SMITH PREDICTOR AND ITS MODIFICATIONS

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Summary

The Smith predictor is a model-based controller that is effective for processes with long dead time. It has an inner loop with a main controller that can be simply designed without the dead time. The effects of load disturbance and modeling error are corrected through an outer loop. The Smith predictor can also be used for processes with significant non-minimum phase dynamics and for high order systems that exhibit apparent dead time.

A modification using a rapid load detector scheme can be applied to further improve the load response. Three modifications of the Smith predictor for open-loop unstable systems are outlined. They are based on a mismatched process model, a static load estimator, and a rapid load detector, respectively, and the main purpose of the modifications is to ensure stability and zero steady-state error to step load disturbances. Simulation results are given to demonstrate the achievable performance.

1. Introduction

In the process industries, the occurrence of “dead time” or “transportation lag” is quite common. For the majority of simple control loops, the amount of dead time is usually not significant when compared to the time constant. For more complicated control loops like those for quality control, dead time can be very significant and may even be longer than the system time constant.

The reasons for this may include analysis delay and the down-stream location of the sampling point for the quality analyzer. Another class of examples is characterized by a multitude of small lags, such as a long bank of heat exchangers, or a distillation column...
with many trays, giving rise to what is called “apparent” dead time.

It has been found in practice that the widely used PID controller would rapidly lose its effectiveness when the process dead time becomes significant. The consequence is that many important control loops such as those for quality control, are either poorly regulated or are left on manual status, which then necessitates the frequent and close attention of the plant operators. From a theoretical viewpoint, it has been established in many standard control textbooks that without any dead time compensation, the gain cross over frequency, \( \omega_c \), has an upper bound, namely

\[
\omega_c < \frac{1}{L}
\]

where \( L \) is the amount of dead time. The achievable speed of response is hence inversely proportional to the dead time.

If the major disturbance to the process could be measured, the most effective way to cope with the problem of long dead time is by means of “feedforward” control. If it cannot be accurately measured and used in feedforward control, dead time compensation will have to be introduced if tight control is desired. The simplest dead time compensation method for a stable, well-damped process is the Smith predictor. For a process with poor damping, the more sophisticated pole-placement or other more advanced control will be more appropriate.

\[
G_1(s) = \frac{k_p e^{-sL}}{1 + s\tau}
\]  

Figure 1. Structure of the Smith predictor.
where \( k_p, L \) and \( \tau \) are the process static gain, dead time, and time constant, respectively. As shown in Figure 1, the Smith predictor is a model-based controller that has two loops. In the inner loop, it uses the process model without the dead time to predict the output \( y_a \) which is fed back to the main controller \( C_1(s) \) to generate the appropriate control signal, \( u \), so that the process output will track the setpoint or reference signal, \( r \).

As this loop does not contain the dead time, the controller gain can be selected to be high to achieve fast and well-damped setpoint responses. The effects of any unmeasurable load disturbance, \( d \), and small modeling errors are then corrected by feeding back the predictor error, \( e_p \), through the outer loop as shown in Figure 1.

### 2. Controller design

The design of the main controller \( C_1(s) \) in the Smith predictor as shown in Figure 1 assumes that the process model parameters \( k_p, L \), and \( \tau \), are known. For the first order process model, \( C_1(s) \) can be simply chosen to be a proportional-integral (PI) controller of the form:

\[
C_1(s) = k_c (1 + \frac{1}{sT_{i1}})
\]  

(2)

Assuming that the desired response without the dead time part should have a time constant \( T_m \), we have

\[
\frac{y(s)}{e^{-sL}} = \frac{1}{1 + sT_m} = \frac{C_1(s)k_p/(1+s\tau)}{1 + C_1(s)k_p/(1+s\tau)}
\]  

(3)

Solving equations (2) and (3), the following controller design is obtained:

\[
T_{i1} = \tau
\]  

(4)

\[
k_{c1} = \frac{\tau}{k_pT_m}
\]  

(5)

The desired \( T_m \) is usually specified as a ratio of \( T_m \) to the process time constant \( \tau \). A suitable range of this ratio taking into account of possible controller saturation and noise sensitivity is 0.2 to 1.

While many real-life processes could be adequately modeled by the first-order plus dead time model of equation (1), it is widely known that some high-order processes could be better approximated by the following second-order plus dead time model:
\[ G_2(s) = \frac{k_p e^{-sL}}{(1 + s\tau_2)^2} \]  \hspace{1cm} (6)

The Smith predictor structure will be the same as Figure 1 except that the first order model will be replaced by the second order model. Assuming the desired response, without the dead time part, should have a natural frequency \( w_o \) and a damping factor \( \xi \), we have

\[ \frac{w_o^2}{s^2 + 2\xi w_o s + w_o^2} = \frac{C_1(s)k_p/(1+s\tau_2)^2}{1+C_1(s)k_p/(1+s\tau_2)^2} \]  \hspace{1cm} (7)

If \( C_1(s) \) is chosen to be a PI controller, a simple solution is

\[ T_{i1} = \tau_2 \]  \hspace{1cm} (8)
\[ k_{c1} = 1/(4\xi^2k_p) \]  \hspace{1cm} (9)

Another practical consideration is the robustness of the controller when there is modeling error, especially a mismatch in the dead time. The controller gain in (5) and (9) may have to be reduced to accommodate a significant mismatch. This can be guided by a simple analytical study. For instance, if \( G_1(s) = G(s) e^{-sL} \) and there is a modeling error of \( \Delta L \) in the dead time, the Smith predictor can be represented by the equivalent structure of Figure 2.

![Figure 2. Structure of the Smith predictor with mismatch in dead time](image)

It is straightforward to obtain:

\[ H(s) = 1 + e^{-s(L + \Delta L)} - e^{-sL} \]  \hspace{1cm} (10)

The controller gain should then be suitably adjusted to ensure that the closed-loop...
transfer function without mismatch has sufficient attenuation, say greater than 6 dB, at the frequencies where $H(j\omega)$ has resonance peaks.

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Biographical Sketch

**Professor HANG CHANG CHIEH** (C.C. Hang) graduated with a First Class Honours Degree in Electrical Engineering from the University of Singapore in 1970. He received the Ph.D. degree in Control Engineering from the University of Warwick, England, in 1973. From 1974 to 1977, he worked as a Computer and Systems Technologist in the Shell Eastern Petroleum Company (Singapore) and the Shell International Petroleum Company (The Netherlands). Since 1977, he has been with the National University of Singapore, serving in various positions including being the Vice-Dean of the Faculty of Engineering and Head of the Department of Electrical Engineering. From 1994 to 2000, he served as the Deputy Vice-Chancellor in charge of research. His major area of research is adaptive control, in which he
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From 1992 to 1999, he served as Principal Editor (Adaptive Control) of the Automatica Journal. He was
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