed of the fringe patterns in a TV monitor are faster than the moving speed of the original grating patterns. The principle of the fringe pattern formation and the analysis of shape, deformation and strain...
are detailed in Section 2. In Section 3, the analysis of the moiré fringe patterns and grating patterns using image processing are explained, especially phase analysis methods of moiré fringe and grating patterns are detailed to obtain high accuracy. In Section 4, the applications for shape measurement by grating projection, real-time phase analysis technique by integrated phase-shifting method, deflection measurement of a beam by sampling moiré method and thermal deformation measurement of an electronic device by moiré interferometry are introduced.

2. Moiré

2.1. Moiré Phenomena

As mentioned in Section 1, moiré fringe patterns are observed by superposing two gratings such as net windows with almost equally spaced parallel lines. Figure 1 shows a photograph of a wind fan which has equally-angle spaced radial lines. It shows a moiré pattern which is different with the original equally-angle spaced radial lines. The moiré fringe pattern appears by superposing a front side grating and a backside grating of a wind fan. In photographs of windows of a building and a tiled roof on newspapers, strange fringe patterns sometimes appear, and the fringe pattern are fundamentally different from the original patterns such as the windows of the buildings and the tiles of the roof. By taking a striped shirt by a TV camera, a fringe pattern appears sometimes on a TV monitor.

Figure 1. Example of moiré phenomenon. Moiré fringe pattern obtained by superposing front side grating and back side grating of wind fan

These phenomena are explained as sampling of an object with a periodical pattern. The moiré appearance by superposition of two net windows is explained as the sampling of
the periodical net pattern of one net by the space of another net. The moiré pattern on the
photograph of tiled roofs printed on a newspaper is explained as the interference between
the periodical roof tiles and the periodical halftone dots of printing of the newspaper. The
moiré pattern on a TV monitor is explained as the sampling of the striped shirt by the
scanning lines of a TV camera.

In sound, when two sound waves with almost the same but a little different frequencies
are simultaneously emitted, the beat of two sounds is generated. The beat frequency is the
frequency difference between the two original sounds as shown in Figure 2(a). If the
frequencies of the waves are $f_A$ and $f_B$, respectively, the frequency of the beat is
$f_A - f_B$ as shown in Figure 2(a). The moiré is similar to the beat of the two sounds. Moiré
fringe patterns appear when two grating are superposed. If the spatial frequencies of the
gratings are $f'_A$ and $f'_B$, respectively, the spatial frequency of the moiré is the
difference $f'_A - f'_B$ of the two spatial frequencies as shown in Figure 2(b).

Figure 2. Moiré similar to beat: (a) Beat by superimposing of two sound waves; (b)
Moiré by superimposing of two grating waves.

By analyzing the fringe pattern, the deformation of the original grating are obtained
quantitatively. In this chapter, the analysis methods of shape, displacement, strain using
moire fringe patterns are introduced. Geometric moiré method using the superposition of
two gratings, scanning moiré method or sampling moiré method using sampling by a TV
camera or an image processor, and moiré interferometry using a high frequency
diffraction grating are explained.

2.2. Geometric Moiré Method

2.2.1. Relationship among Moiré Fringe, Displacement and Strain

Figure 3(a) shows a grating pattern with $x$-directional equally spaced parallel lines. This
grating is called ‘reference grating’ or ‘master grating.’ The same grating is drawn on a
specimen before deformation. The grating on a specimen is called ‘specimen grating’ or
‘model grating.’ When the specimen is deformed in the $x$-direction, the specimen
grating is also deformed as shown in Figure 3(b). If the reference grating is superposed on
the deformed specimen grating, a moiré fringe pattern appears as shown in Figure3(c).
Figure 3. Theoretical explanation of moiré appearance showing equal-displacement contours: (a) Reference grating; (b) Specimen grating; (c) Moiré fringe pattern obtained by superposing Figures 3(a) and 3(b); (d) Moiré fringe pattern obtained by sampling of TV scanning lines.

A white moiré fringe line appears on the parts where a black line of the reference grating and a black line of the specimen grating are superposed or where a white line of the reference grating and a white line of the specimen grating are superposed. A black moiré fringe line appears on the parts where a black line of the reference grating and a white line of the specimen grating are superposed or where a white line of the reference grating and a black line of the specimen grating are superposed. In this case, the moiré fringe lines show equally-displacement lines mentioned as follows.

Let us show the principle for obtaining displacement and strain distributions from this moiré fringe pattern. Each white line from the left edges of the reference grating is numbered as \(k = 0, 1, \ldots\), as shown in Figure 3(a). These numbers are called ‘line number.’ By considering that the deformation is one-dimensional for simplicity, each point moves along the \(x\)-direction, that is, normal to the original grating line. Figure 3(b) shows a deformed grating with this \(x\) directional deformation. The lines of the deformed specimen grating are numbered as \(l = 0, 1, \ldots\), respectively, as shown in Figure 3(b).

Figure 3 (c) shows the moiré fringe pattern shown by superposing Figure 3(a) on Figure 3(b). On a continuous moiré fringe line, the difference between the line number of the reference grating and the line number of the specimen grating is constant. The number \(m\) is the difference of the two line numbers.

\[
m = k - l
\]
It is called ‘fringe order’ of the moiré fringe. When the pitch of the grating lines of the reference grating is $p_0$, the specimen grating is deformed as $mp_0$ along the line normal to the grating lines of the reference grating. That is, on the fringe lines with moiré fringe order $m$, the displacement is $mp_0$. By looking at the moiré fringe pattern, displacement distribution is determined. This is the principle of geometric moiré method to measure deformation.

That is, the $x$-directional displacement $u$ at the place with a fringe order $m$ is expressed as follows:

$$u = mp_0$$

(2)

The $x$-directional strain is defined as differentiation of the $x$-directional displacement with respect to $x$. Then the strain is obtained as follows.

$$\varepsilon = \frac{du}{dx} = \frac{dm}{dx} p_0$$

(3)

On the other hand, the fringe distance $\delta$ between the neighboring fringes shows that the displacement difference between the both ends of the distance is 1 pitch of the reference grating. Therefore the strain at the distance is expressed as follows.

$$\varepsilon = \frac{\text{elongation}}{\text{length before deformation}} = \frac{p_0}{\delta - p_0}$$

$$\approx \frac{p_0}{\delta} \quad \text{(When $\delta$ is sufficiently larger than $p_0$)}$$

(4)

From this equation, strain is obtained from the fringe space $\delta$.

2.2.2. Carrier Patterns

After the reference grating is enlarged or shrunk a little, when it is superposed on the undeformed specimen grating, a moiré fringe pattern appears before deformation as shown in Figure 4. This method is called ‘mismatch method’. Alternatively, when the specimen grating is superposed on the original reference grating rotated a little, a moiré fringe pattern appears as shown in Figure 5. This method is called ‘misalignment method’. If the number of moiré fringe lines is few in a conventional moiré method as mentioned in Figure 3, the fringe number will be increased by using mismatch method or misalignment method. Alternatively, if the number of moiré fringe lines is very large in a conventional moiré method, the fringe number will be decreased by using mismatch method or misalignment method. Like this, the fringe pattern appearing before deformation is called ‘carrier pattern’.

As mentioned in Section 2.2.1, a moiré fringe order is a constant on the same fringe line. However, although it is difficult to detect whether the neighboring fringe order is
increasing or decreasing, it is easy to find the fringe order if many number of lines of the carrier fringe pattern is used so as to increase or decrease monotonically and the fringe order and the neighboring fringe order changes continuously. Then it is easy to count the fringe order. However, the equations calculating displacement and strain will be different from Eqs. (2) to (4).

![Figure 4](image1.png)

Figure 4. Moiré fringe pattern before deformation generated by mismatch method

![Figure 5](image2.png)

Figure 5. Moiré fringe pattern before deformation generated by misalignment method

2.3. Sampling Moiré Method (Scanning Moiré Method)

In the conventional geometric moiré method as mentioned in Section 2.2.1, a moiré fringe pattern appears as the interference between a specimen grating and a reference grating as illustrated in Figure 3. However, a moiré fringe pattern also appears when the specimen grating lines are sampled by the scanning lines of a TV camera or by the pixels of an image processor as shown in Figure 3(d). If the sampling points of the image processor are regarded as the reference grating, the geometric relationship among the specimen grating lines, the sampling points and the resultant moiré fringe lines is obtained in the same way as in the conventional geometric moiré. The phenomenon of the moiré fringe appearance by the sampling can be explained by using the geometric relationship mentioned in Section 2.2.1. Furthermore, although the details are not shown in this Section, the sampling theory of Fourier transform of the image mentioned in Section 2.4 can be explained. Therefore, this method is also called ‘sampling moiré method’ as well as ‘scanning moiré method’.
When the pitch of the sampling points of an image processor is considerably smaller than the pitch of the specimen grating lines shown in this figure, the image shows only the original specimen grating lines and no clearly visible moiré pattern. In this case, if the sampling points are periodically picked up or the sampling points are thinned-out, the image shows a moiré pattern. When the pitch of the sampling points of the image processing is almost equal to that of the specimen grating lines, the sampled image shows a moiré fringe pattern.

By changing the pitch of the picking-up, different moiré patterns can be obtained. This corresponds to the mismatch moiré method. It is easy to change the pitch using image processing. The example is shown using Figure 6.

Figure 6. Moiré fringe appearance by sampling moiré method: (a) Sampling points of camera; (b) Specimen grating; (c) Sampled image of Figure 6(b); (d) Thinned-out image from Figure 6(c); (e) Interpolated image of Figure 6(d)

Figure 6 illustrates the appearance of a moiré fringe by sampling moiré method. Figure 6(a) shows the center position of the sampling points (pixel points) of a camera as black circular points. In this figure, only three horizontal lines are shown. Figure 6(b) shows a specimen grating with periodic lines, which is drawn on a specimen. In Figure 6(b), the pitch of grating is 1.125 times larger than that of the sampling points. The specimen grating is taken at the sampling points shown in Figure 6(a). Although Figure 6(a) shows the center positions of the pixels of the camera, each pixel has a square area shown in Figure 6(c). The output brightness of a pixel is the average brightness value on the square area. Then, the image of Figure 6(b) recorded by the camera is shown in Figure 6(c). The image shows only the original grating, not moiré fringe pattern. The intensity of the grating, however, is not two values (black and white). It has gray values because of the sampling area of the pixels. In Figure 6(c), no moiré fringe pattern can be discerned. However, if every $N_T$-pixel (in this case, $N_T = 4$) from the first sampling point is picked up from Figure 6(c), a moiré fringe pattern is obtained as shown in Figure 6(d). This image processing is called ‘thinning-out’ and $N_T$ is called ‘thinning-out index.’

If all the sampling point images that are thinned-out in Figures 6(d) are interpolated using neighboring data, the image becomes clearer and easy to observe the fringe pattern. Figure 6(e) shows the linear interpolated images from Figures 6(d). Then, a clear moiré fringe pattern is obtained.
2.4. Fourier Transform Moiré Method (FTMM)

As mentioned in Section 2.1, moiré phenomenon is similar to beat of sounds. The spatial frequency of a moiré fringe is the difference between the frequencies of a specimen grating and a reference grating. The frequency of an image can be analyzed by calculating the Fourier transform of the image. Therefore, the frequencies of a deformed grating and a reference grating are obtained by calculating the Fourier transforms of the deformed grating and the reference grating, respectively. Though the conventional spatial frequency \( f \) is the inverse of the grating pitch \( p \left( f = \frac{1}{p} \right) \), here, the spatial frequency \( f \) is defined as the number of pitches in the image size, \( f = \frac{M}{p}, \quad M : \text{pixel size of image} \).

Let us explain this phenomenon for moiré method by using Figure 7. Figures 7(a), (b), (c), (d), and (e) show grating images and the brightness distributions. Figures 7(a'), (b'), (c'), (d'), and (e') show the Fourier spectrum of the corresponding grating images, respectively. The abscissa axis shows the frequency \( f_x \) along the \( x \)-direction and the ordinate axis shows the power \( F(f_x) \) of the frequency \( f_x \). The reference grating or the undeformed specimen grating has equally spaced parallel lines with frequency \( f_0 = 20 \) shown in Figure 7(a). When the brightness distribution is cosinusoidal, the frequency components are only \( I_{-1}, I_0 \) and \( I_1 \) as shown in Figure 7(a'). In this grating, these components, \( I_{-1}, I_0 \) and \( I_1 \) are at a constant frequency \( f_x = -20, 0 \) and \( 20 \), respectively. If the grating brightness distribution is rectangular as shown in Figure 7(b), the frequency has higher harmonics, \( \ldots, I_{-3}, I_{-2}, I_{-1}, I_0, I_1, I_2, I_3, \ldots \), as shown in Figure 7(b'). In the case of the rectangular with the same width of white and black lines, the frequency components of the even terms excepting zero term are zero. That is, \( \ldots, I_{-6}, I_{-4}, I_{-2}, I_2, I_4, I_6, \ldots = 0 \). The gratings are deformed with the same deformation as shown in Figure 7(c) and (d) for cosinusoidal and rectangular, respectively. The frequency components of the deformed gratings of Figures 7(c) and (d) are shown in Figures 7(c') and (d'), respectively.

When the brightness distribution is cosinusoidal, the first and the minus first harmonics are observed at the frequencies \( f_x = -20 \) and \( 20 \) as shown in Figure 7(a'). If the deformation is not constant, each harmonic has a little wide distribution of the frequency according to the deformation as shown in Figure 7(c'). The frequency difference between two gratings (for example, Figures 7(c) and (a)) is obtained by shifting the frequency of the deformed grating by the constant frequency \( f_0 = 20 \) shown in Figure 7(e'). That is, the moiré frequency with low frequencies is obtained from the frequency difference between \( I_1' \) and \( I_1 \), which is shown as \( I_1' \) in Figure 7(e'). After shifting the first harmonic \( I_1' \) of the deformed grating in Figure 7(c) by the reference frequency \( f_0 = 20 \), as shown in Figure 7(e'), the inverse Fourier transform of the shifted first harmonic \( I_1' \) provides a moiré pattern as shown in Figure 7(e). This method is called ‘Fourier transform moiré method (FTMM)’
Figure 7. Relationship of grating and moiré fringe using frequency domain: (a) Cosinusoidal brightness distribution along a horizontal line of undeformed specimen grating; (a’) Fourier spectrum of Figure 7(a); (b) Rectangular brightness distribution along a horizontal line of undeformed specimen grating; (b’) Fourier spectrum of Figure 7(b); (c) Cosinusoidal brightness distribution of deformed specimen grating; (c’) Fourier spectrum of Figure 7(c); (d) Rectangular brightness distribution of deformed specimen grating; (d’) Fourier spectrum of Figure 7(d); (e’) Frequency of moiré fringe i.e., frequency difference between frequencies of specimen grating and reference grating; (e) Inverse Fourier transform of Figure 7(e’), i.e., moiré fringe; (e’’) Phase distribution of moiré fringe of Figure 7(e)

Figure 7(e’’) is the phase distribution which is mentioned in Section 3.3.

In Figure 7, the frequency domain in one-dimensional frequency is shown. If a two-dimensional grating with both of the $x$- and $y$-directional gratings is used, two-dimensional deformation can be analyzed by using two-dimensional Fourier transform. The application is mentioned in Section 4.4.
2.5. Moiré Interferometry (Diffraction Moiré Interferometry)

In geometric moiré method, the sensitivity for displacement is better according to the smaller pitch of the grating. When a smaller pitch less than about 20 mm (50 lines mm⁻¹) is used to give finer sensitivity, it is difficult to observe the fringe pattern because of the effect of diffraction. However, if the high density gating with a fine pitch such as 5~0.5 μm is used, the interference between the two diffraction beams gives a high sensitivity interference fringe pattern. This method is called ‘diffraction moiré interferometry’ or ‘moiré interferometry’. It is applied to various materials and structures such as electronic parts to measure deformation and strain distributions around cracks. This method uses a virtual grating as the reference grating. The virtual grating is obtained as the interference fringe pattern of two diffraction beams obtained by illuminating from two directions. It is easy to measure two-directional small displacement distributions in the order of the wavelength.

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Biographical Sketches

Yoshiharu Morimoto (Nickname: Harry Moiré) was born in Osaka, Japan on February 11, 1944. He graduated Master course of Osaka University, Japan in 1968 and received Dr. Eng. from Osaka University in 1981 for the study on powder compaction. His major field of study is experimental mechanics and image processing.

He is now Vice-President and Executive Director for Research, University Liaison and International Exchange of Wakayama University, Japan. He is also Professor of Dept. of Opto-Mechatronics at Wakayama University. He joined Komatsu Ltd. in 1968. He returned to Osaka University as Associate Researcher in 1974. He moved to Wakayama University in 1993.


His current and previous interests are following measurement methods for shape, deformation, stress and strain:

(1) Scanning moiré method using a TV camera (Sampling moiré method),

(2) Phase analysis methods using Fourier, wavelet or Gabor transforms,

(3) Strain rate distribution measurement by a high-speed video camera,

(4) Real-time integrated phase-shifting method,

(5) High-speed and accurate methods using multi-reference planes, a DMD, a linear image sensor, or a frequency modulated grating,

(6) Real-time simultaneous measurement for two-directional stress or strain distributions, and

(7) Accurate 3D displacement and strain measurement by phase-shifting digital holographic interferometry.
Prof. Morimoto is Fellow of JSME, and he was also Executive Board Member of SEM, President of JSEM and Chairman of Asian Committee for Experimental Mechanics. He received awards, from JSNDI, JSME, JSP SEM and JSEM.

Motoharu Fujigaki was born in Osaka, Japan on April 7, 1967. He graduated Master course of Osaka University, Japan in 1992 and received Ph. D. in Engineering from Osaka University in 2001 for the study on 3D shape measurement. His major field of study is experimental mechanics and opto-mechatronics.

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Dr. Fujigaki is a member of SPIE, SEM, JSEM, JSNDI and JSME. He is also an executive board member of JSEM.