FUNDAMENTALS OF MATHEMATICAL MODELING FOR COMPLEX SYSTEMS

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Summary

The notion about technology of mathematical modeling is introduced as whole instruments, methods, knowledge for mathematical model composition, identification, verification, and exploitation. The notions about 'simple' and 'complex' systems (processes, phenomena) are introduced as connected tightly with mathematical modeling technology development. Simulation models are treated as being on the frontier between 'simple' and 'complex' phenomena (processes systems). This frontier is considered as moving from 'simple' to 'complex' or from 'mathematical' to 'humanities'. Other aspects of interaction between mathematical and humanities methods of prognosis are described. The role of mathematical modeling in the solution of the problem of sustainable development is discussed.

1. Closed Mathematical Models

In the framework of this paper the collocation 'mathematical model' will signify the set of equations (more generally — the set of relations) between the characteristics of some phenomena (processes, systems). Up to section 7 of the paper mathematical modeling will be discussed from a pragmatic point of view as an instrument for prediction the development of phenomena (processes, systems) or their properties. This point of view is rather limited and will be improved later. The unknown characteristics of mathematical models in terms of which the predictions are formulated and which are intended to reveal the means of mathematical modeling are called 'endogenous' (or prognosis, or inner, or phase variables) characteristics. The 'exogenous' (or outer, or parameters) characteristics are also used in many mathematical models. The endogenous characteristics of mathematical models are significantly influenced by their exogenous characteristics, while from the practical point of view (i.e. considering some conditions within the framework of certain correctness) opposite influence does not take place. Mathematical models (i.e. the set of relations) will be called there 'closed', if its endogenous characteristics may be defined by the set of relations as soon as its exogenous characteristics are known.

Thus 'hypothesis about independence' presents the basis of any closed (and therefore capable of prognosis) mathematical model. The essence of hypothesis about independence is presented here: one system of characteristics of phenomena (process, system) depend significantly on other system of characteristics but opposite dependence is absent with practical point of view. Several fundamental principles of independence often serve as the basis of hypothesis about the independence of a mathematical model. For instance, Newton's gravitation law lay at the bottom of one of fundamental principles of independence, which are basic in the mathematical models of motion of cosmic objects. Let there are two bodies with masses m_1 and m_2 and R - distance between 'centers' of the bodies (if the distance considerably exceeds the sizes of the bodies, then there is no need to say 'centers'). The essence of Newton's gravitation law is the gravitational force F act on mass m_1 in the line of mass m_2 and on mass m_2 in the line of mass m_1 so that the ratio FR^2/m_1m_2 is the fundamentally constant, i.e. it does not depend on any characteristic of the real world.

The second Newton's law is also basic for mathematical models of mechanical motion of objects. Let F — the force, which acts on some body, a — acceleration (in an inertial coordinate system) of the body under the action of the force F. The value $m = F/a$ do's not depend on F, a , the body velocity and thus is the internal characteristic of the body, called mass.

The hypotheses about independence arise by observation, by measurements and by measurements processing. However, there exists one more criterion, which influences the forming of hypothesis about independence and, consequently, the forming of corresponding mathematical model. This criterion is the beauty of conceptions, which are the results of modeling. From experience is known, that out of two hypotheses the hypothesis which possesses 'internal beauty', turns out to be more correct then the second one which seems to be more corresponding to the existing measurements.

2. Technology of Mathematical Modeling

Up till now composition of mathematical models had been discussed. But composition of mathematical models presents only the first step in practical process of prediction of phenomena properties (process development, systems behavior). The next steps are:

- identification of models, i.e. measurement (calculation, certain ways of definition) of the values of the exogenous characteristics of models;
- testing mathematical model's closure; elaboration, calculation and processing (usually for computation), of the values of endogenous characteristics under condition that the values of exogenous characteristics are known;
- verification of models, i.e. clearing up conditions ensuring correctness of model's prediction;
- exploitation of the model, i.e. extraction of consequences from model's equations, in particular, realization of calculation of the values of endogenous characteristics.

The special term 'technology of mathematical modeling' will be used in the framework of this paper for designation of the aforementioned steps. This term was introduced in order to avoid introducing additional meaning into the customary term 'mathematical modeling'. In order to comment the aforementioned steps of mathematical modeling two examples of mathematical models will be described. The first example demonstrates the simplest model; it describes the changes of the numbers of population in certain region of a certain States. The proposition is introduced that the number of population in the region is several millions and there is no migration. The second example deals with the simplest model of the motion of Earth's satellite.

3. Examples of the Mathematical Models

3.1. Demography Model

Discussion starts with the simplest model of demography, describing the changes of numbers of population in a region of certain State during some time frame. Let t — initial year of the time frame, T — final one. The current year will be designated as i. Let x_i - number of persons in the region in year i. Then $i = t, t+1, ..., T$. The assumption is introduced that relative changes of the number of persons within one year, i.e. values $(x_{i+1} - x_i)/x_i$ do not depend on i. This assumption presents the hypothesis about independence, which was mentioned earlier. It is very strong and correct only if the social, economic and ecological situation in the region is stable. This assumption written down as a set of equations $(x_{i+1} - x_i)/x_i = \alpha$, $i = t, t+1, ..., T-1$, is the mathematical model of the process under consideration. There α is joint value of all relative changes of numbers of persons in the course of one year. A more convenient form of this model is $x_{i+1} = (1+\alpha)x_i$, $i = t, t+1, ..., T-1$. In order to obtain prognosis of the process under consideration two characteristics of the process have to be known: α and x_t. It is evident that, $x_{t+1} = (1+\alpha)x_t$, $x_{t+2} = (1+\alpha)^2 x_t$ and $x_i = (1+\alpha)^{i-t} x_t$. Thus, this model is closed if characteristics α and x_t are considered to be exogenous characteristics and characteristics x_i, $i = t+1, t+2, ..., T$ are considered to be endogenous. In order to measure the values of exogenous characteristics, i.e. in order to identify the model it is necessary:

- to carry out census of population in the region in the initial year t;
- to have information about people’s birth’s and death’s in a year.

It is obvious that identification of the model under consideration is a special action, which demands organization and huge expenditures. These expenditures exceed all other expenditures needed to obtain in accordance with this model the prognosis of changes of the numbers of population.

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Biographical Sketch

Yury Pavlovsky is Professor, Associated Member of Russian Academy of Science, head of Department 'Simulation Systems' of Computing Center of Russian Academy of Science. He holds his Dr. of Mathematical-Physical Sciences from Moscow Physical-Technical Institute, 1964.
His scientific interests include Simulation Systems, Decomposition Theory, Control Theory. He published over 100 papers and 4 monographs.

Among these the following can be mentioned:

1) The problem of decomposition in Mathematical Modeling. - Moscow: Nauka, 1998 (in Russian, together with T. Smirnova)
2) Simulation models and systems.-Moscow: Phasis, 2000 (in Russian).