CONTROLLER DESIGN

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Keywords: State Feedback Design, SISO Systems, State Feedback Control, stability, controllability, pole placement, Ackermann's formula, dead beat control

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Summary

Based on the state variable description of dynamic systems and its analysis, this chapter presents approaches to the design of closed-loop control. A linear weighted sum of state variables is fed back to the control input of the plant, the resulting control law is called *state feedback*.

This allows the control-system designer to place all of the *n* eigenvalues of the closed-loop system in desirable locations, mostly referred to as *pole placement* or *pole assignment*. The stability can thereby be assured and the transient behavior be adjusted. A frequently used design procedure is described by *Ackermann's formula*.

A special feature of *discrete-time state feedback control* occurs, when the *n* eigenvalues are placed at the origin of the complex plane: the initial state of the system decays to zero after a maximum of *n* steps, which is called *dead-beat* behavior.

1. Objectives and Structure of State Feedback Control

Starting point is the state space description of a dynamic system with one scalar *control input* $u(t)$, one scalar *control output* $y(t)$, and the $(n,1)$-state vector (or *n*-vector of state) $\mathbf{x}(t)$,

State differential equation: $\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{b}u(t)$, (1)

Output equation: $y(t) = \mathbf{c}^{\mathrm{T}}\mathbf{x}(t)$. (2)

Beside the control input, the system is influenced from outside by the vector of initial values, the *initial state*

$$\mathbf{x}(t_0) = \mathbf{x}_0 . \tag{3}$$

In most cases, the initial state will not be known. ***A first objective is therefore to eliminate the effect of this initial state disturbance.*** For this purpose, a feedback is required in order to continuously gather information on the system's state and to generate an appropriate control input $u(t)$. As the system is described in state variable form, it is a good idea to feed back the states to the control input,

$$u = -k(\mathbf{x}(t), t) . \tag{4}$$

At first, $k(.)$ is some function of the state variables $x_1(t),...,x_n(t)$ and of the time t (The conceivable consideration of integrals or time derivatives of state variables within the control law is in fact not required, because from the state and from the control input, the further trajectory $\mathbf{x}(t)$ is uniquely given; derivatives and integrals utilize history values which do not deliver additional information, at least in state feedback control as considered here).

A special case of (4) is *linear time invariant state feedback* of the form

$$u(t) = -k_1 x_1(t) - ... - k_n x_n(t) = -\begin{bmatrix} k_1 & \cdots & k_n \end{bmatrix} \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix} = -\mathbf{k}^{\mathrm{T}} \mathbf{x}(t) . \tag{5}$$

Here, the controller weights the sum of the state variables in a linear manner.

Beside the objective of eliminating initial value disturbances, ***a second objective is to choose the control input*** *u* ***such that the control output*** *y* ***tracks a given reference*** $r(t)$. Therefore, the control input can no longer only be calculated from the states but also from the reference r. Setting up this influence in linear form, the control law (5) is to be extended by an appropriate summand,

$$\begin{aligned} u(t) &= -k_1 x_1(t) - ... - k_n x_n(t) + g\, r(t) \\ &= -\begin{bmatrix} k_1 & \cdots & k_n \end{bmatrix} \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix} = -\mathbf{k}^{\mathrm{T}} \mathbf{x}(t) + g\, r(t) . \end{aligned} \tag{6}$$

It will turn out soon, that with this simple approach a precise tracking of the reference r by the output y will only be achieved for $t \to \infty$ and only if r tends to a constant value as $t \to \infty$.

The extended *control law* (6) will be denoted as *state feedback law* or *state feedback* for the rest of this text. Figure 1 implies the structure of the control plant with state feedback. The block $\mathbf{k}^{\mathrm{T}}$ is also referred to as *state feedback controller*, the block g is a *pre-compensator*.

By substituting the control law (6) into the state differential Eq. (1) of the plant, the *state differential equation of the closed loop system* is obtained:

$$\dot{\mathbf{x}}(t) = (\mathbf{A} - \mathbf{b}\mathbf{k}^{\mathrm{T}})\mathbf{x}(t) + \mathbf{b}gr(t)\,. \qquad (7)$$

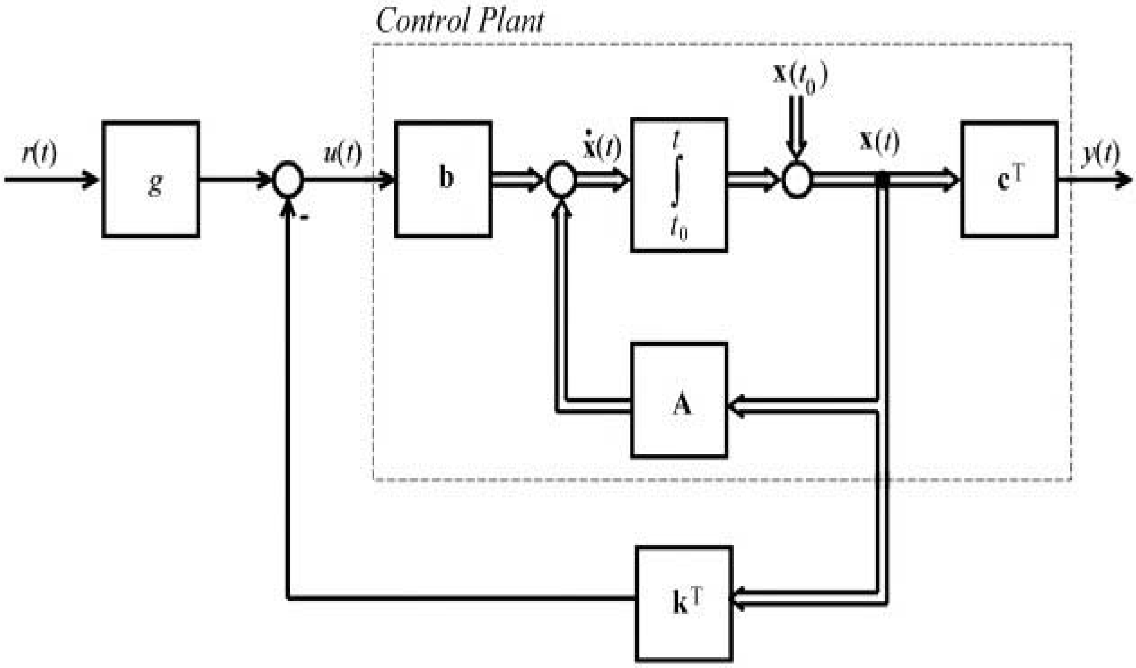

Figure 1: Structure of the control plant with state feedback

It is of the same type as the original state differential equation with the system matrix $(\mathbf{A} - \mathbf{b}\mathbf{k}^{\mathrm{T}})$ instead of **A** and with the vector **b***g* instead of **b**. The trajectory **x**(*t*) of the closed-loop system is therefore given by Eq. (69) in *Description and Analysis of Dynamic Systems in State Space*, if the initial state and the reference signal *r*(*t*) are known,

$$\mathbf{x}(t) \;=\; \int_{t_0}^{t} e^{(\mathbf{A}-\mathbf{b}\mathbf{k}^{\mathrm{T}})(t-\tau)}\mathbf{b}gr(\tau)d\tau \quad + \quad e^{(\mathbf{A}-\mathbf{b}\mathbf{k}^{\mathrm{T}})(t-t_0)}\mathbf{x}_0\,. \qquad (8)$$

We can now formulate the objective of eliminating the influence of initial value disturbances more precisely: The trajectory starting from an arbitrary initial state $\mathbf{x}_0$ is to converge to zero as $t \to \infty$, while *r*(*t*)=0.

Calling in mind the results on stability, we observe that this will happen if the system is *stable*. And the closed loop system (7) will obviously be stable if all of the eigenvalues of $(\mathbf{A} - \mathbf{b}\mathbf{k}^{\mathrm{T}})$ are located in the left half of the complex plane. The specific location of these *closed-loop eigenvalues* (or so-called *control eigenvalues*) determines speed and other characteristics of the transient behavior.

For the practical application it is convenient, if all of the *n* closed-loop eigenvalues can be specified arbitrarily by the control-system designer. This allows to not only stabilize the system but to influence in detail the transient behavior of the closed-loop system. The design objectives can now be summarized as follows:

Consider a *control plant* (1), (2) with the *linear time-invariant state feedback* (6),

$$u(t) = -\mathbf{k}^{\mathrm{T}}\mathbf{x}(t) + g\,r(t)\,. \tag{9}$$

Find a *state feedback controller* $\mathbf{k}^{\mathrm{T}} = \begin{bmatrix} k_1 & \cdots & k_n \end{bmatrix}$ and a *pre-compensator g* such that the closed-loop system

$$\dot{\mathbf{x}}(t) = (\mathbf{A} - \mathbf{b}\mathbf{k}^{\mathrm{T}})\mathbf{x}(t) + \mathbf{b}gr(t)\,, \tag{10}$$

$$y(t) = \mathbf{c}^{\mathrm{T}}\mathbf{x}(t)\,, \tag{11}$$

1. possesses *desired eigenvalues*, specified by the control-system designer, in order to ensure *stability* and to form the dynamic behavior,
2. *avoids steady state error*, i.e. ensures that the control output $y(t)$ tends towards the reference signal $r(t)$ while $t \to \infty$ and with $r(t) = const$.

By Laplace transform of Eqs. (10), (11) and by elimination of the state vector **x**, the input-output behavior of the closed-loop system is obtained by its transfer function $G_{\mathrm{r}}(s)$,

$$Y(s) = \underbrace{\mathbf{c}^{\mathrm{T}}(s\mathbf{I} - \mathbf{A} + \mathbf{b}\mathbf{k}^{\mathrm{T}})^{-1}\mathbf{b}g}_{G_{\mathrm{r}}(s)}\,R(s)\,. \tag{12}$$

This chapter focuses on the design of state feedback (9) in the context of closed loop-systems (10), (11). Relevant assumptions are:

- All state variables are accessible for measurement and feedback,
- Initial state disturbances are the only disturbances occurring, in particular, there are no permanent disturbances,
- The reference signal is constant for most of the time or, at least, changes slowly.

Of course, these assumptions are not always fulfilled in practice, but the resulting problems can be overcome by additional measures like *state estimators* (See *Observer Design*) and *extended control structures* (see *Control of Linear Multivariable Systems* and *Extended Control Structures*).

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MATLAB applications: The following two web-addresses provide introductory examples on how to use the software package MATLAB for control system design purposes: http://tech.buffalostate.edu/ctm/ and http://www.ee.usyd.edu.au/tutorials_online/matlab/index.html

Biographical Sketch

Boris Lohmann received the Dipl.-Ing. and Dr.-Ing. degrees in electrical engineering from the Technical University of Karlsruhe, Germany, in 1987 and 1991 respectively. From 1987 to 1991 he was with the Fraunhofer Institut (IITB) and with the Institute of Control Systems, Karlsruhe, working in the fields of autonomous vehicles control and multi-variable state space design.

From 1991 to 1997 he was with AEG Electrocom Automation Systems in the development department for postal sorting machines, at last as the head of mechanical development. In 1994 he received the 'Habilitation' degree in the field of system dynamics and control from the Universität der Bundeswehr, Hamburg, for his results on model order reduction of nonlinear dynamic systems.

Since 1997 he has been full professor at the University of Bremen, Germany, and head of the Institute of Automation Systems. His fields of research include nonlinear multivariable control theory; system modeling, simplification, and simulation; and image-based control systems, with industrial applications in the fields of autonomous vehicle navigation, active noise reduction, error detection and fault diagnosis.