FORMAL TOOLS FOR EXPLORING COMPLEXITY

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Keywords: complexity, systems theory, fractals, nonlinear dynamics, chaos, entropy

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Summary

Much of the discussion in this paper is concerned with clarifying the concepts of order, disorder, and complexity, and the formulation of their corresponding measures for a complex system. We begin with a review of the concept of entropy because this was the first measure of order in physical systems and understanding the evolution of this concept will also make clearer the evolution of our understanding of complexity. Thus, we start from thermodynamic entropy, along with the ancillary notion of equilibrium, and end with the dynamics of chaos and fractional dimensions. In particular, we use the normalized entropy explicitly to define parameters for order, disorder and complexity. In addition we find that the generalized fractional dimension of a complex dynamical process is intimately related to one of the modern concepts of entropy used as a measure of complexity. We conclude that the generalized fractional dimension and the approximate entropy are two measures of complexity that have demonstrated their value in providing quantitative measures of the complexity of dynamical systems.

1. Introduction

As we grow older the world appears to be more complex and harder to understand, and problems are more difficult to solve. Some students of this phenomenon explain that it is a matter of perception, having to do with aging and therefore it is a subjective rather than objective effect. Others contend that this increased complexity is real and is a consequence of evolution; that cultures, technologies, and biological species either become more complex with age or they die out. The thesis of this paper is that those things which survive do become more complicated with the passage of time, and it is our intent to examine a number of candidate measures that have been proposed to quantify this increase. We shall not be seduced into attempting to understand the mechanisms by which evolution favors increased complexity over time, but merely point to urban sprawl, the internet, the human intellect and democracy as being symptomatic of this phenomenon.

Complexity can not be neatly pigeonholed into a definition from one of the physical sciences, although it is obvious that certain physical phenomena such as hurricanes, oscillating chemical reactions and droughts are complex. However, the specific definitions of complexity used to describe these phenomena are often process-specific and difficult to generalize. Similarly, complexity is also the province of the social and life sciences, and even though examples can be drawn from each, these too are difficult to generalize to all phenomena. To appreciate how the complications of complex phenomena have changed our view of the world we go back a few decades to the work on *systems theory* (ST) pioneered by Von Bertalanffy and subsequently developed by many scientists.

ST adopts the perspective that determinism, at best, provides an inadequate description of nature and a holistic approach is more suitable for understanding phenomena in the social and natural sciences. This methodology casts the scientist in the role of "problem solver" so that in order to extract information from the system the scientist must develop a "heuristic" understanding of the problem to be solved by means of metaphors. The holistic perspective assumes that scientific knowledge is universal in that laws within a given field of study can often, if not always, be mirrored in all other fields of study. The conservation of energy is therefore not just a law of physics, for if the law were only that it would have relatively limited value; its universal importance stems from the fact that energy conservation also applies in biology, sociology and every other area of human investigation.

To facilitate our discussion we suggest a tentative definition of complexity -- one that is found in a number of places, in various related forms: *In general complexity is associated with phenomena we find difficult to understand or equivalently with the difficulty of extracting information from a phenomenon.* One might be tempted to criticize the generality of this definition, but it is that very generality that makes it so appealing. For example, the complexity envisioned here may be applied to all manner of natural and social systems with the clear understanding that it is not objective, but depends on both the system and the observer.

In addition to the subjective nature of this concept of complexity, due to the explicit inclusion of the observer in its definition, there is the additional ingredient of subjectivity having to do with the "questions" the observer asks of the system. Thus, complexity depends on the purposes of the experimenter. For the neurophysiologist the human brain is one of the most complex structures in the universe, but to a butcher it is simply one of a number of kinds of meat. Here the notion of observer is really that of an experiment, but one could also formulate the same definition involving systems containing human beings or other conscious entities. So complexity is in the eye of the beholder.

A system consists of a set of elements together with a defining set of relations among those elements. All the phenomena of interest to us here shall be viewed as systems. It is also possible to study a subset of elements, called a subsystem of the system and continue the telescoping of systems to smaller and smaller, but equally complex, entities. Such nested behavior can be observed in the workings of federal, state, county and city governments, for example. Finally, the system may interact with the observer, who may be a member of the system itself or of the environment. It is also possible, and sometimes necessary, to define an environment of the environment and so on. As already pointed out, the complexity of a system depends on the information sought by the observer, and this depends on the purpose of the study. We imagine that a system may be studied to "understand it", namely to describe and control it or to predict its dynamics. For example, the weather cannot be controlled, but it is very useful to make accurate short-term forecasts. Predicting the trajectory of a hurricane may save millions in dollars, not to mention the saving of lives, even if in principle we cannot know its fundamental nature. It is often crucial to study, whenever possible, the response of a complex system to external perturbations. It is the set of these responses that constitute the information that the observer tries to extract from the system and it is the difficulty encountered in understanding, controlling or predicting these responses that is intuitively used in measures of complexity.

So now we come down to the crucial question: Can we define a measure of complexity that will be useful across a broad spectrum of problems, from the stock market to superconductivity, from social discontent to the laughter of children? Using our tentative definition one may think that it is possible to define a measure, since the concept of "difficulty" admits an ordering relation. In other words it makes sense to say; "problem A is more difficult than problem B". However, a little reflection reveals that this kind of relation is not really objective, since the difficulty of the problem depends on who has to face it and at what time. Establishing objective criteria, for example imagining that the problems have to be faced by machines such as computers or by adopting rigid algorithms, just transfers the subjectivity to the level of the arbitrary criteria adopted.

We make the assumption that complexity is a property of the system and we do not address the difficulties associated with the observer, such as prejudice, limited resources and so on. Even in this restricted context of theory we hope that the measures discussed shall be of some value. In particular, since the measure is inextricably woven into the fabric of complexity, we shall have to be more explicit in what we mean by complexity.

2. Complex Systems

There has been a substantial body of mathematical analysis developed regarding complexity and its measures, and the broad range over which mathematical reasoning and modeling have been applied is rather surprising. One class of problems which defines the limits of applicability of such reasoning is "computational complexity". A problem is said to be computationally complex if to compute the solution one has to write a very long algorithm, essentially one as long as the solution itself. Applications of this quite formal theory can be found in a variety of areas of applied mathematics, but herein we avoid these more formal issues and focus our attention on ST, the influence of nonlinear dynamics in such theories and the subsequent notion of complexity derived from this influence.

It is useful to list the properties associated with the complexity of a system, because we are seeking a quantitative measure that may include an ordinal relation for complexity. We note, however, that in everyday usage, phenomena with complicated and intricate features, having both the characteristics of randomness and order, are called complex. Further, there is no consensus among scientists, poets or philosophers as to what constitutes a good quantitative measure of complexity. Therefore any list of traits of complexity is arbitrary and idiosyncratic, but given that disclaimer the following traits are part of any detailed characterization:

i) A complex system typically contains many elements each one representing a dynamical variable.

ii) A complex system typically contains a large number of relations among its elements. These relations usually constitute the number of independent dynamical equations that determine the evolution of the system.

iii) The relations among the elements are generally nonlinear in nature, often being of a threshold or saturation character or more simply of a coupled, deterministic, nonlinear dynamical form.

iv) The relations among the elements of the system are constrained by the environment and often take the form of being externally driven or having a time-dependent coupling. This coupling is a way for the system to probe the environment and adapt its evolution for maximal survival.

v) A complex system is typically a composite of order and randomness, but with neither being dominant.

vi) Complex systems often exhibit scaling behavior over a wide range of time and/or length scales, indicating that no one or few scales are able to characterize the evolution of the system.

These are among the most common properties selected to characterize complex systems, and, in a set of dynamical equations, these properties can often be theoretically kept under control by one or more parameters. The values of these parameters can sometimes be taken as measures for the complexity of the system. This way of proceeding is however model-dependent and does not allow comparisons between the complexities of distinctly different phenomena, or more precisely between distinctly different models of phenomena.

In the above list we included two of the most subtle concepts entering into our discussion of complexity, and those are the existence and role of randomness. In the present context the idea of randomness may be related to the difference between an "act" and an "event". Following Turner we distinguish between the two by noting that an event has a symmetry in time, in that there is no difference between knowing an event can happen and knowing that an event did happen, so no additional information is gained by the occurrence of an event. On the other hand (here we replace Turner's word "act" with the less value-laden word "action") an action has an asymmetry in time, in that what is known about a process is fundamentally different before and after an action. An event may be predicted by the situation preceding it, an action may not. However, even though we cannot predict the outcome of an action, in retrospect we may say that the action is understandable given the pre-existing situation. Turner also argued that this distinction allows for the notion of "freedom" to be reintroduced into a deterministic universe, and for a clear separation to be made between what a thing is (ontology) and how it is known (epistemology). Note that we use the term "action" to include such "non-conscious" processes as the self-organization made by the formation of stable vortices in turbulent fluid flow, the patterns on butterfly wings, and oscillating chemical reactions in the natural sciences, as well as the "conscious" development of myths, religions and organizations in the social sciences.

From one perspective the unpredictability of free actions has to do with the large number of elements in the system; so many, in fact, that the behavior of the system ceases to be predictable. On the other hand, we now know that having only a few dynamical elements in the system does not insure predictability or knowability. It has been demonstrated that the irregular time series observed in such disciplines as biology, chemical kinetics, economics, logic, physics, physiology, and on and on, are at least in part due to *chaos*. Technically the term chaos may be defined to be a sensitive dependence of the solutions to a set of nonlinear, deterministic, dynamical equations, on initial conditions. Practically chaos means that the solutions to such equations look erratic and may pass all the traditional tests for randomness even though they are deterministic. Therefore, if we think of random time series as complex, then the output of a chaotic generator is complex. However, we know that something as simple as a one-dimensional, quadratic map (logistic equation) can generate a chaotic sequence. Thus, using the traditional definition of complexity, it would appear that chaos implies the generation of complexity from simplicity. This is part of the Poincaré legacy of paradox. Another part of that legacy is the fact that chaos is a generic property of nonlinear dynamical systems, which is to say chaos is ubiquitous; all systems change over time, and because all systems are nonlinear, all systems manifest chaotic behavior to a greater or lesser extent.

A nonlinear system with only a few dynamical variables can generate random patterns and therefore has chaotic solutions. So we encounter the same restrictions on our ability to know and understand a system when there are only a few dynamical elements as when there are a great many dynamical elements, but for very different reasons. Let us refer to the latter random process as noise, the unpredictable influence of the environment on the system of interest. Here the environment is assumed to have an infinite number of elements, all of which we do not know, but they are coupled to the system of interest and perturb it in a random, that is, unknown, way. By way of contrast, chaos is a consequence of the nonlinear, deterministic interactions in an isolated dynamical system, resulting in erratic behavior of at most limited predictability. Chaos is an implicit property of a complex system, whereas noise is a property of the environment in contact with the system of interest. Chaos can therefore be controlled and predicted over short time intervals, whereas noise can neither be predicted nor controlled, except perhaps through the way it interacts with the system.

The above distinction between chaos and noise highlights one of the difficulties in formulating an unambiguous measure of complexity. Since noise can not be predicted or controlled it might be viewed as being complex; thus, systems with many degrees of freedom that manifest randomness might be considered complex. On the other hand, a system with only a few dynamical elements, when it is chaotic, might be considered to be simple. In this way the idea of complexity is ill-posed and a new approach to its definition is required since noise and chaos are often confused with one another.

In the earlier papers on ST it is argued that the increasing complexity of an evolving system can reach a threshold where the system is so complicated that it is impossible to follow the dynamics of the individual elements. At this point new properties often emerge and the new organization undergoes a completely different type of dynamics. The details of the interactions among the individual elements are substantially less important than is the "structure", the geometrical pattern, of the new aggregate.

This is the self-aggregating behavior observed in many biological, physical and social phenomena. Increasing further the number of elements, or alternatively the number of relations, often leads to a complete "disorganization" and the stochastic approach becomes a good description of the system behavior. If randomness (noise) is now considered as something simple, as it is intuitively, one has to seek a measure of complexity that decreases in magnitude in the limit of the system having an infinite number of elements. So a viable measure of complexity must first increase and then decrease with continually increasing numbers of system elements.

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Biographical Sketches

Bruce J. West is a leading thinker in the applications of nonlinear dynamics and in the sciences of complexity. He received his PhD in physics from the University of Rochester in 1970. In 1978 he was a founding member of the La Jolla Institute (LJI), a private, non-profit, research corporation. In 1979 he became the first Associate Director of the Center for Studies of Nonlinear Dynamics (CSND), a division of the LJI. This was the first of the centers devoted to the study of nonlinear phenomena. Dr. West became the Director of the other division of LJI, the Division for Applied Nonlinear Problems (DANP). From 1989 to 1999 he was a Professor of Physics at the University of North Texas where he established the Center for Nonlinear Science on the University of North Texas campus. In June of 1999 he left the university and joined the Army Research Office.

Dr. West's research interests are in the mathematical and physical modeling of complex phenomena, from physical processes whose evolution cannot be described by differential equations of motion, to biomedical phenomena that are so complex that we have little idea of how to describe them mathematically.

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Paolo Grigolini, Ph.D., is a Professor of Physics at the University of North Texas. He received his Ph.D from the University of Pisa, Italy in 1969. Dr. Grigolini, an internationally recognized theorist and member of the Center for Nonlinear Science, is interested in the foundations of quantum mechanics, including wave function collapse and the influence of classical chaos on quantum systems. His research program also examines the foundations of statistical physics. He is also interested in biophysical problems such as DNA sequencing.

Paolo Allegrini took his "Laurea in Fisica" at the University of Pisa, Italy, in 1993 (advisor Dr. Paolo Grigolini), with a theoretical thesis on macroscopic effects of quantum mechanics. He took his Ph.D. at UNT (University of North Texas, Denton, TX, USA) in 1996 with Dr. Bruce J. West, with a thesis on mathematical models and analysis of DNA sequences.

Since he was a graduate student at UNT he has been actively collaborating, as a founding member, with two multidisciplinary scientific groups: IASG (Institute for Advanced Studies, Florence, Italy) and the CNS (Center for Nonlinear Science) at UNT, Denton, Texas.

The main focus of his work is the ideation and implementation of computational tools for multidisciplinary and unified approaches to time-series analysis, from stock market to heart beat, from DNA sequences to natural language. In particular he developed new methods of time-series analysis and a multilingual tool for automatically extracting word senses (semantics) from natural language corpora.

He is currently Visiting Adjunct Professor at UNT and Research Fellow at ILC-CNR (Institute of Computational Linguistics of the National Council of Research), Pisa.