

## MATHEMATICAL SOIL EROSION MODELING

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## Summary

It is becoming increasingly clear that the transport of eroded material from land to water by overland flow is an important environmental problem, promoting the eutrophication of surface waters, damaging freshwater ecosystems and causing microbial contamination of surface water sources. Sediment derived from the soil is a pollutant in its own right: reducing light penetration and physically damaging freshwater ecosystems; it is a carrier of pollutants such as pesticides and phosphorus and many contaminants are associated with soil particle surfaces. As point-sources of pollutants are controlled there is an urgent need to provide the scientific understanding to underpin operational decisions being made with respect to diffuse pollutants. This Chapter reviews the major developments in mathematical soil erosion modeling over the past two decades. In particular, we review progress in finding solutions to the Rose-Hairsine model and their application to experimental data. Because of its unique ability of the Rose-Hairsine model to explicitly recognize the differential behavior of the various sediment particle size classes which comprise natural soils, not only can it provide greater insight into the movement of sediment across both farmlands and other contaminated land, but it is better positioned than any other model to estimate the impact of eroded sediment on water quality of surrounding rivers or streams. Such differential behavior results in the preferential movement of fine sediment with attached compounds such as nutrients, fertilizers and pollutants. Neglecting the size selectivity in the sediment transport and deposition process results in a significant underestimation of the downstream impact of suspended sediment enriched with absorbed chemicals.

### 1. Introduction

Sculpturing of the land surface by erosion, transport and deposition processes has always played a major role in shaping the land surface of the earth. Geomorphologists have long recognized that glaciation is a major erosion agent in cold climates, mass movement is common in steep humid regions, and in many regions both wind and water can play dominant roles in sediment transport.

When human activity substantially reduces the cover provided by vegetation or litter, and when soil is disturbed and loosened, these natural erosion processes can be greatly accelerated. Land management practices found to be necessary or beneficial to the development of agriculture were developed in many countries. Such practices were developed in temperate climatic regions, such as Europe, and typically involved forest clearing and subsequent cultivation. These practices were transferred to other regions of the world which were colonized or conquered, without realizing that the direct transfer of such land management methods may be inappropriate, or at least require modification for sustainable land use in quite different soil and climatic contexts. The scale and rate of expansion of such transferred land management practices was vastly increased by the rapid adoption and power increase in mechanized forms of cultivation.

Thus, early in the last century, especially in countries such as the USA and Australia, European-based agricultural practices were rapidly extended into regions where the soil and climatic contexts were quite different from their European origins. The resultant extensive and accelerated rates of soil erosion which occurred in such countries

provided a major incentive for research into soil erosion, especially in the USA (Hudson, 1981). This is not to infer that land degradation due to water and wind erosion is restricted to such countries (Pimental, 1976; Oldeman, 1994). However, a brief history of water erosion research which follows will be restricted to the USA.

#### *Early development of soil erosion research in the USA*

The United States Department of Agriculture (USDA) declared a policy of land protection in 1907, and from 1915 onwards a number of agencies commenced investigation of the effect of different treatments on runoff and soil erosion from defined plots (Bennett, 1939). This early applied research was expanded and accelerated with the establishment of Federal and State Experiment Stations, and from 1928 to 1953 a period of intensive collection and tabulation of runoff and soil loss data occurred. This work included experiments on mechanical ways of controlling soil loss and runoff from small watersheds. In later years, data using artificial rainfall simulators added to the very large body of collected data.

This substantial empirical database provided guidance on the role of many factors and agronomic treatments in controlling soil loss (Ayres, 1936). However, very few plots were equipped to measure the rate of runoff; only the total runoff and soil loss were recorded. Since rate measurement technology was not the limiting factor, this measurement choice may have come from the mental model held by soil scientists concerned with soil erosion at the plot scale. This model appears to be that “raindrops detach soil and overland flow simply transports this previously removed sediment over the soil surface” (Rose, 1993).

This early emphasis on the role of raindrop impact, and relative neglect of the role of overland flow in soil erosion, appears to have been strengthened by the studies of raindrops and erosion by Laws (1940), Ellison (1947), Ekern (1951), and Hudson (1957).

The very large body of data collected by the USDA and collaborators called for some kind of synthesis, condensation, or generalization. For example, Zingg (1940) developed an empirical equation relating soil erosion to slope and slope length. Also important to subsequent development was Musgrave's (1947) parametric equation which incorporated a rainfall erosivity index as well as other factors. This type of equation was revised and expanded several times to form the Universal Soil Loss Equation (or USLE) of Wischmeier and Smith (1978).

The USLE was developed by applying statistical multivariate regression techniques to the large data bases collected by the USDA Agricultural Research Service, its collaborators and predecessors. The data base included the results of long-term studies of factors believed to affect soil erosion in areas of agricultural significance east of the Rocky Mountains in the USA. Whilst large in size, the data base was for a restricted ecological range, covered slopes of only up to about 7%, and to soils with a low percentage of montmorillonite clay (Morgan and Davidson, 1986).

The factor-product form of data summary provided by the USLE is given by (Wischmeier and Smith, 1978)

$$A = R_f K_f L S_o C_f P_f, \quad (1)$$

where  $A$  is the mass of soil lost from unit area per year, averaged over as many years as is appropriate.  $R_f$ , the rainfall erosivity factor, is calculated using data on both the kinetic energy and intensity of rainfall. The soil erodibility,  $K_f$ , is in practice calculated as the unknown in Equation (1), given values for the slope length  $L$ , slope  $S_o$ , the crop management factor,  $C_f$ , and  $P_f$ , the factor describing any erosion control practice which might be adopted. Experience with calculated values of  $K_f$  for agriculturally important soils in the USA has been summarized in the form of a nomogram, which can be used predictively for such soils (Wischmeier and Smith, 1978).

Wischmeier (1976) took pains to emphasize the limitations of the USLE, stressing that it was particularly designed to address objectives such as the following:

- Give estimates of long-term average annual soil loss from a particular field slope, and with a particular land use and management.
- Provide guidance on the selection of cropping, management systems and conservation practices for specific soils and slopes.
- Provide soil loss estimates for conservationists to use for determining soil conservation needs.

Wischmeier (1976) warned against using the USLE beyond the regions where the basic information was obtained, or to make soil loss estimates for individual erosion events. The USLE applies only to situations where net deposition does not occur. The USLE is based on correlations. Since there is no inclusion in the USLE of factors directly representing physical parameters such as infiltration or overland flow velocity, some factors will be influenced by correlations with effects due to these processes.

Especially in less humid environments, there is a practical limitation in developing locally relevant parameters for use in the USLE methodology. This limitation is that a long time period, possibly several decades, may be required in order to experience an adequate number of erosion events (Edwards, 1987) to reliably estimate these parameter values.

Many modifications have been made to the USLE designed to overcome some of its limitations. Perhaps the most widely accepted modification is RUSLE (Revised USLE) described by Renard *et al.* (1994).

#### *Soil erosion and conservation developments beyond the USLE*

It is clear that the origin and purpose of the USLE was not to describe the *processes* affecting soil erosion. The objective of more recent research on soil erosion has been to describe such processes so that more effective identification and predictability of parameters involved can be achieved. This objective has not been readily obtained, and research to support this objective is still actively in train.

The first general area of advance has been to recognize that a vital role in erosion is played, not only by rainfall, but also by overland flow. Thus, the currently accepted conceptual model, replacing that of earlier researchers previously given, is that: “Raindrops detach soil, and overland flow *both erodes* and transports eroded soil over the land surface” (Marshall, Holmes and Rose, 1996). The common presence of rills in erosion events provides evidence for this statement.

The need to predict excess rainfall from rainfall characteristics requires a robust model of the infiltration process. Especially as scale increases, there is increasing evidence that spatial variability in infiltration rate is common, so that infiltration equations that include this behavior have advantages over one-dimensional infiltration models (Yu *et al.*, 1997, Yu, 1999).

A second major area of development has been recognition of the general importance of the role of sediment deposition as an ongoing process which dynamically accompanies whatever mix of erosion processes is at work. The rate of sediment deposition depends on the settling-velocity characteristic of the sediment involved. Interaction in settling between sediments of quite different size (and therefore settling velocity) appears to be an important factor (Lovell and Rose, 1991).

The range of models developed to describe the series of dynamic processes involved in soil erosion, deposition and transport will be reviewed in subsequent sections. Although the form of description of these processes is not always in complete agreement, Figure 1 illustrates the dynamic form of interaction widely accepted.

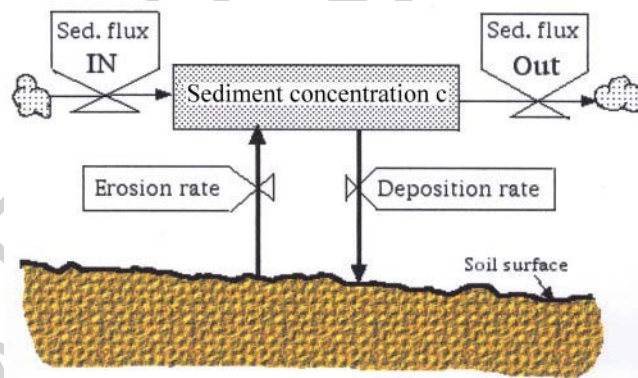


Figure 1: Flow diagram describing the interaction of erosion processes between the sediment flux and the soil surface. Rates of processes exchanging sediment are shown by valve symbols

A common experimental finding is that in any given erosion situation there is an upper limit to the resulting sediment concentration. For flow-driven erosion, Foster (1982) introduced the term ‘transport limit’ to describe this limiting value. A theoretical expression for the transport limit has been derived by Rose and Hairsine (1988), and a corresponding limit for rainfall-driven erosion by Hairsine and Rose (1991).

## 2. Surface Hydrology

Since it is overland flow which transports the suspended sediment, any model of soil erosion processes must first begin with a description of the surface hydrology. The governing equations, obtained from conservation of both mass and momentum, for unsteady one-dimensional non-uniform flow of water down a planar surface of unit width are given by

$$\frac{\partial h}{\partial t} + \frac{\partial(uh)}{\partial x} = R , \quad (2)$$

and

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + g \frac{\partial h}{\partial x} = g(S_o - S_f) - \frac{Ru}{h} . \quad (3)$$

In (2) and (3), which are usually referred to as the St Venant equations,  $h$  is the mean depth of flow,  $u$  is depth averaged velocity,  $R$  is the lateral inflow per unit length,  $g$  is gravity,  $S_o$  is the bed slope,  $S_f$  is the friction slope,  $t$  is time and  $x$  is distance downslope.

In general the St Venant equations need to be solved numerically, however under flow conditions where friction and gravity effects dominate those due to inertial and pressure effects, then (3) has the simple solution  $S_o = S_f$ . Consequently (2) and (3) reduce to the kinematic wave model for overland flow or,

$$\frac{\partial h}{\partial t} + \frac{\partial q}{\partial x} = R , \quad (4)$$

where

$$q = Kh^m , \quad (5)$$

$$K = \frac{S_o^{1/2}}{n} . \quad (6)$$

In (4), (5) and (6),  $q$  is the volumetric flux per unit width,  $n$  is the Manning's roughness coefficient and  $m$  is an exponent having a value of approximately 5/3 for turbulent flow and 3 for laminar flow. Woolhiser and Liggett (1967) have shown that the kinematic wave model is a good approximation to the St Venant equations provided that the kinematic number  $k_e$ , where  $k_e = S_o L / h_n F_r^2$  ( $h_n$  is the normal depth at  $x = L$  and  $F_r$  is the Froude number based on normal flow) is greater than 20. This was however later modified by Morris and Woolhiser (1980) to  $k_e F_r^2 > 5$  when  $F_r < 0.5$  and  $k_e > 20$  when  $F_r > 0.5$ .

The later inflow or excess rainfall rate  $R$  is defined from

$$R = P - I_r \quad , \quad (7)$$

where  $P$  is the rainfall rate and  $I_r$  is the infiltration rate. Due to spatial and temporal variability in  $P$  and  $I_r$ ,  $R$  in general depends on both  $x$  and  $t$  and only numerical solutions to (4) and (5) are possible (Sherman and Singh, 1976). Under the simplifying yet still physical realistic assumptions of a constant rainfall rate or a time varying rainfall rate, analytical solutions to (4) and (5) are possible and can be found by using the method of characteristics.

## 2.1 Analytical Solutions

(a)  $R = \text{constant}$

The solution for a constant excess rainfall rate was first given by Henderson and Wooding (1964) as

$$h = \left( \frac{Rx}{K} \right)^{\frac{1}{m}} \quad x \leq K R^{m-1} t^m \quad , \quad (8)$$

$$h = Rt \quad x \geq K R^{m-1} t^m \quad , \quad (9)$$

for the initial and boundary conditions

$$\begin{aligned} t = 0, \quad x > 0, \quad h = 0 \quad , \\ t > 0, \quad x = 0, \quad h = 0 \quad . \end{aligned} \quad (10)$$

(b)  $R = R(t)$  with  $R(t) \geq 0, t \geq 0$ .

The solution of Henderson and Wooding (1964) was generalized by Parlange et al (1981) for a positive time dependence of  $R$  on  $t$ . For the initial and boundary conditions of (10) the solution is given parametrically by

$$h = \int_{t_0}^t R(t') dt' \quad x \leq x_c \quad , \quad (11)$$

$$x = Km \int_{t_0}^t \left[ \int_{t_0}^{\bar{t}} R(t') dt' \right]^{m-1} d\bar{t} \quad x \leq x_c \quad , \quad (12)$$

with the parameter  $t_0$  in the range  $0 \leq t_0 \leq t$ . The boundary condition (10) is given by  $t_0 = t$  while the initial condition is satisfied by  $t_0 = t = 0$ . For values of  $x$  greater than  $x_c$  where

$$x_c = Km \int_0^t \left[ \int_0^{\bar{t}} R(t') dt' \right]^{m-1} d\bar{t} \quad , \quad (13)$$

( $t_0 = 0$  in (12)), then  $h$  is independent of  $x$  and given by (11) with  $t_0 = 0$ .

(c)  $0 \leq t \leq t^*$ ,  $R(t) \geq 0$ , and  $t > t^*$ ,  $R(t) < 0$ .

When the rainfall rate falls below the infiltration rate then  $R$  becomes negative and neither of the solutions presented in (a) or (b) above apply. Two quite specific solutions for  $R < 0$  have been given in the literature by Cundy and Tonto (1985) and Giraldez and Woolhiser (1996). These were for a constant rainfall rate of finite duration  $t^*$  and a modified Philip infiltration equation (Cundy and Tonto, 1985) or a Smith and Parlange (1978) infiltration equation (Giraldez and Woolhiser). In Sander et al (1990) though, a solution was developed for essentially an arbitrary  $R(t)$  function subject only to the constraint  $0 \leq t \leq t^*$ ,  $R(t) \geq 0$ , and  $t > t^*$ ,  $R(t) < 0$ . This solution incorporates both the Cundy and Tonto (1985) and Giraldez and Woolhiser (1996) solutions.

Since  $R(t) \geq 0$  for  $0 \leq t \leq t^*$ , then the solution for this time period is still given by that of Parlange et al (1981) or (11), (12) and (13). For  $t > t^*$  a drying free surface is formed and begins to move downslope from  $x=0$  so that the  $h=0$  boundary condition no longer occurs at  $x=0$ , but at  $x=x_d(t)$  where

$$x_d(t) = Km \int_{t_1}^t \left[ \int_{t_1}^{\bar{t}} R(t') dt' \right]^{m-1} d\bar{t} \quad , \quad (14)$$

with  $t_1 \leq t^* \leq t$ , and  $t_1$  defined from

$$\int_{t_1}^t R(t') dt' = 0 \quad . \quad (15)$$

Equations (14) and (15) give the time dependence of the edge of the free surface for  $t > t^*$ . In the region  $x > x_d$ , the solution is still given by (11) and (12) but with  $t_0$  restricted to the range  $0 \leq t_0 \leq t_1$ .



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### Biographical Sketches

**Graham Sander** is a Reader in Hydrology in the Department of Civil and Building Engineering at Loughborough University, England. Prior to his current position he was in the Faculty of Science and the Faculty of Environmental Sciences at Griffith University, Australia. His research and teaching interests are in environmental science and engineering and cover predominantly soil erosion modeling, water and solute transport in porous media and unsaturated two-phase flow.

**Professor Calvin Rose** is an Emeritus Professor and founding Dean in the Faculty of Environmental Sciences at Griffith University, Brisbane, Australia. Jointly with a research team he is working on both fundamental and applied aspects of soil and water management. His early book "Agricultural Physics" (1966, Pergamon Press) provided a basic link between physics and the range on agricultural concerns. He has directed research programs involving Australia and Asian countries on soil erosion and the development of sustainable cropping systems for tropical environments. His recent text, "An Introduction to the Environmental Physics of Soil Water and Watersheds" (2004, Cambridge University Press) provides the basic physical knowledge required to understand processes involved in the sustainable use of the earth's land and water resources

**Bill Hogarth** is Pro Vice Chancellor Teaching and Learning and Head of the Faculty of Science and Information Technology at the University of Newcastle, Australia. He was previously Dean of the Faculty of Environmental Sciences at Griffith University, Australia. His teaching and research interests are in environmental modeling with a strong emphasis on the numerical aspects. The particular focus of his research has been on soil processes concentrating on soil infiltration, soil erosion and more recently wind erosion.

**Jean-Yves Parlange** is a Professor of Agricultural and Biological Engineering at Cornell University having previously been a Professor of Applied Mathematics at Griffith University, in Brisbane, Australia. Yves Parlange has been a Fellow of the American Geophysical Union since 1996, received the Hydrology award in 1996, Horton medal in 2002 and elected to the U.S. National Academy of Engineering in 2006. His research interests are many and varied but centre on problems in environmental science and include water movement in porous media, solute transport in soils, surface and subsurface hydrology and erosion and sediment transport.

**Ian Lisle** lectures in Mathematics at the University of Canberra in Australia, having previously worked and studied at Griffith University and the University of British Columbia. His research interests include numerical and analytical solution of problems in soil and water, including solutions based on Lie group methods. Ian is also active in mathematical problems arising in Lie symmetry analysis of differential equations, and algorithmic solution of these problems using computer algebra.