SYSTEM ANALYSIS OF FINANCIAL MARKETS: AN OVERVIEW

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Summary

This article presents an overview of the methods of system analysis of financial markets, methods of investment portfolio's forming and active management, as well as classical methods and modern directions in defining market's statistical characteristics. A notion is given on disseminating the present parameter Value at Risk and methods of

its determination and usage are described.

1. Introduction

The financial market (FM) is a broad field of human activities concerned with the trade of various financial instruments such as home and foreign currencies, governmental, municipal and corporate bonds, corporate stocks and many other derivative securities (futures, options and so on). Millions of people are occupied in this field and the results of their activity influence all aspects of our social and economical life. On the other hand, FM is the tip of an economical and political iceberg and all events in the depth of that iceberg influence FM behaviour. One can present a structural description of FM including banks, stock exchanges over-the-counter market, corporations issuing securities, investment companies, funds and so on. But here we are concerned only with the formal quantitative description of FM behaviour presented by the time series of prices which can be called financial processes. Decision making in FM is investment, quantitative measure of its efficiency is return on the invested capital. The return depends on present prices of securities included in an investors' portfolio and on their prices in the future when the securities are to be sold. System analysis uses mathematical models to forecast prices and returns on securities and to give some recommendation for the choice of portfolios.

2. Classification of Main Approaches

Many different approaches are used in practice for forecasting and decision making on FM. They are traditionally classified as

- methods of fundamental analysis (FA),
- methods of technical analysis (TA),
- methods of system analysis (SA).

2.1 Fundamental Analysis (FA)

Fundamental analysts affirm that security prices are defined by internal factors characterising the activities of those who have issued that security. For example, one can suppose that returns on government bonds depend on the inflation rate, trade balance and so on. A corporate stock price is a result of corporate earnings, dividend policy, and financial reserves. Hence it is reasonable to estimate the influence of such internal fundamental factors, e.g. reports accounting figures or other available information. Fundamental analysts calculate such values as a price-earnings ratio, dividend yield, and liquidity ratios. Bearing in mind the implicit hypothesis that those values will be the same in the future, they try to estimate the profitability of investment in corresponding stocks. As a rule, the classical fundamental analysis gives no formal forecasting algorithms. Moreover, each type of security is considered independently. Generalised FA (GFA) assumes that individual stock behaviour depends on many economical, social and political fundamental factors characterising the security market as a whole and its industrial branches. GFA uses some econometric models in order to forecast the development of industrial branches and to use the results in forecasting price movement for industrial firms if they are attributed to some branch.

2.2 Technical Analysis (TA)

Classical TA (so called chartism) uses graphs (charts) of prices and market volumes to predict some critical points where the price growth is replaced by a depreciation and vice versa. The graphs are individually evaluated by an expertise technique or by a formal extrapolation.

Generalised TA (GTA) admits a formal consideration of price and volume time series of many individual securities simultaneously but GTA asserts that it is unreasonable to take into account other fundamental factors. It claims all useful information is contained in market history only.

2.3 System Analysis (SA)

Modern SA recognises that FA and TA are not competitive but complementary. SA uses both market history and some fundamental factors in security price forecasting. But the most important feature of SA is estimation of risk which is intrinsic to any decision making on FM. From SA point of view, forecasting errors play an essential role in modelling and optimization. On the whole, the basic scheme of SA for FM is the same one as for any other applications of that approach, i.e., hydrology, ecology and so on. It includes:

- design of forecasting models,
- verification of invariant values including in models by statistical procedures,
- formalization of decision making criteria,
- choice of the best solution by a formal optimization procedure, and
- tests of decision making rules.

Nevertheless, SA for FM has some specific elements and some specific methods.

3. Optimal Portfolio Theory (OPT) and Capital Asset Pricing Model (CAPM)

The starting point of SA is an eminent concept proposed by H.Markowitz (Nobel prize,1990) in 50s. He introduced a probabilistic description of security returns and formulated the basic optimization problem, well-known now as optimal portfolio theory (OPT). That theory was modified by D.Tobin (Nobel prize, 1981) who has shown a specific structure of optimal solutions under an additional hypothesis that there exists a risk-free variant of investments. W.Sharpe (Nobel prize, 1990) used that result as a main argument to formulate Capital Asset Pricing Model (CAPM) which gives a relation between the expected behaviour of individual securities and the expected behaviour of the security market as a whole. OPT and CAPM are key stones of the classical SAFM.

3.1 Return as a Random Value

Let $s(t)$ be security price at time t. The security price return for the interval is defined as

$$r(t) = \frac{s(t+1) - s(t)}{s(t)} \qquad (1)$$

In other words, $r(t)$ is a profit or loss obtained by an investor per unit of invested capital under the condition that he buys the security at time t and sells it at time $t+1$. If, while buying the security, investors have a right to obtain some additional payments, such as dividends $d(t)$, then one can calculate $r(t)$ as the market capitalization rate

$$r(t) = \frac{s(t+1) + d(t) - s(t)}{s(t)} \qquad (2)$$

Let an investor have a capital C invested in various securities. It is called portfolio investment. Let x_j be a part of the capital invested in j -th security, so that the investor has each security in a volume N_j, $N_j = \frac{Cx_j}{s_j(t)}, j = 1,...,n,$ where $s_j(t)$ is a price of j -th security.

At time $t+1$, the investor's capital will be equal to

$$\sum_{j=1}^{n} N_j \left[s_j(t+1) + d_j(t) \right] = C \sum_{j=1}^{n} x_j \frac{s_j(t+1) + d_j(t)}{s_j(t)} = C \sum_{j=1}^{n} \left(1 + r_j(t) x_j \right)$$

Hence the return on portfolio investment is $r_p(t)$,

$$r_p(t) = \sum_{j=1}^{n} r_j(t) x_j \qquad (3)$$

It is supposed that the goal of any decision making is the enlargement of the return r_p by a rational allocation $\{x_j\}$ of the capital. If the values $r_j(t)$ are known there exists a simple solution. All of the capital is to be invested in a security in which return is largest. However, the values $r_j(t)$ are principally uncertain. The decision making is to be realised at the present time t but the future prices $s_j(t+1)$ and the future dividends $d_j(t)$ are unknown. The security market behaviour is chaotic and there are no ways of forecasting it exactly.

3.2 Portfolio Optimization

OPT is based on the following hypotheses:

For any fixed t, the returns $r_j(t)$ are random values r_j having known mean values and covariances. Those values do not depend on the decision making of any individual investor. In particular, it means that OPT cannot be applied if big gamblers are present whose actions can change market behaviour.

Any investor avoids unnecessary risk. Having two possible variants of portfolios with equal expected returns, he/she prefers a variant with a smaller uncertainty. The variation of portfolio return random value is used as the uncertainty index.

H.Markowitz gave the following setting of optimal decision making: one has to choose a portfolio which has minimal variation of its return among all admissible portfolios with the same mean values of the return. It is not hard to show that

$$m_p = \sum_{j=1}^{n} m_j x_j$$

$$v_p = \sum_{i=1}^{n}\sum_{j=1}^{n} v_{ij} x_i x_j \qquad (4),$$

where m_p, m_j are mean values of returns r_p, r_j correspondingly, v_p is variation of portfolio return , and v_{ij} are covariances of individual returns , i.e.

$$v_{ij} = E\ \{\ (r_i - m_i)(r_j - m_j)\ \}. \qquad (5)$$

The optimization problem has a form of quadratic programming

$$\min \sum_{i=1}^{n}\sum_{j=1}^{n} v_{ij} x_i x_j \qquad (6)$$

s.t.

$$\sum_{j=1}^{n} m_j x_j = m_p; \sum_{j=1}^{n} x_j = 1, x_j \geq 0,\ j = 1,...,n.$$

Non-negativity condition may be omitted if the so-called "short-sale" operations are permitted. In that case, the absolute value of negative x_j means a debt. It is essential to see that m_p is a parameter. Changing m_p, one can obtain a set of optimal portfolios having minimal variations under any fixed m_p. It can be presented as a convex curve on the plane σ_p, m_p where the standard deviation of the optimal portfolio return, $\sigma_p = \sqrt{v_p^2}$. The more the desired expected return is, the more is minimal risk. D.Tobin took into account that there exists a risk-free investment with a fixed predetermined return. In that case, OPT conditions are changed as follows

$$\sum_{j=1}^{n} m_j x_j + r_0 x_0 = m_p; \sum_{j=0}^{n} x_j = 1, \qquad (7)$$

$$x_j \geq 0,\ j = 1,...,n,$$

where x_0 is a part of capital invested in risk-free asset.

Optimal solutions to the modified problem have some remarkable properties:

- minimal risk linearly depends on the expected value of return,
- optimal structure of risky investments does not depend on the desired expected value of portfolio return.

3.3 CAPM and Market Line

Let all investors have the same information expressed in the same values of expected returns and covariances. Then all of them, being "rational", have to choose the same structure of their own risky portfolios. Their various levels of risk aversion only determine a part of capitals which will be saved in risky-free investments. But it demonstrates that the security market, where only equally informed and rationally behaving investors exist, will have the optimal structure. It is the most important element of the market equilibrium concept called CAPM. A practical conclusion is very simple: the best choice for a reasonable investor is to have a portfolio where all risky securities are in the same proportions as they are in the market. As a formal consequence one can obtain following relation:

$$m_j - r_0 = \beta_j \left(m_m - r_0 \right) \tag{8}$$

i.e. excess return (return above risk-free return) on a security is proportional to the market excess return. Proportionality coefficients β_j (beta-coefficients) play a significant role in the security market analysis and applications.

Following the optimization scheme and "ideal" market assumptions one can obtain the expression for beta coefficients :

$$\beta_j = \frac{\operatorname{cov}\{r_j, r_m\}}{v_m} = \frac{1}{v_m} E\{\left(r_j - m_j\right)\left(r_p - m_p\right)\}. \tag{9}$$

In many cases this expression is considered as the definition of betas, and the principal equation (8) of CAPM is treated as the proportionality between the excess returns and betas.

4. Statistical Verification of Classical Models

In order to develop OPT and CAPM, securities returns were assumed to be random values with some means and covariance matrix . However, the formal results are unuseful if those values are unknown. The estimation of m_j, v_{ij} is to be based on real information available for analysis.

4.1 Statistics of Mean Values and Covariances

Let one have time series of security prices $s_j(k), k = 0,1,...,t$ for some interval up to the time t when the portfolio problem has to be formulated and solved. Then one can calculate the corresponding sequence of returns for unit intervals

$$r_j(k) = \frac{s_j(k+1) - s_j(k)}{s_j(k)}, k = 0,...,t-1, j = 1,...,n.$$

These histories can be used in estimating m_j, v_{ij} but it is necessary to introduce some additional hypotheses. One can use the simplest supposition that the values of $r_j(k)$ are the realizations of the same random constant r_j. After that it is reasonable to design the following estimates

$$\hat{m}_j = \frac{1}{t}\sum_{k=0}^{t-1} r_j(k), \tag{10}$$

$$\hat{v}_{ij} = \frac{1}{t}\sum_{k=0}^{t-1}\left[r_i(k) - \hat{m}_i\right]\left[r_j(k) - \hat{m}_j\right]. \tag{11}$$

Note that the estimates $\hat{v}_{ij}$ are acceptable only if t is large if compared with n. In other words, the time series must be sufficiently long.

However, real time-series are not so long $\left(t \approx 10^2 \div 10^3\right)$ if the time unit is a week or a month but the number of essential securities circulating at the security market is very large. For instance, the well-known Standard & Poor's Index includes 500 securities. Hence, it is necessary to estimate more than 10^5 covariances v_{ij} having the number of observations of the same order. As a rule, all estimates (11) will be insufficient. So that one might overcome that problem, a strengthened variant of the market line equation (8) is commonly used

$$r_j - r_0 = \beta_j\left(r_m - r_0\right) + \varepsilon_j \tag{12}$$

where ε_j are supposed to be i.i.d. (independently and identically distributed) random values with zero mean.

Then

$$v_{ij} = \beta_i \beta_j v_m + v_{\varepsilon i}\delta_{ij} \tag{13}$$

and that's enough to estimate only $2n+1$ values of $\beta_i, v_{\varepsilon i}, v_m$.

4.2 Statistics of Market Line

The model given by (13) is a usual regression model. Therefore, one can try to use least square (LS) methods. The results appear to be very unsatisfactory. To improve the fitness of the model, it is traditionally extended to the form

$$r_j - r_o = \beta_j \left(r_m - r_0 \right) + \alpha_j + \varepsilon_j \tag{14}$$

where α_j (alphas) are some additional constants.

The model (14) is more adequate and is used intensively in practice. The estimates of alphas, betas and the values R-squared,

$$R_j^2 = \frac{\beta_j^2 v_m}{v_{\varepsilon j}},$$

are published regularly in financial press. It should be pointed out that, as a rule, alphas are not equal to zero and it disproves CAPM. Of course it is bad for the theory but it's good for gamblers: there exists a possibility to "beat the market" choosing assets with positive alphas. On the other hand, the relations (13) which were used for simplifying the estimation of v_{ij} , remain valid under the generalized condition (14).

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