RELIABILITY AGAINST FRACTURE AND FATIGUE

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Summary
The elements of the theories of fracture and fatigue are presented with applications to the prediction of the safe fatigue life. The up-to-date approach to the prediction of fatigue crack propagation is developed. Various approaches to the reliability evaluation in the presence of cracks are discussed. The damage-tolerance maintenance is discussed in the terms of reliability theory.

1. Fracture and fatigue in mechanical engineering

Occurrence of most failures in components of mechanical systems is preceded by the process of gradual accumulation of damage in material resulting in the nucleation and growth of cracks. These nuclei are frequently present before the operation being appeared on the manufacturing stage. The cause of failure is the growth of the crack size until the dangerous or undesirable sizes. If a crack is not localized in time, its growth may result into critical or even catastrophic failures. A significant group of failures and limit states is connected with wear and damage of surfaces contacting with operating or ambient environment. Such phenomena as corrosion fatigue, stress corrosion cracking and cracking enhanced by hydrogen embrittlement are frequently met in life support and infrastructure engineering systems.

The term fatigue is used for a wide set of phenomena of damage and fracture. In a narrow sense, fatigue means damage and fracture due to the cyclic, repeatedly applied stresses. In a broad sense, it includes also all the phenomena of delayed damage and fracture under sustained loads and environmental actions. Later, when we say on fatigue without additional comments, we mean the case of cyclic loading and actions. The
notion of fatigue, as one can find in the literature, was introduced into science in 1839 by Poncèlet. A systematic study of fatigue had been initiated later. That was connected with the name of Wöhler, who in period 1858-1860 performed the first systematic experimental study of this phenomenon. In particular, Wöhler introduced the concept of fatigue curve referring the cycle number up to fatigue failure to a characteristic magnitude of the stress process. Until now fatigue curves are used widely and form one of the foundations of the engineering analysis (see *Traditional Approach to Fatigue Analysis*).

Fatigue is a complex phenomenon. In the first line, fatigue is a gradual process of damage accumulation. This process proceeds on various levels beginning from the scale of crystall lattice, dislocations and other objects of solid state physics up to the scales of the order of the size of a structure or its components. At least three stages of fatigue damage are to be distinguished. On the first stage, the damage accumulates on the level of microstructure of the material, e.g. on the level of grains and intergranular layers when a polycrystalline alloy is concerned. This damage is dispersed upon the volume of a specimen or a machinery part, or, at least, within their most stressed parts. At the end of this stage, nuclei of macroscopic crack originate, i.e. such aggregates of microcracks that are acting as strong stress concentrators and, under the following loading, have a tendency to grow. The second stage is a gradual propagation of one, sometimes two or several macroscopic cracks. These cracks grow more rapidly than the others, and at the end become responsible for the integrity of the body. The crack growth, being long enough, results into the third stage – a rapid final fracture due to the sharp stress concentration at the front of a crack and/or the expenditure of the material's resistance to fracture. This stage is in fact a typical instability phenomenon. One could identify a special stage of the fatigue damage situated between the generation of a nucleus of a macrocrack and its regular propagation. Short macroscopic cracks often grow initially in a pattern different from that for completely developed cracks. Hence, one says often on *short fatigue cracks* to distinguish them from macroscopic, *long fatigue cracks*.

A typical picture of fatigue damage of polycrystalline materials looks as follows. Crack nuclei initiate near the surface, in particular, in the area of local stress concentration, and near the damaged or weakest grains. In most metal materials initial slip planes and microcracks are oriented along the planes with maximal shear stresses; the nuclei of macrocracks are oriented, at least approximately, in the same planes. Microcracks go through grains, intergranular boundaries or in a mixed way depending on material properties. Then they merge forming a short macroscopic crack. When the latter becomes sufficiently long, the direction of crack growth changes: the crack propagates in the cross section of the specimen, mostly in the opening mode. The crack intersects in its growth a large number of grains and, therefore, this growth is determined mainly with the averaged material properties.

The ratio between the durations of stages varies in a large scale depending on material properties, type of loading and environmental conditions. The first stage can be absent if a crack propagates from an initially sufficiently sharp crack-like defect or another strong stress concentrator. In that case the position of the macroscopic crack is conditioned beforehand, and it begins to grow after a comparatively small cycle number. Otherwise, for very brittle materials the stage of gradual crack propagation happens to be absent;
the final fracture occurs suddenly, without formation of any stable macroscopic cracks, just as a result of approaching of the microcracks density to a certain critical level.

Pressure vessels are a typical example of mechanical systems subjected to high stresses. These systems are met in almost all fields of industry, in particular, in power engineering and chemical industry. The required service life of pressure vessels is usually long. The vessel walls are subjected to high tensile stresses, frequently at elevated temperatures, in the contact with aggressive environment. To provide the necessary safety, sufficiently high safety factors are needed. However, the wall thickness is usually limited because of economical and engineering considerations. Sometimes the mass of a vessel is to be limited to make the project actualized. Hence, fracture or damage as the result of crack nucleation and growth is a typical mode of failures and limit states for pressure vessels. A similar situation takes place for piping systems under high pressure such as gas and oil pipelines.

The fatigue damage and crack growth are especially important for the machinery parts and joints under vibrations. For example, approximately 60 % of failures of aircraft engines are of mechanical origin, and among those approximately 80 % are associated with damage accumulation, fatigue crack growth and related phenomena. Approximately the same ratio is observed in the cores of nuclear reactors, heat exchangers and other units of power engineering subjected to vibrations. Frequently fatigue is accompanied with plastic deformations, creep, corrosion, hydrogen embrittlement as well as with other forms of material deterioration.

2. Linear Fracture Mechanics

The basis of fracture mechanics has been proposed by Griffith already in the 20s. However, the real interest for this section of mechanics was revived only in the 50s. At that time, due to the development of experimental techniques and non-destructive inspection methods, it was recognized that practically every large-scale structure contains cracks and crack-like flaws which presence, generally, does not mean that a structure is unsafe or unreliable. New trends in design and maintenance in engineering such as fail-safe and damage-tolerance design have stimulated the analysis of crack growth and stability. The aim of fracture mechanics becomes to advance test methods for the proper choice of materials, manufacturing processes and in-service maintenance based on the fracture toughness criteria, as well as to specify the safe crack sizes.

The common model of a crack in fracture mechanics is a mathematical cut in a body of non-damaged material. The dimensions of cracks are assumed large compared with the characteristic sizes of the material structure such as grain sizes. Such cracks are named macroscopic to distinguish them from microscopic cracks with characteristic dimensions of the order of grain sizes. The problem is to find the general features of crack growth for various material properties and loading regimes. The conditions of stability for cracks are of primary importance. A crack is stable when a small increment of its depth, or a small overloading, or a local change of material properties do not imply an intensive crack propagation resulting into the final rupture or approaching a dangerous state.
Linear fracture mechanics is based on the assumption that a material is linear elastic and deformations are small until the final failure. The term linear fracture mechanics is widely accepted because of the above assumptions, although it is not correct: fracture as the whole is a strongly nonlinear phenomenon. The term mechanics of brittle fracture is also used almost as a synonym of the linear fracture mechanics. When small yield zones near the tips are taken into account, one says on mechanics of quasibrittle fracture.

The simplest problem of linear fracture mechanics is the Griffith's problem of a plane opening mode crack in an unbounded medium in the plane-strain state. The crack with the length $2a$ is considered as a mathematical cut, and the far field, applied stresses $\sigma_\infty$ are normal to the plane of the crack (Figure 1). Material deformations follow to Hook's law with Young's modulus $E$ and Poisson's ratio $\nu$. To advance the crack tip from $a$ to $a + da$, the work is to be produced proportional to $da$. Griffith attributed this work to the energy of the surface forces. In reality the main part of the work is spent in plastic deformation and other irreversible phenomena. All these factors may be included into the specific fracture work $\gamma$. Hereafter, we relate the fracture work to the unit area of the crack (not to the unit area of the surface). The specific work $\gamma$ has the dimension $J/m^2$. For engineering materials the unit $kJ/m^2 = kN/m$ is more convenient.

Figure 1. Griffith's problem.
A crack does not propagate if the release of the potential energy of the system $\Pi$ due to the crack tip advancement is less than the required fracture work, i.e. $d\Pi < \gamma da$. At $d\Pi > \gamma da$ the released energy exceeds the required fracture work, and the energy surplus opens a possibility of the dynamic crack growth. As the result, we arrive to Griffith's equation for the critical stress:

$$\sigma_c = \left[ \frac{\gamma E}{\pi a(1-v^2)} \right]^{1/2}$$

An alternative approach was suggested in the 50s by Irwin. The stress field in the vicinity of the crack tip in a linear elastic body has a square-root singularity. Since the fracture process is a local phenomenon, it depends in the first line on the stress distribution close to the tip. The singular terms in formulas for stresses are of the form

$$\sigma_{jk}(r,\theta) = \frac{k_j}{(2\pi r)^{1/2}} f_{jk}(r,\theta)$$

Here $r$ is the polar radius, $\theta$ is the polar angle; indices $j$ and $k$ run for the coordinate notations $x, y, z$. Parameter $K_I$ is named stress intensity factor. In the Griffith's problem it is equal to

$$K_I = \sigma_x (\pi a)^{1/2}$$

The dimension of stress intensity factors is $N \cdot m^{-3/2}$. In practice it is more convenient to measure stress intensity factors in $MN \cdot m^{-3/2} = MPa \cdot m^{1/2}$. In terms of Irwin's approach, a crack does not grow at $K_I < K_{IC}$. The limit condition takes the form

$$K_I = K_{IC}$$

Here $K_{IC}$ is the critical value of the stress intensity factor frequently named fracture toughness. This material parameter is considered now as one of the most important characteristics entering many standards, norms and specifications. For most structural metallic alloys, $K_{IC}$ takes values between 20 and 100 MPa m^{1/2}. Equations (1) and (4) become equivalent if

$$K_{IC} = \left( \frac{\gamma E}{1-v^2} \right)^{1/2}$$

Another approach, also suggested by Irwin, is based on the far-going generalization of Griffith's idea. The system's energy released when a crack advances in a unit of length is $G = -\partial \Pi / \partial a$. 

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This value, named energy release rate, is of the dimension of force if the size in the \( z \)-direction is taken equal to unity. Another name for \( G \) is the force driving the crack tip or, briefly, driving force. For Griffith's crack we have

\[
G = \frac{\pi \sigma^2}{E} a (1 - \nu^2)
\]

(6)

The energy balance condition takes the form

\[
G = G_{IC}
\]

(7)

In the considered problem \( G_{IC} = \gamma \), where \( \gamma \) is the specific fracture work. Thus, we have three equivalent conditions for the critical state of a cracked body, namely equations (1), (4) and (7). It is evident that one of the three related parameters, \( \gamma, K_{IC} \) and \( G_{IC} \) may be used to characterize the fracture toughness of a material.

Figure 2. Modes of fracture: (I) opening mode; (II) fracture shear mode; (III) longitudinal shear mode.

The approach based on the concept of stress intensity factors appears to be the most convenient for practical calculations. For unbounded bodies, the three main problems are distinguished corresponding to the different modes of cracking (Figure 2): mode I (opening mode), mode II (transverse shear mode), and mode III (longitudinal shear, anti-plane mode). Stress intensity factors for these modes are

\[
K_I = \sigma_x (\pi a)^{1/2},
\]

\[
K_{II} = K_{III} = \tau_x (\pi a)^{1/2}
\]

(8)
where $\sigma_\infty$ and $\tau_\infty$ are applied (remote) stresses. Their directions are shown in Figure 2. Singular terms in equations for stresses in the vicinity of crack tips are similar to that in the case of mode I. The explicit equations can be found in any textbook on fracture mechanics (see Bibliography).

Equations (1), (5) and (6) correspond to the plane-strain state. To transfer them upon the plane-stress state (for example, to a thin plate), it is sufficient to replace $1 - \nu^2$ with unity. In general case, stress intensity factors depend on the geometry of a body, the crack shape and position. In particular, the stress intensity factor for the mode I crack is defined as

$$K_I = Y \sigma_\infty (\pi a)^{1/2} \quad (9)$$

with the correction factor $Y$ of the order of unity. If material is anisotropic, the correction factor depends on the ratios of elastic parameters. Reference data on stress intensity factors can be found in many hand- and textbooks.

When the condition given with equations (4) or (7) is attained, a crack in the Griffith problem becomes unstable: it begins to propagate dynamically with velocity limited with the Rayleigh velocity in the considered material. But in general case the attainment of equality in equations (4) or (7) does not necessarily signify instability, dynamic crack propagation, etc. For example, if a crack in an infinite plate is loaded with two forces applied normally to the crack's faces, the equilibrium of the system remains stable even when the forces proceed to grow. The crack size increases with the applied forces to keep the equality between, say, the energy release rate and its critical magnitude.

When $G_C = \text{const}$, the criterion of stability for a single-parameter crack takes the form

$$\frac{\partial G}{\partial a} < 0 \quad (10)$$
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Biographical Sketch

Bolotin Vladimir V., born March 29, 1926, Tambov, Russia. Graduated from the Moscow Institute of Railway Engineers as the Civil Engineer (Bridges and Tunnels) in 1948. Received from the same Institute the Degree of Candidate of Sciences in 1950, and the Degree of Doctor of Sciences in 1952.

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