MATHEMATICAL MODELS OF SOIL IRRIGATION AND SALTING

L.V.Kireycheva,
All-Russian Research Institute of Hydraulic Engineering and Land Reclamation, Russia

Keywords: Irrigation, Underground water, Underground Hydrodynamics, Forecast, Drainage, Soil Salting, Evapotranspiration, Simulation models.

Contents

Summary

Mathematical models of soil irrigation and salting are reviewed. The simplest irrigation models are those based on the balance of water and salt. They were created as a tool for irrigation management with such requirements as short computational time, the minimum of the natural data, and the opportunity to take into consideration a wide range of the factors. It is shown that the models have some serious limitations.

Simulation models for more accurate description of water and salts distribution in soil under irrigation, are discussed. The water and salts transport in soil is described by the differential equations of parabolic type. They are deduced on the basis of the law of mass conservation, the Darcy law, the Fick equation, and the Fourier law. One-dimensional simulation models cannot take into account all possible processes arising in salty soils under irrigation.

Field scale soil heterogeneity, nonlinearity of seepage processes, interaction between root zone and ground waters, and drainage required the development of dimensional models for water, soil solute, nutrients, and heat transport in the “atmosphere – plant – soil - ground water” system. It was demonstrated that the most serious difficulties in the application of models arise from the lack of adequate data for adequately describing natural phenomena. Due to this reason they, as a rule, are applied in scientific researches. However, these models serve as an important tool to achieve the conceptual understanding of complex systems.

It is concluded that further development will be towards more detailed comprehensive description of every process and modular structure and versatility of simulation programs to allow the user to combine individual modules, depending on the calculation purposes, for example, for irrigation and soil salinity management with respect to environmental considerations.

1. Introduction

Mathematical modeling is successfully used as the main quantitative approach to study the processes of transport of water and solutions in irrigation of salty soils. Intensive agriculture requires forecasts for the management of underground hydrodynamics. Knowledge of water and solute transport in soils is important for understanding human influence on the environment. Water and solute transport in soils is determined by evapotranspiration, surface runoff, and deep infiltration. It is one of the key factors in the hydrological cycle. Soil water transfers significant amounts of solute substances including nutrients, mineral salts, and pollutants. Therefore simulation of solute transport in saturated and in unsaturated zones is of great importance for management of plant growth as well as for the environmental aspect of agriculture. Presently there is scarcity of water resources in many countries. Hence environmental problems arise. Irrigation management requires models as a tool for calculations of irrigation rate and timetable, and the depth of soil wetting. Besides, the models allow estimation of the influence of irrigation on underground hydrodynamics of the territory under irrigation and on the crop yield.

For accurate estimation of environmental changes as a result of irrigation and for effective management we need to study the regularities in underground hydrodynamics, namely water transport, salts and nutrients transport in the “water-soil-plant-atmosphere” system. It is very important to be able to forecast these variables and mathematical models are helpful in this respect.

A model is a simplified and formalized representation of a real natural system. For understanding the system behavior from the viewpoint of its intrinsic mechanism simulation models are usually employed. Such a model can be designed by subdividing the system into separated components. Then, the behavior of each component and the connections between components are described. Simulation modeling widely uses system analysis approach that opens major new ways for the study of behaviour of natural objects. During the last thirty years significant advances have taken place in the field of modeling of water and salts transport in soil under irrigation.

The physical basis for mathematical modeling of water and salts transport in soil is as follows. Under irrigation, water is usually delivered on the earth surface. Then, after infiltration a certain water regime forms, which is necessary for agricultural crops planting. The water regime is determined by the following mechanisms of water transport: filtration, convection, diffusion and dispersion, as well as vapor transport and water uptake by the plant roots.

The theory of water transport in soil is based on the experimentally established statement, that water flux is proportional to pressure head gradient:

$$\vec{q} = -K(h)\,\mathrm{grad}H \qquad (1)$$

where $\vec{q}$ is the soil water flux density (cm d^{-1}), which is called usually the filtration rate for total saturation zone; K is the hydraulic conductivity (cm d^{-1}), h is the soil water pressure head (cm), H is the soil water hydraulic head (cm).

Mass-transfer processes modeling in a soil uses the continuity hypothesis, which supposes the space and time continuity of the parameters (humidity, pressure etc) of continuum under consideration. The equation of water mass conservation can be presented as

$$\frac{\partial \theta}{\partial t} + \mathrm{div}\vec{q} = -J\,, \qquad (2)$$

where θ is the volumetric water content (cm^3cm^{-3}), t is time in days (d), J is the intensity of internal uptake (by plants roots, by evaporation in soil) (g $cm^{-3}d^{-1}$).
In the Cartesian coordinates the hydraulic head is equal to:

$H = \psi - z$, where $\psi = \frac{p}{\rho_{sol}}$ is the potential of capillary and sorption forces for unsaturated zone (cm), p is the water suction pressure in the unsaturated zone of soil; ρ_{sol} is the solute density (g cm^{-2}); z is vertical coordinate (cm).

The relationship between the water content θ, the pressure head h and the hydraulic conductivity K are generally summarized in the retention function $\theta(h)$ and the unsaturated hydraulic function $K(\theta)$. These soil hydraulic functions need to be specified for each soil layer individually.

The salt transport formation is a complex of processes. The three main solute transport mechanisms are diffusion, convection and dispersion. Diffusion is solute transport, which is caused by the solute gradient. Thermal motions of the solute molecules within the soil solution cause a net transport of molecules from high to low concentrations. The solute flux J_{dif} (g cm^{-2} d^{-1}) is generally described by the first Fick law:

$$J_{dif} = -\theta D_{dif} \frac{\partial C}{\partial z} \qquad (3)$$

with the diffusion coefficient D_{dif} (cm^2 d^{-1}) and the solute concentration C in soil water (g cm^{-3}). The coefficient D_{dif} is very sensitive to the actual water content, as it strongly affects the solute transport path and the effective cross-sectional transport area.

When describing water flow, it needs to consider the water velocity variation between pores of different size and geometry and also the water velocity variation inside a pore itself. The variety of water velocities causes some solutes to advance faster than the average solute front, and other solutes to advance slower. The overall effect will be that steep solute fronts tends to smoothen or to disperse. Besides, there is the ion exchange between moving and stationary (bypass pores) states, and interface exchange by sorption-desorption, dissolution and leaching.
Taking into account physical-chemical interaction of pore solution with porous environment the Fick equation in its general form is used:

$$q_{salt} = -D\,\mathrm{grad}C + \vec{q}C, \qquad (4)$$

where q_{salt} is the solute mass (g cm^{-2}d^{-1}); C is the salt concentration in water (g cm^{-3}); D is the convective diffusion coefficient

In this case the continuity relation of solute transport can be expressed as:

$$\frac{\partial \theta C}{\partial t} + \frac{\partial N}{\partial t} + \mathrm{div}\vec{q} = -S, \qquad (5)$$

where N is the salt concentration in solid phase surface (g cm^{-2}), S is the root water extraction rate (cm^3 cm^{-3}d^{-1}).

The mass exchange rate between solid phase and soil solution can be determined using the kinetic equation of salts dissolution:

$$\frac{\partial N}{\partial t} = \bar{\gamma}\left(C - C_s\right), \qquad (6)$$

where $\bar{\gamma}$ is the parameter of dissolution rate (d^{-1}); C_s is the solubility.

In soils of unsaturated zone the solutions transport takes place at the equilibrium and nonequilibrium solid phase and soil solution concentrations, the nonequilibrium conditions being reversible or irreversible. In unsaturated soils the solute transport phenomena is more complicated than in saturated ones because of the intensively changing water flux parameters in soil near earth surface.

Ion exchange modeling has great practical importance. Ion exchange is a physical-chemical process of the exchange between solution ions and ions in exchangeable form of solid.

To develop a general model, which considers all details of water-salt transport in the system “water-soil-plant-atmosphere” and is good for practical calculation, is difficult.

A comprehensive resolution of this question is hardly possible. At the present time the development of mathematical models is carried out using simplified phenomenological equations for solute water and salts in soils. Depending on the requirements the developed models consider the elemental processes in complicated system in the different degrees of approximation.

The simplest balance models are used for the calculation of irrigation rate and water application timetable. Whereas empirical models are applied for the estimation and the forecast of agricultural crops yield. As it was demonstrated by practice, to describe water transport in the porous environment, it is very reasonable to use deterministic or stochastic models, which reflect the physical basis of the transport processes. The deterministic models are better developed now. They are applied more often both for the researches and practical purposes.

To describe the solute transport in general ion transport should be considered in the background of all physical and chemical phenomena. For this purpose the kinetic or thermodynamic approaches are used. Both consist in the description of energy parameters of the solutions. These two approaches are non-alternative because of their correlation.

Mathematical models of water and salts transport under irrigation consider the soil as a homogeneous or heterogeneous medium. The heterogeneity is estimated using the similar media scaling method or the model of building-blocks media with double capacity (porosity). There are the variants and modifications of heterogeneous model depending on the method of estimation of the solutions transport in inter-block space.

One-dimensional water-salt models, which are used for the irrigation management in non-drained soils, are well developed. Under drainage the water and solutions transport can be considered in two-dimensional profile. The upper boundary of the investigated area is considered on the earth surface, or at any depth of an unsaturated zone. The bottom boundary is a confining overlying layer. Under more complex boundary conditions the regional models can be applied, for example, by setting the pseudo-two-dimensional bottom boundary conditions (drainage conditions and transport from deeper aquifer).

The mathematical models are considered below in the order of their sophistication.

2. Balance models of calculation of the irrigation regime and crops productivity

The most simple irrigation models are the models based on balance water and salt volumes ratio. The simplified models were created as a tool for irrigation management. The requirements upon them were short computational time, the minimum of the natural data, and the opportunity to take into consideration a wide range of the factors, which influence on return waters quality from irrigated fields. According to this philosophy the two following independent versions were developed: "forecast" and "development". The model "development" uses preliminary given water application time-table, including time, depth of wetting, and irrigating water volume and quality for every water application, to determine the influence of this regime on water and salt quantity and which inflow to unsaturated zone. The model "forecast" differs from the first one and defines the depth of wetting for every water application so that salts concentration in a soil solution would not exceed the allowable limit. It is based on the concept of constant accumulation of salts in a soil profile.

The models based on simple water and salts balance have the distinct restrictions, which are connected with the procedure of extrapolation of point field data to total area under consideration. Under this approach, the total area is considered as a separate soil column with the mean parameters. As a result only average parameters of soil salinity can be predicted.

2. 1. The calculation of actual time and water delivery

For the calculation of actual time and water delivery the model RELREG was developed by Teixeira and Pereira in 1995. This model is intended to support farmer decisions on when and how much to irrigate based on actual observations of parameters relative to crop water use. The model utilizes the water balance routines developed for the ISAREG simulation model. This considers the evolution of soil water reserves in conditions of optimal or non-optimal yield as influenced by the effective rainfall, the crop evapotranspiration, the irrigation volumes and the upward and downward fluxes below the root zone.

The model uses two field files including the characteristics of the crop and the soil, a meteorological file relative to precipitation and evapotranspiration, an irrigation file storing the on-going irrigation scheduling, and a ground water contribution file.

The model performs a daily water balance since the beginning of crop establishment until the actual day, together with a forecast for the next five days. Thus the user gets the soil water balance information required to decide when to irrigate.

The soil water balance equation can be written in the form:

$$\Delta R=(P_e+V_Z+I+GW-ET_a-D_r)\Delta t, \qquad (7)$$

where ΔR is the variation of the soil water reserve (mm), P_e the effective precipitation (mm), V_Z the water stored in the deeper soil layers (mm), I the irrigation depth (mm), GW the groundwater contribution (mm), D_r the deep percolation losses (mm), Δt the time interval (days), ET_a the actual sum water consumption (mm).

The calculations are carried out for three zones: (*i*) – overlogging, (*ii*) – optimal humidity, (*iii*) – insufficient humidity.

For the optimal zone $D_r=0;\ GW=0;\ ET_a=ET_m$, where ET_m is the sum water consumption at maximal crop yield.Then

$$R(t)=R_i+(P_e+V_Z-ET_m)\Delta t\ , \qquad (8)$$

where R_i is the initial R value for the stress zone $R<R_{min}$. Thus, $ET_a<ET_m$ and R depends on available water in root zone:

$$R(t) = \frac{P_e + V_z + G}{\alpha}(R_i - \frac{P_e + V_z + G}{\alpha})e^{-\alpha t}, \qquad (9)$$

where $\alpha = \frac{ET_{min} + GC}{R_{min}}$, GC is the potential of capillary rise.

The model RELREG seems to satisfy the objectives, which were identified as:

- the simulation of the soil water balance has the accuracy required to compute irrigation depths (mm) and dates of application;
- the capabilities to forecast water deficits within the next five days are useful for planning irrigation applications in real time.

2.2. Water and salt balance calculation under slow drainage

Typical water balance calculations determine the fall in level of soil water available for crop growth. One of them is the model created by M.Parkes, R. Bailey, D. Williams and Y.Li in 1995. The model is based on the water balance for root zone, slow drainage and sum evaporation being taken into account. Changes in water level or 'deficit' are calculated from gains by irrigation or rainfall together with losses due to soil evaporation, crop transpiration, drainage or runoff.

The unsaturated hydraulic conductivity functions:

$$K_i=Ke^{\alpha\,(\theta-\theta max)}, \qquad (10)$$

where α is the constant in the exponential function, K the hydraulic conductivity (mm h^{-1}), θ the water content (mm m^{-1}), θ_{max} the maximal water content (mm m^{-1}).
For daily time increments the cumulative drainage flow is estimated by:

$$\sum W_L = \sum_{i=1}^{\infty} m(Z_{i-1} - ET_a)\left[\frac{Z_{i-1} - ET_a}{Z}\right]^{\frac{1}{m}}, \qquad (11)$$

where W_L is the drainage flow volume at the depth L_m (mm), i the number of days after irrigation, ET_a the actual sum water consumption on day i (mm), $Z_{i\text{-}1}$ the profile water content i-1 days after irrigation (mm), Z the profile water content 1 day after irrigation (mm), m the constant for the soil profile, L_{max} the maximum rooting depth (mm).

Stores and rules of operation for root zone are identified for mobile (preferential macropore) and immobile water separately.

- Slow mobile water store (excluding bypass pore space) has losses by evapotranspiration ET_a and slow drainage W_L with drainage from maximum rooting depth. The bypass flow, beyond the root zone, which occurs when 24-

hour rainfall causes the slow mobile water store to overfill, can be also considered.

- Immobile water store has ET_a losses only; it is filled first by rainfall; has the balance taken from it when ET_a, losses exceed intercepted drainage; with ET_a losses applying to current rooting depth, depending on extent of crop cover % (*CCi*) and soil type.
- Input data

Five functions must be determined daily for overall water balance calculations, including the following:

- actual evapotranspiration on day *i*, depending on crop and soil coefficients
- rainfall and irrigation on day *i*
- drainage flux on day *i*, which depends on time and maximal root depth *L*
- immobile water loss up to and including day *i*.
- slow mobile water store contents on day *i*.

Actual evapotranspiration is calculated by

$$ET_a=ET_{a0}[K_c(K_s)+K_w(0{,}9- K_c(K_s))], \qquad K_c(K_s) = 0.9, \qquad (12)$$

where ET_{a0} is the reference evapotranspiration of a short cut grass crop adequately supplied with water (mm d^{-1}), K_c the crop coefficient related to % crop cover on *i*-day, K_s the crop stress coefficient, K_w the coefficient equal to 0.8, 0.5 and 0.3 for the first, second and third day respectively following rain or irrigation.

Output data: Water deficient and actual time and water delivery.

Model advantages and limitations:

The model gives the good description of infiltration and seepage conditions during cold rainy season with low intensity rainfall, which provides the slow drainage, as well as in the case of deep drainaged soils, when upper soil and subprofile layers are unsaturated.

-
-
-

Bibliography

Van Dam J.C., Huygen J., Wesseling J.G.R.A., Feddes R.A., Kabat P., Van Walsum P.E.V., Groenendijk P. And Van Diepen C.A.(1997). *Theory of SWAP version 2.0. Simulation of water flow, solute transport and plant growth in the Soil-Water-Atmosphere-Plant environment.* Wageningen Agricultural University and DLO Winand Staring Centre. Technical Document 45. 168 pp[This manual describes the theoretical background and modeling concepts that were used for soil water flow, solute transport, heat flow, evapotranspiration, crop growth, multi-level drainage and interaction between fields water balance and surface water management].

Pereira L.S., Van den Broek B.J., Kabat P. and Allen R.G.(1995). *Crop-Water-Simulation Models in Practice.* Selected papers of the 2nd Workshop on Crop-Water-Models organized by the ICID Working Group. WAGENINGEN Pers, Wageningen. The Netherlands. 339 pp. [The papers included in this volume cover a large spectrum of modeling objectives, approaches, and applications].

Tyagi, N.K., Kamra, S.K., Minhas, P.S. and Singh N.T. (1993). *Sustainable Irrigation in Saline Environment*, Proc. of National Workshop on Sustainable Irrigation, Karnal, 229 pp. [Papers in this volume deal with research solutions to the problems of irrigation sustainability in saline environment].

Yakirevich A.M. (1989). *The water quality estimation by mathematical modeling* Melioratsiya I vodnoe khozyaistvo, N10, in Russian, 42-43 p. [The paper describes the one-dimensional model of water and salt transport for the unsaturated-saturated zone including ion exchange, adsorption and salts sedimentation-dissolution processes].

Faibishenko B.A. (1986) *Water-salt regime of soils under irrigation*, Agropromizdat, Moscow, in Russian, 304 pp. [The regularities in the water-salt regime formation in soil for the irrigated, drained and near canal territories are discribed. The water and salts transport models are considered to control and to make the forecast of the water-salt regime in the irrigated lands].

Pachebsky Ya.A. (1980) *The matematical models for simulation of physical and chemical phenomena in soils*, Moscow, Nauka, in Russian, 293 pp. [This book contains a comprehensive description of the mathematical models widely used in the soil science. The models presented correspond to different soils].

Biographical Sketch

Kireycheva L.V. is Head of environmentally sustainable technologies department in All-Russian Research Institute of Hydraulic Engineering and Land Reclamation, Russia. She was born in 1944, graduated from M.V.Lomonosov Moscow state university, and authored more than 120 scientific publications and three books. She is an expert in the field of irrigation, soil salinity, drainage, the modeling of water-salt transport in soil under irrigation with respect to environmental considerations, water and soil cleaning from pollutants (pesticides, heavy metals, petrol products etc).