NUCLEAR REACTOR DESIGN

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Contents

Summary

In a nuclear reactor fission of uranium is induced by the absorption of neutrons by the uranium nuclei. Each nucleus splits into two fragments and, on average, two or three neutrons are released. All these products fly apart with high kinetic energy which is subsequently degraded into heat by collisions with other nuclei. If one of these neutrons, on average, goes on to produce fission of another nucleus, a nuclear chain reaction is established. If there are multiple parallel fission reactions then the ratio of the number of neutrons in the current generation to the number in the previous generation is unity and the reactor is said to be critical.

Neutrons produced from fission have high energy and consequently a high velocity. These are known as fast neutrons. To increase the probability of them being absorbed in the fuel (U-235) they are moderated by a suitable medium where many scattering collisions with nuclei reduce their energy to a state of thermal equilibrium within the medium. These are known as thermal neutrons or slow neutrons. At this lower velocity there is a very much higher probability of being absorbed by the fuel (U-235) to cause another fission.

To promote and maintain this process, the neutron cycle must be carefully balanced and controlled. The number of fast neutrons released by fission per neutron absorbed in the fuel (reproduction factor) depends on the concentration of the fissile material (U-235) and type of fissile material (U-235 or Pu-239). The number of additional fast neutrons produced by fast fission of fissionable material (U-238) (fast fission factor) depends on the reactor configuration. The number of slow neutrons remaining after passing through the resonance region of the fuel (U-238) (resonance escape probability) also depends upon the reactor configuration. Finally the number of slow neutrons absorbed by the fuel over those absorbed by the whole reactor including the fuel (thermal utilization factor) depends upon the structural materials of the reactor as well as on the amount of neutron absorbing materials inserted into the reactor to maintain a stable and continuous nuclear chain reaction. The four factors mentioned above make up the well known four factor formula. When leakage of fast and slow neutrons from the reactor is taken into account two additional factors (fast neutron non-leakage probability and slow neutron non-leakage probability) are incorporated to make up the six factor formula.

In the design of a reactor, leakage of neutrons from the reactor is reduced by employing a reflector around the reactor. This allows for neutron diffusion in all directions (with minimal absorption) within this zone and roughly half diffuse back into the reactor while the remaining half diffuse out and are lost. The reactor design also separates the fuel from the neutron moderator so that during the moderating or slowing down process the neutrons are away from the fuel and less likely to be absorbed by the strong resonance absorption peaks of the non-fissile fuel (U-238). This heterogeneous arrangement also facilitates heat removal from the fuel by a coolant which, in many reactors, also serves as a moderator.

During operation of the reactor, the absorption of neutrons can be controlled by the insertion or withdrawal of neutron absorbing control rods to maintain critical conditions on a short term basis. In the long term, fuel is consumed and the concentration of fissile material (U-235) decreases. This can be corrected periodically by refueling the reactor and restoring the concentration.

1. Basic Principles

1.1. Fission Chain Reaction

When the nucleus of a fissile material such as Uranium-235 is induced to fission by the absorption of a neutron, two to three neutrons, on average, are released as the two fission fragments fly apart. These neutrons are able to go on to cause fissions in other fissile nuclei, however, some may be absorbed in other materials or may leak from the system. If the number of neutrons lost by absorption and leakage is carefully controlled such that, on average, only one neutron from each fission goes on to produce another fission, then a stable self sustaining chain reaction has been established. Considering multiple fission processes in parallel throughout the reactor it is convenient to define the neutron multiplication factor k as follows:

$$k = \textit{number of neutrons in current generation} \;/\; \textit{number of neutrons in previous generation}$$

A nuclear reactor in such a stable self sustaining condition is said to be *critical* and k is equal to unity. Any deviation in the value of k from unity will lead to an increase or decrease in the number of neutrons and hence in the number of fissions. This in turn will change the rate of heat release in the reactor and hence its power output. Generally a nuclear reactor is designed such that k is slightly greater than unity but can be brought back to unity (or made less than unity) by the insertion into the reactor core of additional neutron absorbing material. Such material is usually in the form of *control rods* which can be inserted into or withdrawn from the reactor core. Fine adjustments of the degree of insertion of the control rods allow the value of k to be maintained at unity and the reactor power level kept constant.

1.2. Neutron Absorption Characteristics

Neutrons released by a fission reaction have a high energy of about 2 MeV on average. At this high energy the probability of absorption of a neutron in a fissile nucleus to cause fission is low. At a low energy however this probability is much higher. It is therefore advantageous to reduce the energy of the neutrons to increase the probability of fission. If this were not done, a much higher concentration of fissile fuel would be required.

To effect this reduction in energy, a *moderator* is incorporated in the reactor core. As the neutrons are scattered within the moderator by multiple collisions their energy is reduced. A good moderator has a high scattering cross section and a low absorption cross section. Although a few neutrons are absorbed, the great benefit is the substantially increased fission cross section of the fissile material for neutrons of low energy.

1.3. Heat Removal

The heat generated by fission must be removed continuously by a suitable coolant flowing through the reactor core. For effective heat removal, multiple coolant channels are employed in most nuclear reactors. As is the case for a moderator, the coolant should have a low neutron absorption cross section. In many nuclear reactors the same fluid is used as moderator and coolant. This naturally simplifies construction. Similarly, also to promote heat removal from the reactor core, the fissile material is usually arranged in small diameter fuel rods which are surrounded by the coolant contained in the coolant channels.

1.4. Basic Reactor Core Design

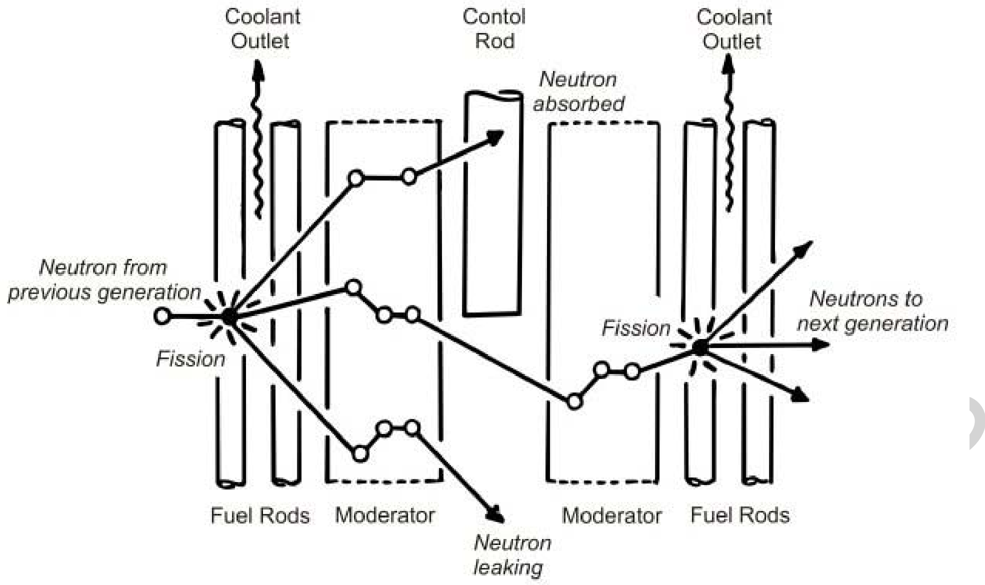

Figure 1. Nuclear reactor components

The reactor core thus consists of four basic and discrete elements as shown in Figure 1.

- Fuel
- Moderator
- Coolant
- Control Rods

The fuel contains the fissile material, the moderator reduces the energy of the neutrons, the coolant removes the heat from fission and the control rods absorb excess neutrons to maintain the desired value of k.

2. Basic Theory

2.1. Neutron Diffusion Equation

The neutron diffusion equation based on Fick's Law in three dimensions and Cartesian co-ordinates is given as follows:

$$J = -D\nabla\phi$$

Here J is neutron flow, D the diffusion coefficient and ϕ the neutron flux. Considering the leakage of neutrons from a unit volume the following is obtained where L_{total} is the leakage in all directions:

$$L_{\text{total}} = -D\nabla^2\phi$$

The absorption of neutrons is given by the macroscopic neutron absorption cross section Σ_a multiplied by the neutron flux ϕ. If the production of neutrons is defined as S then a neutron balance equation can be written as follows:

$$Change = Production - Absorption - Leakage$$

$$Change = S - \Sigma_a\phi + D\nabla^2\phi$$

For steady state conditions the change is zero so the steady state diffusion equation is obtained

$$D\nabla^2\phi - \Sigma_a\phi = -S \quad (1)$$

2.2. One Group Reactor Equation

For a reactor of infinite size (no leakage) the production of neutrons S is equal to the absorption multiplied by the neutron multiplication factor k_∞.

$$S = k_\infty \Sigma_a \phi$$

Substituting this into the steady state diffusion equation gives the following:

$$D\nabla^2\phi + (k_\infty - 1)\Sigma_a\phi = 0$$

The diffusion length L and buckling B^2 are defined as follows:

$$L^2 = D/\Sigma_a$$

$$B^2 = (k_\infty - 1)/L^2$$

Incorporating these into the above equation gives the one group steady state reactor equation:

$$\nabla^2\phi + B^2\phi = 0 \quad (2)$$

The first term is related to the leakage of neutrons from the reactor core while the second is related to the properties Σ_a and D of the reactor core. As the shape and size of the reactor core change so must the nuclear properties be altered to maintain a critical configuration. The thermal neutron diffusion parameters of common moderators are given in Table 1.

Moderator	**Density** g cm^{-3}	$D_{thermal}$ cm	$\Sigma_{a\ thermal}$ cm^{-1}	L_t^2 cm^2	L_t cm
H_2O	1.00	0.16	0.0197	8.1	2.85

D_2O^*	1.10	0.87	9.3×10^{-5}	9.4×10^{3}	97
Be	1.85	0.50	1.04×10^{-3}	480	21
Graphite	1.60	0.84	2.4×10^{-4}	3.5×10^{3}	59

*D_2O containing 0.25% by weight of H_2O.

Data obtained from Lamarsh and Baratta, Introduction to Nuclear Engineering, Prentice Hall, 2001.

Table 1. Thermal neutron diffusion parameters of common moderators

2.3. Reactor Design Considerations

The above basic theory can be applied to a nuclear reactor of simple geometry and homogeneous structure. This will however lead to erroneous and impractical results. Firstly, some important nuclear characteristics will have been neglected and, secondly, certain practical design constraints have yet to be introduced.

In the first category of nuclear characteristics there are the following important considerations:

- As neutrons lose energy during moderation, the probability of neutron interaction changes, so Σ_a and D are different at different neutron energies.
- At high neutron energies, U-238, which is always present, becomes fissionable and additional neutrons are produced by fast fission.
- All fuel contains substantial amounts of U-238, which has very high resonance absorption peaks at certain neutron energies, and some neutrons are absorbed.

In the second category of design constraints there are certain modifications that are made to the basic geometry and structure of the reactor to overcome practical constraints and to enhance operational performance. Some of these are as follows:

- Neutron leakage from the reactor core is reduced by the use of a reflector, which allows some neutrons to diffuse back into the core.
- The characteristic sinusoidal neutron flux profile in the reactor core is altered to increase the ϕ_{ave} / ϕ_{max} ratio and so obtain an increased power output.
- All reactor cores are heterogeneous rather than homogeneous in structure, that is, the fuel and moderator are separated but arranged to form a uniform matrix.
- Control rods inserted into the reactor core to maintain steady state critical conditions distort the neutron flux profile.

In the following sections these factors will be considered with some indication of how their effects can be taken into account.

3. Neutron Energy Production

3.1. Group Diffusion Method

Neutrons, when slowing down, pass through a range of energies. Within each energy range there is a certain probability of scattering or absorption within the medium. The general diffusion equation may still be used but the diffusion characteristics change as the neutron energy varies. The parameters for thermal neutrons are therefore not applicable to neutrons at the higher energies occurring immediately after fission.

In order to solve the general diffusion equation accurately, the neutron energy range has to be divided up into a number of groups with each group having its particular diffusion characteristics. A series of diffusion equations, each with different parameters, is therefore obtained and applied to the neutrons within the appropriate energy range.

A complication is that not all neutrons are created with the same energy. The source term must therefore be divided up so that the appropriate number of neutrons enter each energy group. A further complication is that neutrons do not all lose the same amount of energy in a collision so they do not pass progressively from a particular energy group to the next lower energy group. They may miss one or more groups should they lose a large amount of energy in a single scattering collision or may remain within a single group for a number of scattering collisions if the energy loss in collisions is small. The net result is that any particular group will receive neutrons from any higher energy group including the source and lose neutrons to any lower energy group. All such transfers have a certain probability Σ_{t} specified by the group transfer cross section into the group $\Sigma_{\mathrm{t}(h\rightarrow i)}$ and out of the group $\Sigma_{\mathrm{t}(i\rightarrow j)}$ where i is the group being considered and h and j range respectively from 1 to i-1 and from $i+1$ to n so representing the number of the particular higher and lower energy groups with n being the total number of groups.

This type of analysis is quite complicated and suited to computer analysis. In such cases four to six energy groups may be considered. Even in a four group calculation any group will be subject to the transfer of neutrons from or to three other groups.

3.2. Two Group Calculations

Reasonably accurate results may be obtained by considering just two groups of neutrons namely the higher energy group arising from fission and the thermal energy group consisting of those having lost most of their energy. Two diffusion equations are therefore required for fast and thermal neutrons respectively. If fast neutrons are created by fission due to thermal neutrons and are converted to thermal neutrons within the medium these equations are respectively:

$$D_{\mathrm{fast}}\nabla^2\phi_{\mathrm{fast}} - \Sigma_{\mathrm{t(fast\rightarrow thermal)}}\phi_{\mathrm{fast}} = - k_{\infty}\Sigma_{\mathrm{a\,thermal}}\phi_{\mathrm{thermal}}$$

$$D_{\mathrm{thermal}}\ \nabla^2\phi_{\mathrm{thermal}} - \Sigma_{\mathrm{a\,thermal}}\phi_{\mathrm{thermal}} = - \Sigma_{\mathrm{t(fast\rightarrow thermal)}}\phi_{\mathrm{fast}}$$

Note that the transfer cross section term of the fast neutron diffusion equation $\Sigma_{\mathrm{t(fast\rightarrow thermal)}}\phi_{\mathrm{fast}}$ becomes the source term of the thermal neutron diffusion equation.

With further analysis of these equations it can be shown that the non-leakage probability P_{L} is given by:

$$P_{\mathrm{L}} = 1/(B^4 L_{\mathrm{f}}{}^2 L_{\mathrm{t}}{}^2 + B^2 L_{\mathrm{f}}{}^2 + B^2 L_{\mathrm{t}}{}^2 + 1)$$

Here L_{f} is the diffusion length for fast neutrons and L_{t} that for thermal neutrons. $L_{\mathrm{f}}{}^2$ is also known as the neutron age τ since it is a function of how long it takes a neutron to thermalize. For large reactor cores with small values of buckling this equation reduces to the following:

$$\begin{aligned} P_{\mathrm{L}} &= 1/\{B^2(L_{\mathrm{f}}{}^2 + L_{\mathrm{t}}{}^2) + 1\} \\ &= 1/(B^2 M^2 + 1) \end{aligned}$$

Here M is the migration length defined as:

$$M^2 = L_{\mathrm{f}}{}^2 + L_{\mathrm{t}}{}^2$$

From this it follows that:

$$k_\infty = 1 + B^2 M^2 \qquad (3)$$

Calculations may thus be carried out as for the one group reactor theory. This is known as the modified one group theory. If $B^4 L_{\mathrm{f}}{}^2 L_{\mathrm{t}}{}^2$ is not neglected then it remains as the two group theory. Fast neutron diffusion parameters of common moderators are given in Table 2.

Moderator	D_{fast} cm	$\Sigma_{\mathrm{a\ fast}}$ cm^{-1}	$L_{\mathrm{f}}{}^2$ cm^2	L_{f} cm
H_2O	1.13	0.0419	~27	~5
D_2O	1.29	0.00985	131	11
Be	0.562	0.00551	102	10
Graphite	1.016	0.00276	368	19

Data obtained from Lamarsh and Baratta, Introduction to Nuclear Engineering, Prentice Hall, 2001.

Table 2. Fast neutron diffusion parameters of common moderators

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Biographical Sketch

Dr. Robin A. Chaplin obtained a B.Sc. and M.Sc. in mechanical engineering from University of Cape Town in 1965 and 1968 respectively. Between these two periods of study he spent two years gaining experience in the operation and maintenance of coal fired power plants in South Africa. He subsequently spent a further year gaining experience on research and prototype nuclear reactors in South Africa and the United Kingdom and obtained M.Sc. in nuclear engineering from Imperial College of London University in 1971. On returning and taking up a position in the head office of Eskom he spent some twelve years initially in project management and then as head of steam turbine specialists. During this period he was involved with the construction of Ruacana Hydro Power Station in Namibia and Koeberg Nuclear Power Station in South Africa being responsible for the underground mechanical equipment and civil structures and for the mechanical balance-of-plant equipment at the respective plants. Continuing his interests in power plant modeling and simulation he obtained a Ph.D. in mechanical engineering from Queen's University in Canada in 1986 and was subsequently appointed as Chair in Power Plant Engineering at the University of New Brunswick. Here he teaches thermodynamics and fluid mechanics and specialized courses in nuclear and power plant engineering in the Department of Chemical Engineering. An important function is involvement in the plant operator and shift supervisor training programs at Point Lepreau Nuclear Generating Station. This includes the development of material and the teaching of courses in both nuclear and non-nuclear aspects of the program. He has recently been appointed as Chair of the Department of Chemical Engineering.