MODELING OF PROCESSES OF SNOW COVER FORMATION AND SNOWMELT

Lev S. Kuchment
Water Problems Institute of the Russian Academy of Sciences, Moscow 119333, Russia

Keywords: Snow cover, snow metamorphosis, modeling of heat and moisture transfer in snow, snow melt, spatial variation of snow cover.

Contents

Summary

The chapter contains the general information on snow properties and main processes which occur in snow pack during cold period and during snowmelt. The models of hydrothermal processes and heat and moisture transfer in the dry and melting snow pack are described. The methods of calculation of snowmelt rate based on the energy budget of snow pack have been reviewed. To interpolate the spatial distributions of snow characteristics and to take into account the variation of statistic parameters of snow characteristics inside of areas of different sizes, the snow covers are considered as random fields. Variograms and correlograms of these fields constructed for a number of the river basins of various sizes have shown that for these fields in many cases can be accepted the hypotheses of homogeneity and isotropy which are allowed to use for snow interpolation the methods of kriging or optimal interpolation. The conditions of statistical self- similarity of random fields are presented and estimation of fractal dimensions of the snow depth and the snow water equivalent fields of different scales has been made.

1. Introduction

A permanent snow cover is formed on about 20% of the Northern hemisphere and about 15% of the Southern hemisphere. A significant part of land is covered by snow several times during cold period. Changing the heat balance of the land, the snow cover has a considerable effect on the climate. The presence of a snow cover on a drainage basin

also influences very greatly the runoff generation. In many parts of the world river runoff consists mainly of water yielded by the melting of snow. The snowmelt spring runoff of most large plain rivers of Russia and Canada exceeds half of annual runoff; at the same time, the portion of snowmelt runoff from mountain area in the arid regions can be significantly larger.
Snowfall over an area is more uniform than rainfall; however, snow accumulation is largely a function of elevation, slope, exposure, and vegetative cover. Snow spatial redistribution is strongly affected by the interaction of wind and topography as well as by interaction of wind and vegetation. Gullies and surface depressions are filled up by snow first of all and can accumulate a considerable portion of the total river basin snow resources. In forests much of the intercepted snow is blown off and accumulates on the open areas. During blowing and transport of snow significant evaporation may occur (the evaporation losses may reach 40-50% of annual snowfall). The snow water equivalent (the depth of water which would result from the melting of the snow) in forest areas is usually 10-40% more than in the open areas (in some cases, a general increase of precipitation in the forest is possible). Snow accumulation generally increases with elevation because of the combined effect of the prevailing lower temperatures and the increased frequency of precipitation events caused by orographic effects.

The small-scale variations of snow cover, caused by spatial change of relief, vegetation, and local meteorological conditions, are superimposed on large- scale variations associated with physiographic and climatic tonality. It leads to very large spatial variability of snow cover characteristics, and they are often considered as random values. The coefficients of spatial variation of the snow water equivalent range from 0.15-0.20 in the forest zone to 0.30-0.60 in the steppe zone.

After snowfall the snow pack undergoes significant transformation (metamorphosis) caused by compaction, action of the thermal gradients, and change of the crystal structure resulting from interactions of ice, liquid water, and water vapor. The air humidity in the snow pack is usually close to saturation; however at rapid changes of air temperature there may be a significant transfer of water vapor from the lower layers of snow (thermodiffusion of water vapor) and its considerable sublimation on the ice crystals. In addition to the changes brought about by vapor the freezing together of two or ice crystals takes place, especially in the presence of liquid water in a snow pack. This transformation of snow structure is called constructive metamorphosis. The term “destructive metamorphosis” applies to the transformation of the form of snowflakes, which takes place during the first one-two days period after deposition as a result of mechanical interaction of snowflakes. Because of surface tension, sharp edges and abrupt angles of stellar crystals are unstable and the common tendency of crystals is reduction of their overall surface area.

Constructive and destructive metamorphosis results in producing a uniform and coarse structure of the snow (this process is called snow ripening). In a fully ripe snow ice-crystal size is so large that at the beginning of the melt it can contain only from 3 to 5% of liquid water. The metamorphosis of snow significantly changes density and other physical properties of snow. Snow at the time of fall may have a density as low as 0.01 to as high as 0.15 gcm^{-3}; snowfall in the form of dry snow may vary in density between

0.07 and 0.15 gcm^{-3}; average wind-toughened snow has a density about 0.28-0.30 gcm^{-3}. Ripe snow has a uniform density of 0.4-0.5 gcm^{-3}. The influence of constructive metamorphosis on the snow density is usually small; however this process can considerably change the mechanical properties of snowpack and plays an important role in the heat transfer in snow. During melting period the snow density changes as a result of formation of liquid water and transformation of structure of ice crystals (especially when there is night refreezing of melt water).

The greatest density which can be attained by shifting the snow grains around, is about 0.55 gcm^{-3}. Further densification, which can occur under the action of deformation, refreezing, recrystallization, produces a compact, dense material called firn. At a density of between 0.82 and 0.84 gcm^{-3}, the air spaces disappear and the material becomes impermeable to air and water. This material can be defined as ice. The old ice has the density about 0.90 gcm^{-3}; the theoretical density of pure ice is 0.92 gcm^{-3}. Accumulation on land of ice resulted from recrystallization of snow or other forms of precipitation leads to formation of glaciers. Typical peculiarities of the glacier are the presence of an area where snow or ice accumulates in excess of melting, and another area where the wastage of snow or ice exceeds the accumulation, as well as a slow transfer of mass from the first area to the second.

Being a porous medium, the snow pack has much in common with the soil. In the dry snow, liquid water is retained mostly by film tension and capillary forces. The porosity of snow varies from 0.80-0.87 (for new snow) to 0.50-0.70 (for old coarse-grained snow). The liquid -water holding capability of snow (the maximum value of liquid water content beyond which water will drain by gravity action) is about 0.13- 0.15. The movement of water through snow pack begins when snow pack is saturated by liquid water beyond these values. In the period of snowmelt, a part of the liquid water may refreeze.

The proportion of solar radiation falling on snow cover and then reflected (albedo) is high compared with soil and vegetation and varies over the winter. The new snow has an albedo 0.75 - 0.90 and after ripening the albedo can reach 0.35-0.40. The empirical dependence can usually be constructed between the albedo and the snow density as a characteristic of snow ripening. A close relationship also exists between albedo and the accumulated daily maximum temperature after the last snowfall. The sensible heat is the second important energy budget member. Sometimes, the precipitation heat can be a considerable contribution to positive snow pack energy balance. However in most cases effects of rainfall on the ripening snow and a decrease of albedo are more important.

The latent heat of vaporization is extremely large, approximately 2.83 MJ kg^{-1} of snow. The energy required to sublimate 1 kg of snow therefore is equivalent to that required to raise the temperature of 10 kg of liquid water by 67° C. Vaporization is reversible, as this energy is released to the environment upon recrystallization of vapor to ice. The latent heat of fusion is large, approximately 333 kJ kg^{-1} of snow. The energy required to melt 1 kg of snow (already at 0°C) therefore is equivalent to that required to raise the temperature of l kg of water to 79° C. Latent heat is released to the environment during freezing, when liquid water crystallizes.

The thermal conductivity of a snow cover is low compared with soil surfaces and varies with the density and liquid water content of the snow cover. A typical value of thermal conductivity for dry snow with a density of 100 kg m^{-3} is 0.045 W m^{-1} K^{-1}, over six times less than that for soil.

2. Modeling of Hydrothermal Processes in the Dry Snow Pack

Temperature regime in dry snow pack is exceedingly complex and is controlled by a balance of the energy fluxes at the top and bottom of the snow pack, radiation penetration, amount of snowfall, thermal conductivity of snow layers and metamorphosis processes.

Snow during winter periods with small influence of thaws can be considered as a dry porous medium where the ice crystals are fixed and vertical water transfer is going on in the form of vapor flux. Under these assumptions, it is possible to describe hydrothermal processes in the dry snow pack including constructive metamorphosis as follows

$$\frac{\partial}{\partial t}\left[\rho_{\mathrm{v}}\left(1-I\right)\right]-\rho_{\mathrm{v}}\frac{\partial I}{\partial t}=-\frac{\partial q_{\mathrm{v}}}{\partial z} \qquad (1)$$

(mass balance equation)

$$C_{\mathrm{ef}}\frac{\partial T_{\mathrm{s}}}{\partial t}=-C_{\mathrm{v}}q_{\mathrm{v}}\frac{\partial T_{\mathrm{s}}}{\partial z}+\rho_{\mathrm{I}}L_{\mathrm{s}}\frac{\partial I}{\partial t}-\frac{\partial q_{\mathrm{T}}}{\partial z} \qquad (2)$$

(energy balance equation)

where ρ_{v}, q_{v}, and C_{v} are the density, unit flux and heat capacity of vapor, respectively, C_{ef} is the effective heat capacity of snow pack, which is equal to $\rho_{\mathrm{I}}C_{\mathrm{I}}I+\left(\rho_{\mathrm{a}}C_{\mathrm{a}}+\rho_{\mathrm{v}}c_{\mathrm{v}}\right)\left(1-I\right)$, ρ_{a} and C_{a} are the density and heat capacity of air, ρ_{I} is the ice density, L_{s} is the latent heat of sublimation, q_{T} is the conductive heat transfer in snow pack, I is the volumetric content of ice in snow pack, T_{s} is the snow temperature.

Assuming

$$q_{\mathrm{v}}=-D_{\mathrm{v}}\frac{\partial\rho_{\mathrm{v}}}{\partial z} \qquad (3)$$

where D_{v} is the effective coefficient of vapor diffusion and substituting this expression in the mass balance equation gives

$$\left(\rho_{\mathrm{I}}-\rho_{\mathrm{v}}\right)\frac{\partial I}{\partial t}=\frac{1}{\partial z}\left(D_{\mathrm{v}}\frac{\partial\rho_{\mathrm{v}}}{\partial t}\right)+\left(I-1\right)\frac{\partial\rho_{\mathrm{v}}}{\partial z} \qquad (4)$$

Treating ρ_{v} and D_{v} as functions of the snow temperature T_{s} and taking into account that $\rho_{\mathrm{I}} \gg \rho_{\mathrm{v}}$ gives the mass balance equation in the following form

$$\rho_{\mathrm{v}}\frac{\partial I}{\partial t}+\left(1-I\right)\frac{\partial\rho_{\mathrm{v}}}{\partial T_{\mathrm{s}}}\frac{\partial T_{\mathrm{s}}}{\partial t}=D_{\mathrm{v}}\frac{\partial\rho_{\mathrm{v}}}{\partial T_{\mathrm{s}}}+\frac{\partial}{\partial z}\left(D_{\mathrm{v}}\frac{\partial\rho_{\mathrm{v}}}{\partial T}\right)\left(\frac{\partial T_{\mathrm{s}}}{\partial z}\right)^{2} \tag{5}$$

Assuming that

$$q_{\mathrm{T}}=-\lambda_{\mathrm{s}}\left(\rho_{\mathrm{s}}\right)\frac{\partial T_{\mathrm{s}}}{\partial z} \tag{6}$$

where $\lambda_{\mathrm{s}}\left(\rho_{\mathrm{s}}\right)$ is the heat transfer coefficient of snow enables to transform the energy balance equation to the following form:

$$\begin{aligned}&\left[C_{\mathrm{ef}}+\left(1-I\right)\frac{\partial\rho_{\mathrm{v}}}{\partial T}L_{\mathrm{s}}\right]\frac{\partial T_{\mathrm{s}}}{\partial t}=\left(\lambda_{\mathrm{s}}+L_{\mathrm{s}}D_{\mathrm{v}}\frac{\partial\rho_{\mathrm{s}}}{\partial T_{\mathrm{s}}}\right)\frac{\partial^{2}T_{\mathrm{s}}}{\partial z^{2}}+\\&+\left[C_{\mathrm{v}}D_{\mathrm{v}}\frac{\partial\rho_{\mathrm{v}}}{\partial T_{\mathrm{s}}}+L_{\mathrm{s}}\frac{\partial}{\partial T_{\mathrm{s}}}\left(D_{\mathrm{v}}\frac{\partial\rho_{\mathrm{v}}}{\partial T_{\mathrm{s}}}\right)\left(\frac{\partial T_{\mathrm{s}}}{\partial z}\right)^{2}+\frac{\partial\lambda_{\mathrm{s}}}{\partial T_{\mathrm{s}}}\frac{\partial\rho_{\mathrm{s}}}{\partial z}\frac{\partial T_{\mathrm{s}}}{\partial z}\right]\end{aligned} \tag{7}$$

Comparing the terms of this equation shows that the terms

$\left(1-I\right)\frac{\partial\rho_{\mathrm{v}}}{\partial T_{\mathrm{r}}}L_{\mathrm{s}}$ and $C_{\mathrm{v}}=D_{\mathrm{v}}\frac{\partial\rho_{\mathrm{v}}}{\partial T_{\mathrm{s}}}$ can be neglected and instead of (7) we can use the following equation:

$$C_{\mathrm{ef}}\frac{\partial T_{\mathrm{s}}}{\partial t}=(\lambda_{\mathrm{s}}+L_{\mathrm{s}}D_{\mathrm{v}}\frac{\partial\rho_{\mathrm{s}}}{\partial T_{\mathrm{s}}})\frac{\partial^{2}T_{\mathrm{s}}}{\partial z^{2}}+L_{\mathrm{s}}\frac{\partial}{\partial T_{\mathrm{s}}}(D_{\mathrm{v}}\frac{\partial\rho_{\mathrm{v}}}{\partial T_{\mathrm{s}}})(\frac{\partial T_{\mathrm{s}}}{\partial z})^{2}+\frac{\partial\lambda_{\mathrm{s}}}{\partial T_{\mathrm{s}}}\frac{\partial\rho_{\mathrm{s}}}{\partial z}\frac{\partial T_{\mathrm{s}}}{\partial z}. \tag{8}$$

The effective coefficient of vapor diffusion D_{v} depends mainly on snow temperature and, in less extent, on snow density.

To calculate heat transfer in dry snow for comparatively short period (to 2-3 months) Eqs. (5) and (8) can be simplified. At ρ_{s} > 0.05 g/cm^{3} s the term $\rho_{\mathrm{I}}C_{\mathrm{I}}I$ is significantly larger than the other terms including in the coefficient at $\frac{\partial T_{\mathrm{s}}}{\partial t}$. It is also possible to neglect heat and mass transfer by vapor and to use instead of the snow thermal conductivity the effective snow thermal conductivity λ_{ef} which accounts for the heat transfer by vapor. As a result, instead of the system (5-8), we have

$$\frac{\partial I}{\partial t}=0$$

$$C_I \rho_I I \frac{\partial T_s}{\partial t} = \frac{\partial}{\partial z}\left(\lambda_{ef} \frac{\partial T_s}{\partial z} \right) \tag{9}$$

Kuchment et. al. (1983) combining the experimental data of a number of researchers obtained the empirical relationship

$$\lambda_{ef}(\rho_s) = 0.00005 + 0.004\rho_s^2 \tag{10}$$

(λ_{ef} in cal/cm sec ^{o}C; ρ_s in g/cm^3).

At low negative temperature the role of water vapor in heat transfer increases and the value of λ_{ef} decreases.

For describing the densification of snow cover caused by destructive metamorphosis, Anderson (1976) suggested to use the following equations.

$$\frac{1}{\rho_s}\frac{\partial \rho_s}{\partial t} = C_3 \exp[-C_4(T_0 - T_s)] \text{ for } \rho_s < \rho_d \tag{11}$$

$$\frac{1}{\rho_s}\frac{\partial \rho_s}{\partial t} = C_3 \exp[-C_4(T_0 - T_s)\exp[-46(\rho_s - \rho_d)] \text{ for } \rho_s > \rho_d$$

where ρ_d is a calibrated critical value of snow density, C_3 and C_4 are calibrated coefficients, T_s is in K, T_0 = 273K .The numerical experiments carried out using this solution for a 1.2m snow layer in (Kuchment et al., 1983) showed that the terms of equations which accounted for the vapor–ice phase transitions affect mainly the snow temperature, increasing effective snow thermal conductivity. Differences in the snow temperatures calculated with accounting for phase transitions vapor –ice and without these transitions can reach 10%.

-
-
-

Bibliography

Anderson E. A. (1976). A point energy and mass balance model of snow cover.—In: NOAA Technical report. NWS19. Washington,—150 p. [The paper contains the results of investigations of energy and mass balance of snow pack and as a result, the relationships for calculation of energy components are suggested].

Colbeck, S. C. (1972). A theory of water percolation in snow. *J. Glaciol.*, vol. 11, N 3, p. 369—385. [The equations of moisture transfer in snow during melting are given].

Kuchment L.S., V.N. Demidov, and Y.G. Motovilov. (1983). *River Runoff Formation (Physically-Based models)*. Nauka, Moscow (in Russ.). [The book contains the description of models of snow cover formation and snowmelt. Numerical methods for simulations of models are given. The results of snow model simulation are compared to laboratory and field measurements. Application of snow cover and snow melt models for simulation of snowmelt runoff are shown].

Kuchment L.S. and A.N. Gelfan (2001). Statistical self-similarity of spatial variations of snow-cover: verification of the hypothesis and application for runoff modelling. *Hydrological processes*, 2001, No. 12. [The methods of verification of the hypothesis on statistical self-similarity of spatial variations of snow-cover are described and opportunities of use this hypothesis for accounting for subgrid effects in distributed snowmelt runoff generation models are presented].

Kuusisto E. (1984). *Snow accumulation and snowmelt in Finland*. - Publ. Water Resources Institute, Helsinki, 151p. [The book includes results of analysis of comprehensive material of field observations of snow accumulation and snowmelt energy and mass components in Finland].

Kuzmin, P.P. (1961). Melting of Snow Cover. Hydrometeoizdat. 179 p. (In Russ.) [Methods of calculations of snow melt based on the energy budget and standard meteorological observation are presented.].

Kuzmin PP. (1963). Formirovanie Snezhnogo Pokrova I Metody Opredeleniya Snegozapasov (Snow Cover and Snow Reserves). [English Translation by Israel Program for Scientific Translation, Jerusalem]. 139 p. [The results detailed field experimental investigations of energy and mass components of snow pack dynamics are presented].

Male D.H., and D. M. Gray. (1981). *Handbook of Snow. Principles, Processes, Management and Use*, Pergamon press, 745p. [The book contains the main information on physical properties of snow, formation of snow cover and spatial distribution, snow transport, snow melt and ablation, role snow in the hydrological cycle and climate. A significant part of the book dedicated to role of snow in human society and snow management].

Morris, E.M. (1983). Modeling the Flow of Mass and Energy within a Snow Pack. *Annals of Glaciology* 4 p.198-203. [The equations describing the heat and mass transfer in snow pack with application for hydrological forecasting are presented].

Pomeroy, J. W. and D. M. Gray. (1995). Snowcover: accumulation, relocation and management. NHRI Science Report No. 7. National Hydrology Research Institute, Sascatoon, Saskatchewan, Canada. 135 p. [The scientific report on snow accumulation, relocation and management in Canada which include comprehensive data for a better understanding the natural behaviour of snow].

Tuteja, N., and K.C. Cunnane. (1997). Modelling coupled transport of mass and energy into the snowpack-model development, validation and sensitivity analysis. *Journal of Hydrology,* 195, p.232-255 [Numerical methods of solution of the differential equations describing the transport of mass and energy into the snow pack and validation of snow pack models].

Biographical Sketch

Lev S. Kuchment has held the position of Head of the Laboratory of the Hydrological Cycle at the Water Problems Institute of the Russian Academy of Sciences, Moscow, since 1977. Professor Kuchment is the author of 175 scientific publications, including 9 books. He is an expert in the modeling of hydrological processes and application of these models in hydrological forecasting and design. His research interests have included: unsteady flow in river channels, snow cover formation; heat and moisture transfer in soil; evapotranspiration; overland, subsurface and groundwater flow; interaction of surface water and groundwater; water quality formation; the hydrological cycle as a whole. He has also dealt with estimation of human induced changes of the hydrological cycle and possible hydrological impacts of global climate change. In recent years, his main research fields are risk assessment of catastrophic floods .Prof. L.S. Kuchment is the Honorable Scientist of Russian Federation. His main publications are: Kuchment L.S. (1980) Models of runoff formation processes ,Leningrad, Gidrometeoizdat, 143 p. *(In Russ.).* Kuchment L.S., Demidov V.N., Startzeva Z.P. (2005) Couple modeling of the hydrological and

carbon cycles in the soil-vegetation-atmosphere system. Journal of Hydrology, vol.313, Nov., 2005. Kuchment L.S. and Gelfan A. N. (2002) Estimation of extreme flood characteristics using physically based models of runoff generation and stochastic meteorological inputs. Water International, vol.27, no.1, p.77-87. Kuchment L.S., Gelfan A. N. (2001) Statistical self-similarity of spatial variations of snow-cover: verification of the hypothesis and application for runoff modelling. Hydrological processes, 2001, No. 12, pp.21-37. Kuchment L.S., Demidov V.N., and A. N. Gelfan (2000) A Distributed Model of Runoff Generation in the Permafrost Regions// Journal of Hydrology, Vol 240, N 1-2, pp. 1-22. Kuchment L.S. River runoff (generation, modeling, prediction). Institute of Water Problems of RAS, Moscow, 2008, 394 p. *(in Russ.).*

L.S. Kuchment can be contacted at the Water Problems Institute of Russian Academy of Sciences, 3, Gubkin Str., Moscow119333, Russia. Email: kuchment@mail.ru.