OPTIMIZATION METHODS FOR ENERGY SYSTEMS

C. A. Frangopoulos
National Technical University of Athens (NTUA), Greece

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Contents

Summary

In the past, the design and operation of energy systems, either stationary (power plants, co-generation systems, chemical plants, air-conditioning systems, etc.) or mobile (propulsion plants, car engines, etc.) was based primarily on experience. The complexity of contemporary systems and the need to satisfy several objectives, often conflicting with each other (low cost, high efficiency, low emission of pollutants, etc.) make it necessary to aid the work of the system designer and operator with mathematical optimization techniques.

An overview of available methods is presented, divided in two main categories: (a) general mathematical methods, applicable also to energy systems, and (b) special methods for energy system optimization. There is still need of further research and development in the field; a few hints for the work ahead are given.

1. Introduction

In designing an energy system that covers certain needs, the first concern is to reach a workable system, i.e. a system that performs the assigned task within the imposed constraints. The main steps in achieving a workable system are (1) to select the concept to be used (system configuration), and (2) to specify all the technical characteristics of the system components so that the requirements and constraints are satisfied. In most of the cases there are more than one workable system. The task is then to identify the optimum system based on a defined criterion, e.g., efficiency, size, weight, cost.

In operating an energy system, there may be various combinations of components and their operating points (e.g., load factors) that satisfy the needs, while the needs usually change with time. The challenge at this stage is to identify the best operating conditions of the system, i.e. the conditions that satisfy a certain criterion.

In both stages (design, operation) the variations of workable systems and/or of operating conditions that satisfy the needs may be so many that an evaluation of each and every one in order to select the best one is practically impossible.

Consequently, there is need of a systematic procedure to determine the optimum system and the optimum mode of operation. The procedure is called by the general name "optimization." Methods appropriate for optimization of energy systems are presented in brief in this article. For a thorough knowledge of the subject, a study of the relevant literature is necessary.

2. Definition of Optimization

A goal is specified and expressed as a mathematical function of certain variables, which is called an "objective function." Optimization can be defined as follows:

Optimization is the process of finding the conditions, i.e. the values of variables, that give the minimum (or maximum) of the objective function.

In the literature on energy systems, the word "optimization" is often used in cases where the proper word is "improvement." The two words do not have the same meaning and care should be exercised in their use.

3. Formulation of the Optimization Problem

3.1 Mathematical Statement of the Optimization Problem

The general optimization problem consists of a determination of the extremum (minimum or maximum) of an objective function under certain constraints. It is usually stated mathematically as follows:

$$\text{minimize } f(\mathbf{x}) \tag{1}$$

$$\text{with respect to } \mathbf{x} = (x_1, x_2, \ldots, x_n) \tag{2}$$

subject to the constraints:

$$h_i(\mathbf{x}) = 0, \qquad i = 1, 2, \ldots, I \tag{3}$$

$$g_j(\mathbf{x}) \leq 0, \qquad j = 1, 2, \ldots, J \tag{4}$$

where

$\mathbf{x}$ vector of independent variables,
$f(\mathbf{x})$ objective function,
$h_i(\mathbf{x})$ equality constraint functions,
$g_j(\mathbf{x})$ inequality constraint functions.

Maximization is also covered by Eq. (1), since:

$$\min f(\mathbf{x}) = \max \{-f(\mathbf{x})\}$$

3.2. Objective Functions

The decision regarding which criterion is to be optimized is of crucial importance and the answer depends on the particular application: in an aircraft or space vehicle, it may be the minimum weight of the system; in an automobile, it may be the minimum size of the system; in a stationary power plant it may be the minimum life cycle cost (LCC) of the system. Examples of other objective functions for energy systems include:

maximization of efficiency, minimization of fuel consumption, maximization of the net power density, minimization of emitted pollutants, maximization of the internal rate of return (IRR), minimization of the payback period (PBP), etc. Some of these are pure technical objectives, while the rest are (thermo)economic objectives.

In a complex world, a single objective may result in a system that does not satisfy other requirements. Consequently, the final design may deviate from, for example, the least cost one, in order to take environmental, social, aesthetic or other aspects into consideration. Methods have been developed under the name "multiobjective optimization," which attempt to take two or more objectives into consideration simultaneously. The optimum point they reach does not satisfy each objective in isolation but it corresponds to a compromise, often subjective, of the various objectives.

3.3. Independent Variables

Each component and the system as a whole is defined by a set of quantities. Certain of those are fixed by external conditions (e.g., environmental pressure and temperature, fuel price) and are called "parameters." The remaining are variables, i.e. their value may change during the optimization procedure. Those variables, the values of which do not depend on other variables or parameters, are called "independent variables." The rest can be determined by the solution of the system of equality constraints and they are called 'dependent variables'. The number of dependent variables is equal to the number of equality constraints. Thus, the task of the optimization procedure is to determine the values of the independent variables ($\mathbf{x}$ in Eqs. (1)–(4)). Of course, if the number of equality constraints is higher than the number of all the variables, then the problem is over-specified and there is no room for optimization.

3.4. Equality and Inequality Constraints

The functions appearing in Eqs. (3) and (4) are expressions involving design characteristics and operating parameters or variables of the components as well as the system as a whole. For example, the required mass flow rate of steam in a steam turbine is given as a function of the power output and the properties of steam at the inlet and outlet of the turbine. On the other hand, the safety and operability of the system impose inequality constraints such as the following: speed of revolution not higher than a certain limit; quality (dryness) of steam at the exit of the steam turbine not lower than a certain limit, etc.

The set of equality and inequality constraints is derived by an analysis of the system and constitutes the mathematical model of the system. Models may initially be developed at the level of each component, which are then integrated to form the model of the whole system.

A word of caution: Describing reality by mathematics is not an easy task and it is often accompanied by simplifying assumptions, which introduce inaccuracies. This is mentioned not in order to deter one from applying modeling and optimization techniques, but to make it clear that the solution (design or operation point) reached is optimal only under the assumptions made in modeling the system; and it is as close to the real optimum as any discrepancies between model and reality allow. However, most probably, if a care has been taken, it is closer than a decision based only on past experience or similar preceding designs.

4. Levels of Optimization of Energy Systems

Optimization of an energy system can be considered at three levels:

(A) *Synthesis optimization*. The term "synthesis" implies the components appearing in a system and their interconnections. If the synthesis of a system is known, then the flow diagram of the system can be drawn.

(B) *Design optimization.* The word "design" here is used to imply the technical characteristics (specifications) of the components and the properties of the substances entering and exiting each component at the nominal load of the system. The nominal load is usually called the "design point" of the system. One may argue that design includes synthesis too. However in order to distinguish the various levels of optimization and due to the lack of a better term, the word 'design' will be used with the particular meaning given here.

(C) *Operation optimization.* For a given system (i.e. the synthesis and design are known) under specified conditions, the optimal operating point is requested, as it is defined by the operating properties of components and substances in the system (speed of revolution, power output, mass flow rates, pressures, temperatures, composition of fluids, etc.).

Of course if complete optimization is the goal, each level cannot be considered in complete isolation from the others. Consequently, the complete optimization problem can be stated by the following question:

What is the synthesis of the system, the design characteristics of the components and the operating strategy that lead to an overall optimum?

The degree of freedom increases when the task of the system, i.e. its production rates, are not pre-specified but are to be determined by the optimization procedure. Time-dependent optimization adds one more dimension, which increases the complexity of the problem.

5. Mathematical Methods for Solution of the Optimization Problem

In spite of their apparent generality, there is no single method available for solving efficiently all the optimization problems stated by Eqs. (1)–(4). A number of methods have been developed for solving different types of optimization problems. They are known as *mathematical programming methods* and they are usually available in the form of a mathematical programming algorithm.

5.1 Classes of Mathematical Optimization Methods

Optimization problems and the techniques developed for their solution can be classified in several ways, depending on the criterion. The classification is very useful from the computational point of view, because there are many special methods available for the efficient solution of particular classes of problems.

5.1.1. Constrained and Unconstrained Programming

Any optimization problem can be classified as constrained or unconstrained, depending on whether or not constraints exist in the problem.

5.1.2. Search and Calculus Methods

A search method uses values of the objective function in order to locate the optimum point, with no use of derivatives. On the contrary, calculus methods use first and (some of them) second derivatives. Search methods calculate the values of the objective function at a number of combinations of values of the independent variables and seek for the optimum point. The search may be random or systematic, the second one usually being more efficient.

If the objective function is continuous, by applying a search method the exact optimum can only be approached, not reached, by a finite number of trials, because only discrete points are examined. However, the region, in which the optimum point is located, can be reduced to a satisfactorily small size at the end of the procedure. On the other hand, there are problems for which search methods may be superior to calculus methods, as for example in optimization of systems with components available only in finite sizes.

5.1.3. Linear, Nonlinear, Geometric, and Quadratic Programming

This classification is based on the nature of the equations involved. If the objective function and all the constraints are linear functions of the independent variables, then a linear programming (LP) problem is at hand. If at least one of the functions (no matter whether it is the objective function or one of the constraint functions) is nonlinear, then the problem is a nonlinear programming (NLP) problem.

A geometric programming (GMP) problem is one in which the objective function and the constraints are expressed as posynomials in **x**. A function f(**x**) is called a *posynomial*, if it has the form:

$$f(\mathbf{x}) = c_1 x_1^{a_{11}} x_2^{a_{12}} \ldots x_n^{a_{1n}} + \ldots + c_N x_1^{a_{N1}} x_2^{a_{N2}} \ldots x_n^{a_{Nn}} \qquad (5)$$

where c_i and a_{ij} are constants and $c_i > 0$, $x_j > 0$.

A quadratic programming (QP) problem is a nonlinear programming problem with a quadratic objective function and linear constraints.

5.1.4. Integer- and Real-valued Programming

This classification is based on the values permitted for the independent variables. If some or all of the independent variables of an optimization problem are restricted to take on only integer (or discrete) values, then the problem is called an *integer programming* (IP) problem. If all the independent variables are permitted to take any real value, then the optimization problem is called a *real-valued programming* problem.

The existence of integer variables in linear and nonlinear programming problems leads to *mixed integer linear programming* (MILP) and *mixed integer nonlinear programming* (MINLP) problems, respectively.

5.1.5. Deterministic and Stochastic Programming

If some or all of the prespecified parameters and/or independent variables are probabilistic (nondeterministic or stochastic), then the optimization problem is *a stochastic programming problem*. Otherwise, it is a *deterministic programming problem*.

5.1.6. Separable Programming

A function f(**x**), $\mathbf{x} = (x_1, x_2, \ldots, x_n)$, is called *separable* if it can be expressed as the sum of n single-variable functions:

$$f(\mathbf{x}) = \sum_{i=1}^{n} f_i(x_i) \tag{6}$$

A separable programming problem is one in which the objective function and the constraints are separable functions.

5.1.7. Single and Multiobjective Programming

Depending on the number of objective functions, optimization problems can be classified as *single-objective* or *multiobjective* programming problems. In most of the problems there is no single point $\mathbf{x}^*$ that satisfies all the objectives simultaneously. Therefore, there is usually need of a compromise, often subjective.

5.1.8. Dynamic Programming and Calculus of Variations

Dynamic programming (DP) or calculus of variations (COV) is applied when an optimal function rather than an optimal point is sought. The calculus of variations seeks a function that optimizes an integral; in a single variable, the problem is stated as

$$\min_{y} I = \int_{x_1}^{x_2} F(x, y, y', y'')\,dx \tag{7}$$

where y = y(x) is the function sought, F is a known function and

$$y' = \frac{dy}{dx}, \qquad y'' = \frac{d^2y}{dx^2}$$

Dynamic programming is applicable to staged processes or to continuous functions that can be approximated by staged processes. Thus, the decision variables are sought for which, for a specified input to stage n and a specified output from stage 1, the summation $\sum_{i=1}^{n} F_i(y)$ is optimum.

COV and DP are both methods to determine y(x). Which method is precise and which is an approximation depends on the problem. If, for example, the velocity of a vehicle is continuously adjusted during a trip to minimize the total fuel consumption, COV is a precise representation and DP is an approximation (since it would represent the varying speed as a series of steps). If, however, the problem were to optimize the sizes of a series of heat exchangers, DP would be the precise method and COV an approximation.

5.1.9 Genetic Algorithms

Genetic Algorithms (GAs) have been developed by J. Holland in an attempt to simulate growth and decay of living organisms in a natural environment. Even though originally designed as simulators, GAs proved to be a robust optimization technique. Philosophically GAs are based on the concepts of biological evolution (natural genetics and natural selection) and Darwin's theory of survival of the fittest. The basic elements of natural genetics, i.e., reproduction, crossover and mutation, are used in the genetic search procedure. The main characteristics of the GAs, which highlight also their differences from the traditional methods of optimization, are the following:

- A population of points (instead of a single point) inside the optimization space, selected randomly, is used to start the procedure. Since several points are used as candidate solutions, GAs are less likely to be trapped at a local optimum.
- The GAs use only the values of the objective function. The derivatives are not used.
- In GAs the decision variables are represented as strings of binary variables, that correspond to the chromosomes in natural genetics. Any type of variables, either discrete (e.g. integers) or continuous, can be handled. For continuous variables, the string can be selected so that the desired resolution is achieved.
- The value of the objective function of each string in a population plays the role of fitness in natural genetics.
- A new population is generated (reproduction) by applying randomized crossover and mutation on the old one. The value of the objective function is used so that "weak" strings are dropped out, while "strong" strings give more offsprings in the new population. The procedure is repeated until no further improvement is achieved.

The aforementioned show that GAs are appropriate for problems with mixed discrete-continuous variables and discontinuous and non-convex decision spaces. Furthermore, in most cases they have a high probability in finding the global optimum.

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Biographical Sketch

Christos A. Frangopoulos is Professor at the Department of Naval Architecture and Marine Engineering, National Technical University of Athens (NTUA), Greece. He received the Diploma in Mechanical and Electrical Engineering from the NTUA in 1971. After his military service (1971-1973), he worked as Superintendent Engineer of ship-owning companies, and as Head of the Diagnostic Center of a ship repairing company in Greece (1973-1979).He performed graduate studies in Mechanical Engineering with major in Thermal Sciences at the Georgia Institute of Technology, Atlanta, Ga., USA, leading to the M.Sc. degree (1980) and Ph.D. degree (1983). He joined the Department of Naval Architecture and Marine Engineering (NTUA) as a faculty member in 1985. He lectures on marine engineering, as well as marine and land-based energy systems in both undergraduate and inter-departmental graduate courses. His research activity is related to the development and application of methods for analysis, evaluation and optimal synthesis, design and operation of energy systems (power plants, propulsion plants, heat recovery systems, cogeneration systems, etc.) by combining thermodynamic, economic and environmental considerations. Second Law (exergetic) analysis and internalization of environmental externalities are two particular subjects of this work. He has often given invited lectures on the results of his research in several countries. Among his publications are more than forty papers in journals and international conferences and one book on cogeneration (in Greek).