INfiltration AND PONDing

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Summary

The original model of Green and Ampt is used as a basic tool to obtain quantitative understanding of infiltration and associated soil properties. Indeed, their result is a limiting case of any description of soil-water behavior.

At the next level of approximation, most soils can be adequately described by interpolating between two limiting behaviors, one being the Green and Ampt soil, the other being that of a Gardner soil. For the latter, soil profiles do not have to be discontinuous as in the original Green and Ampt model. Also, the time condensation approximation appears as an exact result for this case.

Finally, a third level of approximation gives further corrections leading to better descriptions of soil-water profiles and ponding times that can be used to assess the errors of the time condensation approximation.

1. Introduction

Accurate description of infiltration and post-ponding runoff remains a fundamental problem in hydrology. Water added to an unsaturated soil will be absorbed until it ponds at the surface. The prediction of ponding time is especially crucial as the time condensation approximation (TCA) remains a widespread practical tool. Recall that the TCA basically assumes that at ponding there is a unique relationship between the cumulative infiltration and the flux, independent of previous history. In order to estimate the portion of precipitation that enters the soil and that which becomes overland flow, calculations require estimates of ponding time as the first step. Subsequently, infiltration after ponding is a key quantity that needs to be quantified. It is often handled by empirical results that must be used carefully as their application cannot be universal depending as they do on local conditions.

The Green and Ampt (1911) model is the earliest physically based conceptual infiltration model and to this day can be considered as having played a fundamental role in the description of infiltration. Although it possesses well known shortcomings, these can be redressed and incorporated into new models, which remain both physically based and usable in practice. Existing models have been reviewed extensively in recent times; see, for example, Haverkamp et al. (1988), Parlange and Haverkamp (1989), Clausnitzer et al. (1998), Skula et al. (2003) and Mishra et al. (2003).

Besides the Green and Ampt model, the main models that have been reviewed include those of Kostiakov (1932), Mezencev (1948), Philip (1957; 1969), Talsma and Parlange (1972), Schwarzendruber (1974), Smith and Parlange (1978), Parlange et al. (1982) and Parlange et al. (1985). Based on these and similar papers, general conclusions can be drawn. For any particular case, empirical and physically based models can be of comparable precision. Not surprisingly, the Green and Ampt model has been one of the most used and widely studied. It is amenable to practical applications, and by extension one can deduce that it has a theoretical basis that captures behavior of soil water movement.
However, there is little doubt that its practical value is limited by its rather drastic physical assumptions. Apart from its simplicity, its great importance is that all physically based models must reduce to it when the same physical assumptions are made. Consequently, it is a valid limiting case that cannot be ignored.

In the next sections, we shall look again at the Green and Ampt model in detail and extensions that improve its theoretical and practical value. One of its key simplifications is the assumption of a sharp infiltrating front, i.e., of piston flow. We shall relax this constraint using the standard model of flow in unsaturated soil due to Richards (1931). In particular, we show that by writing Richards’ equation in integral form, it can be solved with approximate analytical techniques, subsequently giving rise to physically based infiltration equations.

Exact solutions of Richards’ equation are not of primary interest here, except when they directly affect the discussion. This is not to minimize their importance; on the contrary, exact solutions have been crucial both to test numerical schemes and as a guide to obtain approximate solutions. A sample of exact solutions can be found elsewhere (e.g., Parlange and Braddock, 1980; Parlange et al., 1980a; Clothier et al., 1981; Rogers, 1983; Broadbridge and White, 1987, 1988; Sander et al., 1988a; Barry and Sander, 1991; Barry et al., 2002). Our focus is also limited to the archetypal case of one-dimensional flow in a semi-infinite medium, with the soil surface at \( z = 0 \) (\( z \) positive downwards) and uniform initial water content (which can always be taken as zero by taking the excess water as the variable). Also, we consider only capillarity and gravity, these being the two main forces affecting infiltration and redistribution of water in the soil profile. Other forces can be very important in specific cases. Amongst other processes one can mention there are (i) the effects of air movement and entrapment (Sander et al., 1988b,c; Culligan et al., 2000; Hammecker et al., 2003), (ii) soil layering and surface sealing (Parlange et al., 1984; Römkens et al., 1986; Baumhardt et al., 1990, 1991; Vandervaere et al., 1998; Corradini et al., 2000; Assouline, 2004.), (iii) flow instability (Hill and Parlange, 1972; Philip, 1972; Raats, 1973; Parlange and Hill, 1976; Baker and Hill, 1990; Selker et al., 1992) and its relationship to water repellency (Bond, 1964; Bauters et al., 1998; Bauters et al., 2000; DiCarlo et al., 2000) and (iv) hysteresis (Liu et al., 1995; DiCarlo et al., 1999). Several fundamental studies of hysteresis which are relevant here can also be mentioned (e.g., Parlange, 1976; Hogarth et al., 1988; Viaene et al., 1994; Si and Kachanoski, 2000; Braddock et al., 2001).

2. The Green and Ampt (1911) Model

Despite being published nearly 100 years ago, this model remains, from a theoretical point of view, the most basic of physically based infiltration models. As it can be manipulated relatively easily it is, in spite of its physical limitations, still widely used in practice. The practical TCA method can be justified using the Green and Ampt model, following the fundamental paper of Mein and Larson (1973), see also Poulosvassilis et al. (1991), Liu et al. (1998) and Brutsaert (2005). This feature has made it even more attractive as a practical hydrological tool.

Mathematical properties of the Green and Ampt solution, for instance its description as a branch of the Lambert W-function and its relationship as a solution of Richards’ equa-
tion have been demonstrated (Barry et al., 1993, 2005; Parlange et al., 2002). However, its practical use requires the determination of physical parameters. These are curve-fitted in most cases to infiltration and soil data and therefore tend to apply primarily for the conditions of the fitting, e.g., see Aggelides and Youngs (1978), McCuen et al. (1981) and Rawls et al. (1983).

The original Green and Ampt solution has also proved to be quite flexible in field applications when other processes have additionally to be considered: solute transport, non-aqueous flow, overland flow and the possibility of erosion, presence of a water table, spatial variability, layered and crusted soils, initial water content and structure varying with depth, air entrapment (e.g., see Thooyamali and Norum, 1987; Charbeneau and Asgian, 1991; Huang and van Genuchten, 1995; Kao and Hunt, 1996; Chu, 1997; Yu et al., 1997; Vandervaere et al., 1998; Selker et al., 1999; Wang et al., 1999; Fiedler and Ramirez, 2000; Govindaraju et al., 2001; Hammecker et al., 2003; Nahar et al., 2004). Note that, although the model has been applied to redistribution it has some difficulty in integrating hysteresis effects (Ogden and Saghafian, 1997; Nielsen and Perrochet, 2000).

2.1. Derivation

Consideration of Darcy’s law for piston flow in a soil yields (e.g., Neuman, 1976):

\[ q = K_s \left( H_s + I / \theta_s - H_f \right) \theta_s / I, \]  

(1)

where \( q [L \ T^{-1}] \) is the Darcy flux, \( K_s [L \ T^{-1}] \) the surface (here saturated) conductivity, \( H_s [L] \) the ponded water thickness, \( \theta_s \) the surface water content (here saturated and measured relative to the constant initial water content), \( I [L] \) is the cumulative infiltration and \( H_f [L] \) is the negative pressure at the wetting front. In addition to piston flow, Eq. (1) assumes that the soil-water conductivity \( K [L \ T^{-1}] \) is independent of the soil water pressure. As we shall see later, this last assumption is the main reason for the difficulties associated with Eq. (1). In the limit of small time, \( t \to 0, I \to 0 \) and \( q \to \infty \); Eq. (1) then shows that:

\[ I q \to K_s \theta_s (H_s - H_f). \]  

(2)

This quantity can be associated to the sorptivity \( S [L \ T^{-1/2}] \) (Parlange, 1975; Neuman, 1976) by

\[ S^2 = 2K_s \theta_s (H_s - H_f), \]  

(3)

thus relating \( H_f \) to a physical parameter \( S \), with \( I \to S \sqrt{t} \) and \( 2q \to S / \sqrt{t} \) for short times. Physically, \( S \) quantifies the capillary forces affecting water movement in the soil; these forces of course are affected by the soil’s moisture status and surface boundary condition. Equation (3) not only shows the relation between \( H_f \) and \( S \), but also predicts the dependence of \( S \) on \( H_s \), whereby increasing \( H_s \) increases the rate at which water initially enters the soil, as would be expected intuitively. Indeed, this prediction,
noted explicitly by Green and Ampt (1911), is extremely accurate as shown by more detailed analyses (Parlange et al., 1988, 1992; Broadbridge, 1990). In the particular case when \( H_s \) is constant, Eq. (1) is integrable since \( q \equiv dl/dt \), giving

\[
K_s t = I - (2S/ 2K_s) \ln \left(1 + 2IK_s/S^2\right).
\]  

(4)

The short time expansion of Eq. (4) is,

\[
I = S\sqrt{t} + \frac{2}{3} K_s t + ...
\]  

(5)

The second term \( 2K_s t/3 \) in Eq. (5) is a consequence of assuming that \( K \) is independent of soil-water pressure. Gardner (1958) postulated a very different behavior to be discussed in the following. It leads, even for a piston flow, to an infiltration law (Talsma and Parlange, 1972) where the second term in the short time expansion is \( K_s t^3/3 \) instead of \( 2K_s t/3 \). This result is also consistent with the measurements of Talsma (1969). In spite of this fundamental difficulty, the success of the Green and Ampt model in field applications can be partially attributed to the large scatter of field observations. Agreement between the Green and Ampt model predictions and observations can also be artificially improved by using varying properties (Ahuja and Tsuji, 1976; Haverkamp et al., 1988).


Heaslet, M.A. and Alksne, A., 1961. Diffusion from a fixed surface with a concentration-dependent coef-


Nielsen, P. and Perrochet, P., 2000. Watertable dynamics under capillary fringes: Experiments and mod-
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2401-2406. [A discussion of TCA]


Talsma, T., 1969. Infiltration from semi-circular furrows in the field. Australian Journal of Soil Research, 7(3): 277-84. [Short time measurements of infiltration]


Biographical Sketches

D. A. Barry, carries out research on porous media, flow and transport processes, particularly the modeling of such processes. He has been involved in many collaborative projects involving combinations of laboratory, field and theoretical work, including infiltration modeling. For example, he has collaborated in the development of a biogeochemical transport models for predicting transport and fate of contaminants in complex subsurface environments. Other computer-modeling efforts include distributed catchment modeling and modeling of on-shore/off-shore sediment transport on ocean beaches. He is Editor of the journal Advances in Water Resources.

J.-Y. Parlange's field of specialization and special interests are in environmental engineering and applications of nonlinear mathematics to water and solute movement in porous media, surface and subsurface hydrology, watershed modeling, sediment transport and erosion. He is a member of the National Academy of Engineering.

Meng-Chia Liu is currently a Ph.D. candidate in the Department of Biological and Environmental Engineering at Cornell University, Ithaca, New York, USA. He previously worked for several consulting companies in Taiwan as an environmental engineer and a project manager. He also participated in projects associated with Taiwan’s 4th nuclear power plant. His research interests include soil physics, infiltration modeling, and water resources planning.

Graham Sander, is a Reader in Hydrology in the Department of Civil and Building Engineering at Loughborough University, England. Prior to his current position he was in the Faculty of Science and the Faculty of Environmental Sciences at Griffith University, Australia. His research and teaching interests are in environmental science and engineering and cover predominantly soil erosion modeling, water and solute transport in porous media and unsaturated two-phase flow. He is on the editorial board for Advances in Water Resources and is Associate Editor of Water Resources Research.

Marc B. Parlange, is Professor in Environmental Engineering at the Ecole Polytechnique Fédérale de Lausanne (EPFL). His research interests include hydrology, environmental fluid mechanics, evaporation into the atmosphere, the structure of the atmospheric boundary layer and the development of instruments to probe the lower atmosphere and shallow soil and water surfaces. He is Editor-in-Chief of Water Resources Research.

David Lockington, is the Head of Environmental Engineering in the School of Engineering, as well as the Director of the Centre for Water Studies, at the University of Queensland, Australia. He is also the Director of the Engineering, Technology and Design Research Program in the national Cooperative Research Centre for Sustainable Tourism. His research interests center on modeling fluid flow in unsaturated porous media, variable density groundwater flow and contaminant transport. A particular interest is the use of these models in quantifying coastal catchment processes such as: subsurface estuary dynamics; submarine groundwater discharge; tidal marsh, wetlands and island hydrology; and seawater intrusion. He is a member of the editorial board of Advances in Water Resources and is a Fellow of the Institute of Mathematics and its Applications.

Frank Stagnitti, a mathematician, soil physicist and environmental scientist, is an expert in the study of agricultural and hydrological systems, particularly in the fields of solute and contaminant transport, bioremediation and ecotoxicology, groundwater flow, coastal processes and wetlands technology. Dr Stag-
nitti has established an international research profile in the field of environmental systems modeling. In the last five years, he has published in excess of 50 manuscripts in international, high-impact journals and invited to present keynote addresses at conferences in Europe, USA and Asia. He has attracted significant research funding within Australia, mainly through the Australian Research Council and has participated in a number of international projects, mainly funded by the EU under the 5th and 6th Framework programs. Dr. Stagnitti is a member of several scientific advisory committees and boards including the Australian Research Council, Center of Excellence in Light Metals and the Victorian Partnership in Advanced Computing. He currently holds the position of Associate Dean Research for the Faculty of Science and Technology at Deakin University and is a full Professor and Chair in Aquatic Science in the School of Life and Environmental Science. Dr Stagnitti is a Fellow of Australian Mathematics Society, a Fellow of Institute of Mathematics and its Applications (UK), a member of American Mathematics Society, a Chartered Mathematician and a Chartered Scientist registered in the UK.

Shmuel Assouline, is a senior research scientist at the Agricultural Research Organisation (Volcani Center) of Israel. His research includes flow and transport processes in porous media. He is also involved in projects dealing with new irrigation practices and technologies and soil and water conservation methods. Dr. Assouline has authored and co-authored about 60 refereed publications. He is Associate Editor of Vadose Zone Journal.

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Tammo Steenhuis, is a professor in the Department of Biological and Environmental Engineering at Cornell University. He works with a group of 25 graduate students, postdoctoral researchers and research associates to see if it is possible to do research in hydrology from the nano to watershed scale years. He has collaborated extensively with Yves Parlange during the last 20 years on finding new ways for describing spatial variability in flow fields both above and below the ground.

Ling Li, is a Professor and Chair in Environmental Engineering in School of Engineering at the University of Queensland, Australia. His research and teaching interests are in environmental science and engineering with a particular focus on modeling of environmental systems. His current research work addresses interactions between the ocean and coastal aquifers, and pathways and fluxes of chemicals to coastal waters via submarine groundwater discharge. He is on the editorial board for Advances in Water Resources and is Associate Editor of Hydrogeology Journal.

R. Haverkamp, is “Directeur de Recherche” at CNRS (French National Research Center) and works at the research laboratory LTHE in Grenoble, France. His research is focused on flow processes in the vadose zone with a particular interest on the scaling of these flow processes. His work involves field and theoretical work including large scale watershed modeling. As is clear from his numerous co-authored publications, most of his research is carried out in close collaboration with colleagues from various international research laboratories. He has been involved in national and international projects in many parts of the world. He initiated a spin-off computer software company working on hydroinformatics.

Bill Hogarth, is Pro Vice Chancellor Teaching and Learning and Head of the Faculty of Science and Information Technology at the University of Newcastle, Australia. He was previously Dean of the Faculty of Environmental Sciences at Griffith University, Australia. His teaching and research interests are in environmental modeling with a strong emphasis on the numerical aspects. The particular focus of his research has been on soil processes concentrating on soil infiltration, soil erosion and more recently wind erosion.